Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/1.1.2.3-a+b-x^2-^p-c+d-x^2-^q

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [158]. This is test number [7].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (158)	0.00 (0)
Mathematica	100.00 (158)	0.00 (0)
Fricas	89.24 (141)	10.76 (17)
Maple	83.54 (132)	16.46 (26)
Giac	68.35 (108)	31.65 (50)
IntegrateAlgebraic	52.53 (83)	47.47 (75)
Maxima	50.00 (79)	50.00 (79)
Mupad	41.14 (65)	58.86 (93)
Sympy	39.24 (62)	% 60.76 (96)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

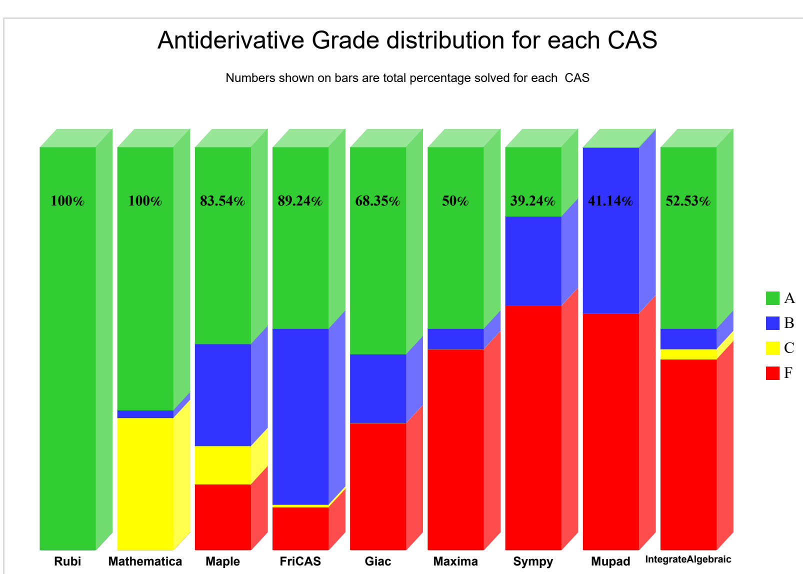
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

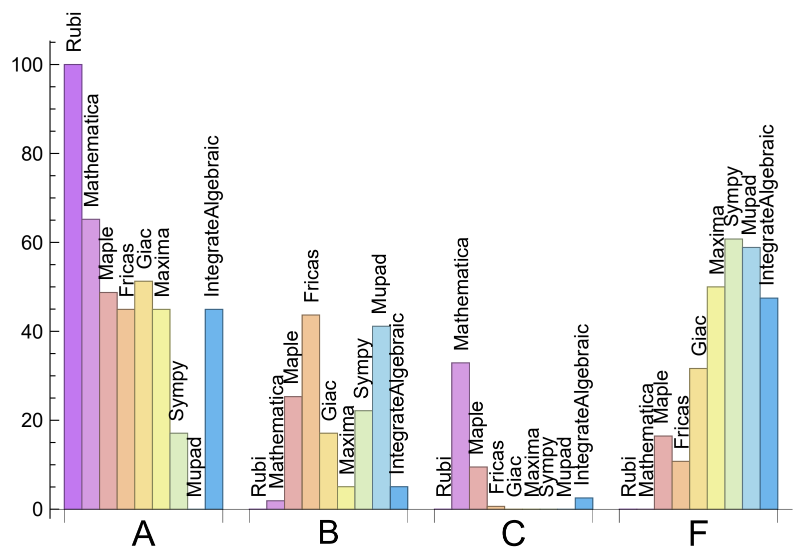
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	65.19	1.90	32.91	0.00
Giac	51.27	17.09	0.00	31.65
Maple	48.73	25.32	9.49	16.46
Fricas	44.94	43.67	0.63	10.76
IntegrateAlgebraic	44.94	5.06	2.53	47.47
Maxima	44.94	5.06	0.00	50.00
Sympy	17.09	22.15	0.00	60.76
Mupad	N/A	41.14	0.00	58.86

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	26	96.15 %	3.85 %	0.00 %
Fricas	17	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	75	96.00 %	4.00 %	0.00 %
Giac	50	96.00 %	0.00 %	4.00 %
Maxima	79	100.00 %	0.00 %	0.00 %
Sympy	96	79.17 %	20.83 %	0.00 %
Mupad	93	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

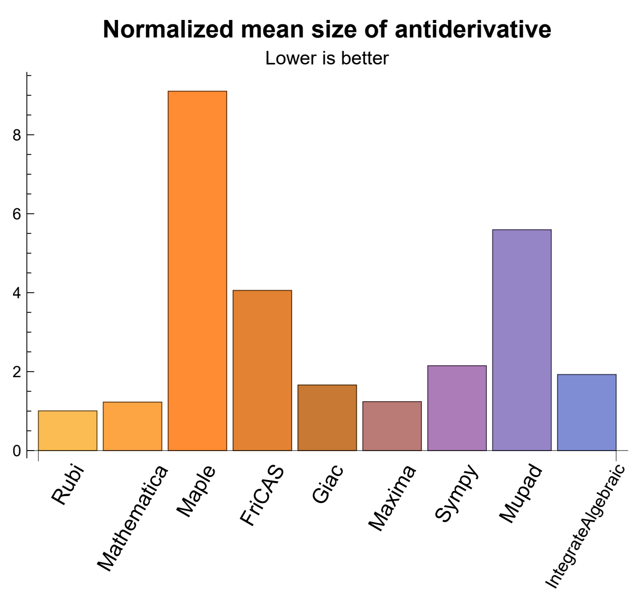
1.3 Performance

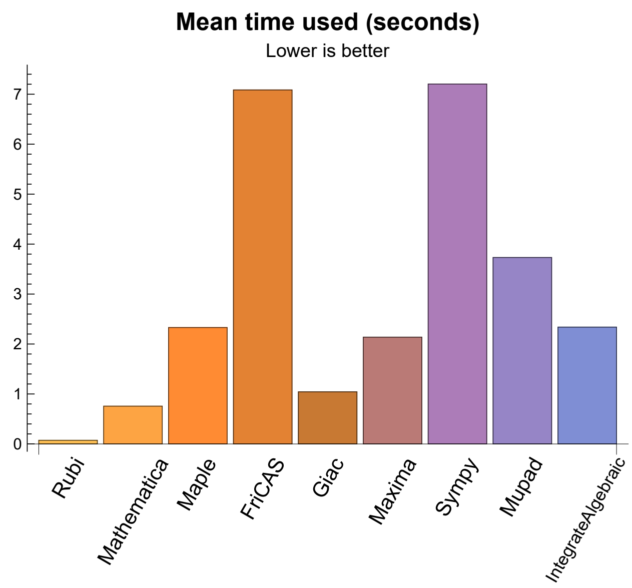
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	119.44	1.00	109.00	1.00
Mathematica	0.76	140.68	1.23	135.50	1.00
Maple	2.33	1498.48	9.10	186.50	1.43
Maxima	2.14	157.72	1.24	116.00	1.16
Fricas	7.09	530.65	4.06	302.00	3.27
Sympy	7.20	225.27	2.15	150.00	2.10
Giac	1.04	225.66	1.66	129.00	1.14
Mupad	3.73	925.98	5.59	88.00	1.12
IntegrateAlgebraic	2.34	286.06	1.93	133.00	1.07

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
7
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {46, 50, 54, 59, 63, 69, 75, 83, 86, 87, 88, 91, 94, 101, 102, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

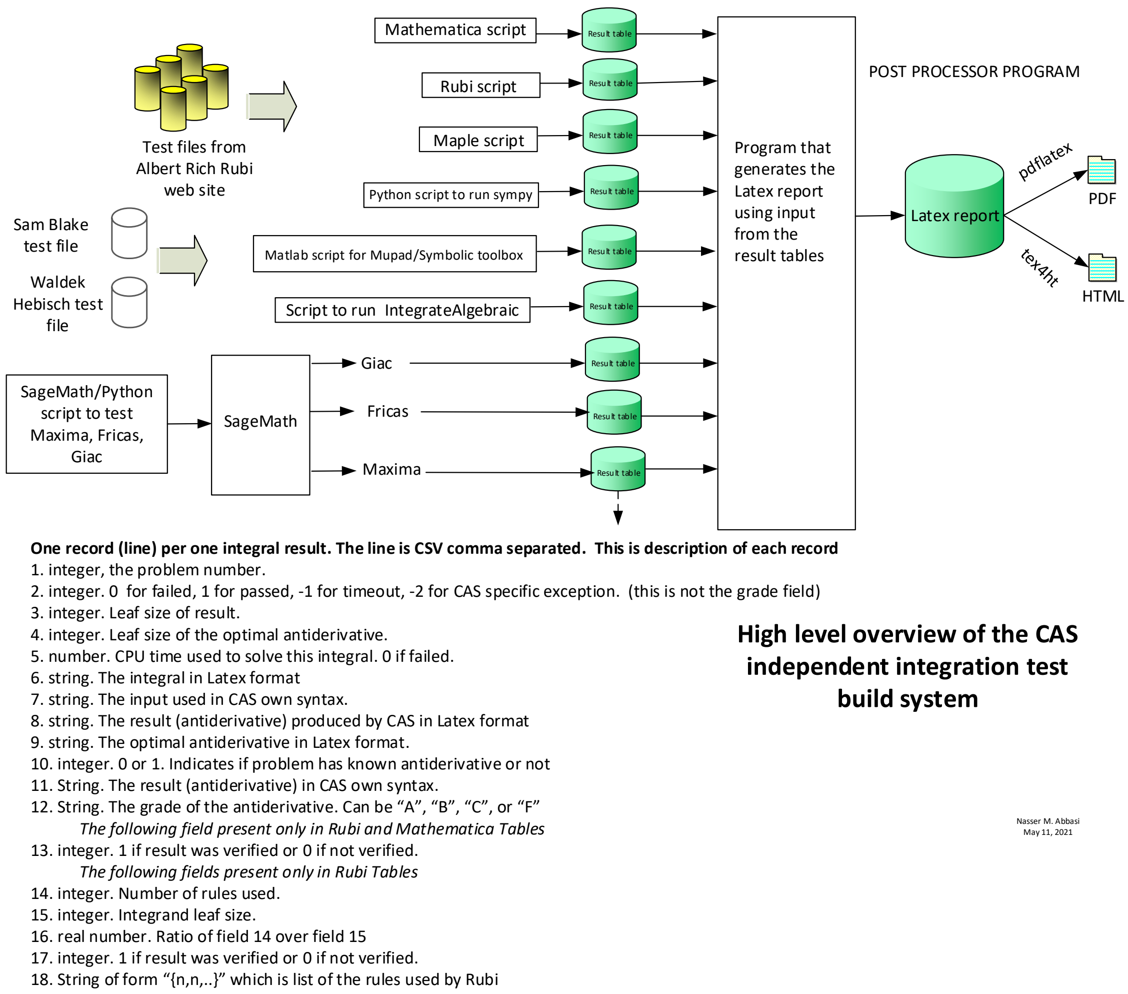
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 76, 77, 78, 79, 80, 81, 82, 84, 85, 89, 90, 92, 93, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 110, 133, 134, 135, 136, 138, 139, 140, 158 }

B grade: { 50, 72, 137 }

C grade: { 46, 54, 63, 75, 83, 86, 87, 88, 91, 94, 101, 102, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 30, 31, 32, 33, 34, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 62, 63, 64, 65, 71, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 97, 98, 99, 100, 105, 106, 108, 110, 133, 134, 135, 137, 138, 139, 140, 158 }

B grade: { 18, 19, 26, 27, 28, 29, 35, 36, 37, 40, 49, 50, 51, 52, 57, 58, 59, 60, 61, 66, 67, 68, 69, 70, 72, 73, 78, 79, 80, 86, 87, 88, 94, 95, 96, 101, 102, 103, 104, 107 }

C grade: { 116, 118, 119, 126, 127, 128, 129, 130, 132, 136, 141, 142, 149, 150, 157 }

F grade: { 109, 111, 112, 113, 114, 115, 117, 120, 121, 122, 123, 124, 125, 131, 143, 144, 145, 146, 147, 148, 151, 152, 153, 154, 155, 156 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 62, 63, 64, 65, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 98, 99, 100, 108, 110, 133 }

B grade: { 26, 33, 34, 41, 42, 72, 73, 97 }

C grade: { }

F grade: { 49, 50, 51, 52, 57, 58, 59, 60, 61, 66, 67, 68, 69, 70, 71, 78, 79, 80, 86, 87, 88, 94, 95, 96, 101, 102, 103, 104, 105, 106, 107, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 30, 31, 32, 39, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 66, 67, 71, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 97, 98, 99, 100, 105, 106, 110, 133, 134, 135, 139, 158 }

B grade: { 12, 13, 18, 19, 26, 27, 28, 29, 33, 34, 35, 36, 37, 38, 40, 41, 42, 50, 51, 52, 59, 60, 61, 68, 69, 70, 72, 73, 78, 79, 80, 86, 87, 88, 94, 95, 96, 101, 102, 103, 104, 107, 108, 116, 117, 118, 119, 130, 132, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157 }

C grade: { 150 }

F grade: { 109, 111, 112, 113, 114, 115, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 14, 15, 16, 39, 43, 44, 47, 48, 56, 65, 74, 75, 76, 77, 84, 85, 136, 137, 138 }

B grade: { 5, 6, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 35, 36, 37, 38, 45, 46, 53, 54, 55, 62, 63, 64, 92, 93, 99, 100, 108 }

C grade: { }

F grade: { 25, 26, 32, 33, 34, 40, 41, 42, 49, 50, 51, 52, 57, 58, 59, 60, 61, 66, 67, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 62, 63, 64, 65, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 105, 107, 108, 136 }

B grade: { 50, 51, 52, 58, 59, 60, 61, 67, 68, 69, 70, 71, 72, 73, 79, 80, 87, 88, 94, 95, 96, 102, 103, 104, 106, 137, 138 }

C grade: { }

F grade: { 49, 57, 66, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 48, 56, 65, 71, 72, 73, 76, 77, 84, 85, 92, 93, 97, 98, 99, 100, 105, 106, 108, 110, 158 }

C grade: { }

F grade: { 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 74, 75, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 94, 95, 96, 101, 102, 103, 104, 107, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157 }

2.1.9 IntegrateAlgebraic

A grade: { 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 66, 67, 68, 71, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 108, 118, 119, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157 }

B grade: { 59, 60, 61, 69, 72, 78, 88, 107 }

C grade: { 73, 133, 134, 135 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 52, 70, 104, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 136, 137, 138, 139, 140, 158 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	94	97	96	98	107	98	88	0
N.S.	1	1.00	1.00	1.03	1.02	1.04	1.14	1.04	0.94	0.00
time (sec)	N/A	0.064	0.021	0.002	1.355	0.486	0.085	0.567	4.775	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	73	70	73	76	73	65	0
N.S.	1	1.00	1.00	1.04	1.00	1.04	1.09	1.04	0.93	0.00
time (sec)	N/A	0.042	0.015	0.002	1.372	0.803	0.082	0.565	4.753	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	49	48	50	53	50	48	0
N.S.	1	1.00	1.00	0.98	0.96	1.00	1.06	1.00	0.96	0.00
time (sec)	N/A	0.027	0.011	0.001	1.352	0.403	0.074	0.570	0.047	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	25	24	26	26	26	25	0
N.S.	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89	0.00
time (sec)	N/A	0.013	0.005	0.001	1.349	0.767	0.065	0.561	0.036	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	45	34	99	82	34	31	0
N.S.	1	1.00	1.00	1.12	0.85	2.48	2.05	0.85	0.78	0.00
time (sec)	N/A	0.020	0.023	0.007	2.998	0.799	0.280	0.571	0.060	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	68	57	182	112	57	51	0
N.S.	1	1.00	1.00	1.08	0.90	2.89	1.78	0.90	0.81	0.00
time (sec)	N/A	0.020	0.047	0.008	3.115	0.822	0.392	0.583	5.006	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	82	90	92	300	150	78	82	0
N.S.	1	1.00	0.89	0.98	1.00	3.26	1.63	0.85	0.89	0.00
time (sec)	N/A	0.031	0.059	0.010	2.963	0.819	0.541	0.587	5.061	0.001

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	122	125	124	131	136	131	116	0
N.S.	1	1.00	1.00	1.02	1.02	1.07	1.11	1.07	0.95	0.00
time (sec)	N/A	0.074	0.023	0.002	1.395	0.742	0.091	0.564	4.945	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	87	82	91	97	91	75	0
N.S.	1	1.00	1.00	1.06	1.00	1.11	1.18	1.11	0.91	0.00
time (sec)	N/A	0.046	0.016	0.000	1.327	0.657	0.081	0.572	0.047	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	49	48	50	53	50	48	0
N.S.	1	1.00	1.00	0.98	0.96	1.00	1.06	1.00	0.96	0.00
time (sec)	N/A	0.028	0.007	0.000	1.320	0.476	0.073	0.569	0.046	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	59	95	68	179	172	72	90	0
N.S.	1	1.00	0.94	1.51	1.08	2.84	2.73	1.14	1.43	0.00
time (sec)	N/A	0.043	0.050	0.003	2.886	0.867	0.420	0.585	0.088	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	89	129	96	302	236	95	124	0
N.S.	1	1.00	1.09	1.57	1.17	3.68	2.88	1.16	1.51	0.00
time (sec)	N/A	0.099	0.060	0.009	2.840	0.870	0.698	0.570	5.021	0.001

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	121	147	138	449	223	126	130	0
N.S.	1	1.00	1.04	1.27	1.19	3.87	1.92	1.09	1.12	0.00
time (sec)	N/A	0.072	0.097	0.009	3.084	0.753	0.984	0.585	5.030	0.001

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	161	177	167	187	189	187	152	0
N.S.	1	1.00	1.05	1.15	1.08	1.21	1.23	1.21	0.99	0.00
time (sec)	N/A	0.103	0.030	0.003	1.378	0.663	0.100	0.567	4.905	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	122	125	124	131	136	131	116	0
N.S.	1	1.00	1.00	1.02	1.02	1.07	1.11	1.07	0.95	0.00
time (sec)	N/A	0.070	0.022	0.002	1.362	0.738	0.091	0.591	4.875	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	73	70	73	76	73	65	0
N.S.	1	1.00	1.00	1.04	1.00	1.04	1.09	1.04	0.93	0.00
time (sec)	N/A	0.044	0.012	0.001	1.294	0.490	0.078	0.565	0.034	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	93	161	122	290	238	130	145	0
N.S.	1	1.00	0.95	1.64	1.24	2.96	2.43	1.33	1.48	0.00
time (sec)	N/A	0.064	0.061	0.004	3.025	0.567	0.573	0.577	4.870	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	107	205	147	444	314	152	181	0
N.S.	1	1.00	1.00	1.92	1.37	4.15	2.93	1.42	1.69	0.00
time (sec)	N/A	0.096	0.059	0.010	2.991	0.819	1.054	0.581	0.100	0.001

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	141	266	187	618	422	180	240	0
N.S.	1	1.00	1.08	2.05	1.44	4.75	3.25	1.38	1.85	0.00
time (sec)	N/A	0.165	0.081	0.011	2.979	0.789	1.813	0.587	4.956	0.001

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	136	246	187	428	326	198	216	0
N.S.	1	1.00	0.96	1.73	1.32	3.01	2.30	1.39	1.52	0.00
time (sec)	N/A	0.093	0.091	0.006	3.033	0.633	0.757	0.579	4.861	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	92	161	122	292	238	129	146	0
N.S.	1	1.00	0.94	1.64	1.24	2.98	2.43	1.32	1.49	0.00
time (sec)	N/A	0.059	0.067	0.004	2.911	0.804	0.591	0.576	0.076	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	59	95	69	181	172	72	90	0
N.S.	1	1.00	0.94	1.51	1.10	2.87	2.73	1.14	1.43	0.00
time (sec)	N/A	0.041	0.051	0.004	2.934	0.913	0.435	0.568	4.902	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	40	45	33	98	82	33	32	0
N.S.	1	1.00	1.03	1.15	0.85	2.51	2.10	0.85	0.82	0.00
time (sec)	N/A	0.015	0.026	0.005	2.991	1.053	0.276	0.571	0.055	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	61	55	54	292	712	54	135	0
N.S.	1	1.00	0.87	0.79	0.77	4.17	10.17	0.77	1.93	0.00
time (sec)	N/A	0.027	0.045	0.008	2.985	0.923	2.798	0.585	0.320	0.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	95	144	133	711	0	122	3637	0
N.S.	1	1.00	0.87	1.32	1.22	6.52	0.00	1.12	33.37	0.00
time (sec)	N/A	0.084	0.171	0.011	3.045	1.036	0.000	0.569	5.688	0.001

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	158	310	277	1585	0	217	6033	0
N.S.	1	1.00	0.99	1.94	1.73	9.91	0.00	1.36	37.71	0.00
time (sec)	N/A	0.192	0.238	0.013	3.075	2.417	0.000	0.578	6.869	0.001

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	192	402	294	810	502	306	386	0
N.S.	1	1.00	1.00	2.09	1.53	4.22	2.61	1.59	2.01	0.00
time (sec)	N/A	0.163	0.098	0.012	3.026	0.680	1.910	0.579	5.024	0.001

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	142	296	213	612	403	220	261	0
N.S.	1	1.00	1.00	2.08	1.50	4.31	2.84	1.55	1.84	0.00
time (sec)	N/A	0.120	0.090	0.012	2.997	0.859	1.475	0.576	5.054	0.001

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	106	205	147	442	314	152	182	0
N.S.	1	1.00	1.00	1.93	1.39	4.17	2.96	1.43	1.72	0.00
time (sec)	N/A	0.093	0.062	0.010	2.990	0.848	1.075	0.568	0.102	0.001

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	88	129	95	297	236	94	124	0
N.S.	1	1.00	1.07	1.57	1.16	3.62	2.88	1.15	1.51	0.00
time (sec)	N/A	0.104	0.063	0.009	2.839	0.954	0.723	0.584	5.062	0.001

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	68	57	181	112	57	51	0
N.S.	1	1.00	1.00	1.08	0.90	2.87	1.78	0.90	0.81	0.00
time (sec)	N/A	0.021	0.047	0.009	3.114	0.582	0.399	0.572	5.042	0.001

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	109	144	132	699	0	121	3649	0
N.S.	1	1.00	1.01	1.33	1.22	6.47	0.00	1.12	33.79	0.00
time (sec)	N/A	0.081	0.146	0.010	2.915	1.119	0.000	0.576	5.766	0.001

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	136	238	294	1681	0	232	6183	0
N.S.	1	1.00	0.81	1.43	1.76	10.07	0.00	1.39	37.02	0.00
time (sec)	N/A	0.201	0.319	0.015	3.140	2.414	0.000	0.583	6.875	0.001

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	197	403	529	3239	0	332	8649	0
N.S.	1	1.00	0.86	1.75	2.30	14.08	0.00	1.44	37.60	0.00
time (sec)	N/A	0.309	0.419	0.017	3.409	8.316	0.000	0.582	7.793	0.001

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	196	484	334	1044	615	340	409	0
N.S.	1	1.00	1.00	2.47	1.70	5.33	3.14	1.73	2.09	0.00
time (sec)	N/A	0.227	0.126	0.015	3.037	0.919	4.406	0.580	5.023	0.001

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	160	367	253	817	515	254	318	0
N.S.	1	1.00	1.00	2.29	1.58	5.11	3.22	1.59	1.99	0.00
time (sec)	N/A	0.197	0.097	0.013	3.073	0.867	2.811	0.583	0.135	0.001

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	139	266	185	606	422	178	240	0
N.S.	1	1.00	1.07	2.05	1.42	4.66	3.25	1.37	1.85	0.00
time (sec)	N/A	0.166	0.084	0.011	2.938	0.642	1.890	0.571	5.052	0.001

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	124	147	138	449	223	126	130	0
N.S.	1	1.00	1.07	1.27	1.19	3.87	1.92	1.09	1.12	0.00
time (sec)	N/A	0.077	0.096	0.010	3.010	0.811	1.049	0.569	5.023	0.001

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	84	89	92	301	150	78	81	0
N.S.	1	1.00	0.91	0.97	1.00	3.27	1.63	0.85	0.88	0.00
time (sec)	N/A	0.033	0.064	0.009	3.002	0.793	0.585	0.576	5.016	0.001

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	158	309	278	1587	0	218	6033	0
N.S.	1	1.00	0.98	1.92	1.73	9.86	0.00	1.35	37.47	0.00
time (sec)	N/A	0.197	0.281	0.012	3.161	2.484	0.000	0.578	6.892	0.001

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	197	403	530	3251	0	333	8635	0
N.S.	1	1.00	0.83	1.71	2.25	13.78	0.00	1.41	36.59	0.00
time (sec)	N/A	0.311	0.418	0.014	3.177	8.542	0.000	0.612	7.855	0.001

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	315	315	233	568	820	5070	0	574	11150	0
N.S.	1	1.00	0.74	1.80	2.60	16.10	0.00	1.82	35.40	0.00
time (sec)	N/A	0.451	0.928	0.018	3.395	32.351	0.000	0.620	8.555	0.001

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	23	33	33	31	20	31	0
N.S.	1	1.00	0.71	0.68	0.97	0.97	0.91	0.59	0.91	0.00
time (sec)	N/A	0.010	0.008	0.007	1.323	1.040	0.132	0.578	4.995	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	41	33	48	67	46	54	47	0
N.S.	1	1.00	0.87	0.70	1.02	1.43	0.98	1.15	1.00	0.00
time (sec)	N/A	0.016	0.013	0.007	2.994	1.019	0.172	0.580	0.042	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	181	310	281	398	484	201	-1	202
N.S.	1	1.00	0.78	1.34	1.22	1.72	2.10	0.87	-0.00	0.87
time (sec)	N/A	0.179	5.114	0.016	1.395	0.848	20.381	0.642	0.000	0.295

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	149	149	160	190	168	264	291	129	-1	132
N.S.	1	1.00	1.07	1.28	1.13	1.77	1.95	0.87	-0.01	0.89
time (sec)	N/A	0.088	2.673	0.010	1.347	0.976	11.393	0.607	0.000	0.189

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	85	96	81	158	144	70	-1	77
N.S.	1	1.00	0.98	1.10	0.93	1.82	1.66	0.80	-0.01	0.89
time (sec)	N/A	0.028	0.157	0.005	1.350	1.156	5.681	0.600	0.000	0.099

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	49	36	28	94	41	37	35	48
N.S.	1	1.00	1.07	0.78	0.61	2.04	0.89	0.80	0.76	1.04
time (sec)	N/A	0.010	0.021	0.000	1.364	0.753	1.855	0.576	4.712	0.001

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	84	932	0	596	0	0	-1	138
N.S.	1	1.00	1.02	11.37	0.00	7.27	0.00	0.00	-0.01	1.68
time (sec)	N/A	0.054	0.044	0.045	0.000	1.118	0.000	0.000	0.000	0.225

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	82	82	165	2521	0	369	0	217	-1	145
N.S.	1	1.00	2.01	30.74	0.00	4.50	0.00	2.65	-0.01	1.77
time (sec)	N/A	0.034	0.234	0.022	0.000	1.223	0.000	1.641	0.000	0.393

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	176	5101	0	698	0	487	-1	154
N.S.	1	1.00	1.18	34.23	0.00	4.68	0.00	3.27	-0.01	1.03
time (sec)	N/A	0.094	0.566	0.025	0.000	1.199	0.000	3.746	0.000	1.167

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	227	7922	0	1220	0	958	-1	0
N.S.	1	1.00	1.09	38.09	0.00	5.87	0.00	4.61	-0.00	0.00
time (sec)	N/A	0.213	0.990	0.029	0.000	2.589	0.000	2.870	0.000	180.007

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	272	272	220	393	364	502	665	260	-1	259
N.S.	1	1.00	0.81	1.44	1.34	1.85	2.44	0.96	-0.00	0.95
time (sec)	N/A	0.218	5.129	0.016	1.496	1.355	52.753	0.662	0.000	0.406

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	196	196	157	249	227	344	440	175	-1	173
N.S.	1	1.00	0.80	1.27	1.16	1.76	2.24	0.89	-0.01	0.88
time (sec)	N/A	0.116	2.712	0.007	1.424	1.307	29.419	0.660	0.000	0.273

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	109	131	116	210	253	103	-1	102
N.S.	1	1.00	0.92	1.11	0.98	1.78	2.14	0.87	-0.01	0.86
time (sec)	N/A	0.040	0.204	0.004	1.372	0.933	14.712	0.613	0.000	0.165

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	51	43	124	70	49	37	60
N.S.	1	1.00	1.00	0.78	0.66	1.91	1.08	0.75	0.57	0.92
time (sec)	N/A	0.016	0.090	0.001	1.336	0.703	2.915	0.612	4.709	0.001

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	110	1845	0	721	0	0	-1	148
N.S.	1	1.00	0.97	16.33	0.00	6.38	0.00	0.00	-0.01	1.31
time (sec)	N/A	0.108	0.200	0.016	0.000	1.445	0.000	0.000	0.000	0.325

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	142	4621	0	907	0	317	-1	155
N.S.	1	1.00	1.08	35.27	0.00	6.92	0.00	2.42	-0.01	1.18
time (sec)	N/A	0.090	0.128	0.021	0.000	1.364	0.000	0.682	0.000	0.624

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	113	113	163	9059	0	526	0	451	-1	1323
N.S.	1	1.00	1.44	80.17	0.00	4.65	0.00	3.99	-0.01	11.71
time (sec)	N/A	0.057	0.693	0.025	0.000	1.205	0.000	3.723	0.000	3.188

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	247	13766	0	972	0	919	-1	1786
N.S.	1	1.00	1.24	69.18	0.00	4.88	0.00	4.62	-0.01	8.97
time (sec)	N/A	0.115	0.817	0.032	0.000	1.893	0.000	2.893	0.000	43.503

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	300	300	362	18791	0	1604	0	1557	-1	2592
N.S.	1	1.00	1.21	62.64	0.00	5.35	0.00	5.19	-0.00	8.64
time (sec)	N/A	0.365	1.385	0.046	0.000	5.358	0.000	9.597	0.000	43.482

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	349	349	270	476	447	608	796	321	-1	318
N.S.	1	1.00	0.77	1.36	1.28	1.74	2.28	0.92	-0.00	0.91
time (sec)	N/A	0.247	5.179	0.019	1.472	2.030	102.666	0.680	0.000	0.541

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	241	241	158	308	286	420	537	221	-1	214
N.S.	1	1.00	0.66	1.28	1.19	1.74	2.23	0.92	-0.00	0.89
time (sec)	N/A	0.148	2.802	0.010	1.388	1.794	58.630	0.656	0.000	0.384

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	130	166	151	260	316	135	-1	126
N.S.	1	1.00	0.87	1.11	1.01	1.74	2.12	0.91	-0.01	0.85
time (sec)	N/A	0.052	0.228	0.006	1.401	1.931	29.528	0.632	0.000	0.231

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	76	66	58	146	97	63	37	71
N.S.	1	1.00	0.90	0.79	0.69	1.74	1.15	0.75	0.44	0.85
time (sec)	N/A	0.023	0.112	0.003	1.328	1.714	4.282	0.608	4.693	0.001

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	140	3053	0	935	0	0	-1	199
N.S.	1	1.00	0.89	19.45	0.00	5.96	0.00	0.00	-0.01	1.27
time (sec)	N/A	0.199	0.122	0.019	0.000	3.399	0.000	0.000	0.000	0.455

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	144	7345	0	1236	0	405	-1	210
N.S.	1	1.00	0.82	41.97	0.00	7.06	0.00	2.31	-0.01	1.20
time (sec)	N/A	0.226	0.164	0.023	0.000	2.680	0.000	0.689	0.000	0.807

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	194	184	14133	0	1517	0	659	-1	311
N.S.	1	1.00	0.95	72.85	0.00	7.82	0.00	3.40	-0.01	1.60
time (sec)	N/A	0.194	0.203	0.030	0.000	3.101	0.000	0.733	0.000	1.908

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	144	144	201	21220	0	706	0	846	-1	1180
N.S.	1	1.00	1.40	147.36	0.00	4.90	0.00	5.88	-0.01	8.19
time (sec)	N/A	0.073	0.807	0.040	0.000	1.513	0.000	2.858	0.000	20.707

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	249	306	28625	0	1258	0	1448	-1	0
N.S.	1	1.00	1.23	114.96	0.00	5.05	0.00	5.82	-0.00	0.00
time (sec)	N/A	0.137	1.094	0.053	0.000	2.682	0.000	9.315	0.000	180.008

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	33	0	42	0	95	83	54
N.S.	1	1.00	1.00	1.10	0.00	1.40	0.00	3.17	2.77	1.80
time (sec)	N/A	0.014	0.029	0.023	0.000	0.850	0.000	0.608	0.394	0.099

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	64	84	59	67	0	70	59	57
N.S.	1	1.00	2.37	3.11	2.19	2.48	0.00	2.59	2.19	2.11
time (sec)	N/A	0.014	0.026	0.012	2.978	0.639	0.000	0.620	0.167	0.114

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	187	110	74	0	118	85	55
N.S.	1	1.00	1.00	7.48	4.40	2.96	0.00	4.72	3.40	2.20
time (sec)	N/A	0.013	0.011	0.043	3.074	1.129	0.000	0.611	5.347	0.068

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	140	228	199	300	400	150	-1	148
N.S.	1	1.00	0.83	1.35	1.18	1.78	2.37	0.89	-0.01	0.88
time (sec)	N/A	0.145	5.095	0.014	1.360	1.180	13.273	0.639	0.000	0.174

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	108	108	160	131	109	192	238	90	-1	95
N.S.	1	1.00	1.48	1.21	1.01	1.78	2.20	0.83	-0.01	0.88
time (sec)	N/A	0.056	2.489	0.008	1.469	1.311	6.943	0.619	0.000	0.106

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	57	62	47	113	126	49	86	59
N.S.	1	1.00	0.98	1.07	0.81	1.95	2.17	0.84	1.48	1.02
time (sec)	N/A	0.017	0.021	0.006	1.329	0.674	2.780	0.599	5.515	0.058

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	21	13	59	17	23	20	28
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	0.92	0.80	1.12
time (sec)	N/A	0.006	0.004	0.002	1.284	1.248	0.997	0.596	0.121	0.001

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	300	0	241	0	70	-1	112
N.S.	1	1.00	1.00	6.12	0.00	4.92	0.00	1.43	-0.02	2.29
time (sec)	N/A	0.022	0.017	0.015	0.000	1.452	0.000	0.603	0.000	0.128

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	126	809	0	463	0	242	-1	132
N.S.	1	1.00	1.25	8.01	0.00	4.58	0.00	2.40	-0.01	1.31
time (sec)	N/A	0.049	0.302	0.019	0.000	1.625	0.000	0.627	0.000	0.332

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	192	1815	0	864	0	538	-1	180
N.S.	1	1.00	1.18	11.13	0.00	5.30	0.00	3.30	-0.01	1.10
time (sec)	N/A	0.119	0.668	0.021	0.000	1.969	0.000	3.505	0.000	0.912

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	257	257	172	340	311	584	0	235	-1	229
N.S.	1	1.00	0.67	1.32	1.21	2.27	0.00	0.91	-0.00	0.89
time (sec)	N/A	0.259	5.203	0.019	1.382	1.386	0.000	0.670	0.000	0.478

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	122	219	197	416	0	157	-1	156
N.S.	1	1.00	0.72	1.30	1.17	2.46	0.00	0.93	-0.01	0.92
time (sec)	N/A	0.197	5.100	0.008	1.397	0.888	0.000	0.644	0.000	0.281

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	90	105	160	123	108	276	0	92	-1	99
N.S.	1	1.17	1.78	1.37	1.20	3.07	0.00	1.02	-0.01	1.10
time (sec)	N/A	0.062	2.529	0.008	1.377	0.942	0.000	0.632	0.000	0.166

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	70	54	46	167	60	50	53	58
N.S.	1	1.00	1.30	1.00	0.85	3.09	1.11	0.93	0.98	1.07
time (sec)	N/A	0.017	0.061	0.005	1.302	0.889	5.168	0.643	5.117	0.089

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	23	17	14	14	16
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	0.88	1.00
time (sec)	N/A	0.002	0.004	0.003	1.324	0.957	0.614	0.596	0.040	0.001

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	79	79	309	618	0	441	0	107	-1	143
N.S.	1	1.00	3.91	7.82	0.00	5.58	0.00	1.35	-0.01	1.81
time (sec)	N/A	0.039	0.720	0.017	0.000	1.221	0.000	0.619	0.000	0.241

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	143	143	758	1439	0	864	0	318	-1	173
N.S.	1	1.00	5.30	10.06	0.00	6.04	0.00	2.22	-0.01	1.21
time (sec)	N/A	0.109	2.673	0.022	0.000	2.223	0.000	1.902	0.000	0.698

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	225	225	1392	2919	0	1482	0	643	-1	6883
N.S.	1	1.00	6.19	12.97	0.00	6.59	0.00	2.86	-0.00	30.59
time (sec)	N/A	0.243	5.017	0.026	0.000	3.782	0.000	2.699	0.000	56.773

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	157	351	392	684	0	237	-1	237
N.S.	1	1.00	0.62	1.38	1.54	2.68	0.00	0.93	-0.00	0.93
time (sec)	N/A	0.245	5.168	0.017	1.442	1.308	0.000	0.679	0.000	0.411

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	125	228	254	486	0	158	-1	162
N.S.	1	1.00	0.73	1.33	1.48	2.83	0.00	0.92	-0.01	0.94
time (sec)	N/A	0.157	5.101	0.009	1.493	1.213	0.000	0.682	0.000	0.273

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	105	105	214	136	147	318	0	103	-1	107
N.S.	1	1.00	2.04	1.30	1.40	3.03	0.00	0.98	-0.01	1.02
time (sec)	N/A	0.050	4.146	0.007	1.389	1.126	0.000	0.632	0.000	0.212

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	37	34	68	54	144	40	33	37
N.S.	1	1.00	0.79	0.72	1.45	1.15	3.06	0.85	0.70	0.79
time (sec)	N/A	0.010	0.016	0.004	1.366	0.954	11.030	0.618	4.785	0.096

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	29	26	31	47	95	27	28	29
N.S.	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	0.72	0.74
time (sec)	N/A	0.006	0.006	0.000	1.332	0.580	0.821	0.609	4.754	0.001

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	122	122	775	1070	0	764	0	320	-1	180
N.S.	1	1.00	6.35	8.77	0.00	6.26	0.00	2.62	-0.01	1.48
time (sec)	N/A	0.103	2.750	0.020	0.000	1.741	0.000	0.639	0.000	0.415

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	170	2371	0	1440	0	620	-1	261
N.S.	1	1.00	0.84	11.74	0.00	7.13	0.00	3.07	-0.00	1.29
time (sec)	N/A	0.228	5.498	0.027	0.000	3.590	0.000	2.070	0.000	1.591

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	313	221	4495	0	2250	0	1010	-1	516
N.S.	1	1.00	0.71	14.36	0.00	7.19	0.00	3.23	-0.00	1.65
time (sec)	N/A	0.400	5.662	0.029	0.000	12.491	0.000	3.760	0.000	3.662

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	163	190	465	229	0	218	326	193
N.S.	1	1.00	0.73	0.85	2.08	1.02	0.00	0.97	1.46	0.86
time (sec)	N/A	0.101	0.103	0.007	1.513	1.615	0.000	0.677	5.152	0.424

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	107	115	249	151	0	138	176	118
N.S.	1	1.00	0.61	0.66	1.43	0.87	0.00	0.79	1.01	0.68
time (sec)	N/A	0.071	0.068	0.008	1.500	0.968	0.000	0.641	4.987	0.230

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	59	57	103	87	566	72	87	60
N.S.	1	1.00	0.65	0.63	1.13	0.96	6.22	0.79	0.96	0.66
time (sec)	N/A	0.029	0.023	0.004	1.453	0.749	27.986	0.618	4.849	0.134

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	29	26	31	47	95	27	28	29
N.S.	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	0.72	0.74
time (sec)	N/A	0.006	0.009	0.003	1.338	0.872	0.826	0.603	4.787	0.056

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	79	79	236	628	0	442	0	107	-1	133
N.S.	1	1.00	2.99	7.95	0.00	5.59	0.00	1.35	-0.01	1.68
time (sec)	N/A	0.046	2.716	0.042	0.000	1.407	0.000	0.617	0.000	0.260

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	100	100	405	823	0	459	0	225	-1	122
N.S.	1	1.00	4.05	8.23	0.00	4.59	0.00	2.25	-0.01	1.22
time (sec)	N/A	0.055	0.779	0.025	0.000	1.350	0.000	0.623	0.000	0.359

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	130	5177	0	698	0	487	-1	154
N.S.	1	1.00	0.87	34.74	0.00	4.68	0.00	3.27	-0.01	1.03
time (sec)	N/A	0.079	5.169	0.033	0.000	1.560	0.000	3.132	0.000	1.253

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	179	13964	0	972	0	919	-1	0
N.S.	1	1.00	0.90	70.17	0.00	4.88	0.00	4.62	-0.01	0.00
time (sec)	N/A	0.113	5.270	0.049	0.000	2.228	0.000	2.812	0.000	180.016

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	19	0	27	0	18	18	20
N.S.	1	1.00	1.00	0.95	0.00	1.35	0.00	0.90	0.90	1.00
time (sec)	N/A	0.005	0.008	0.003	0.000	0.868	0.000	0.604	4.771	0.076

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	28	0	23	0	51	79	33
N.S.	1	1.00	1.00	1.12	0.00	0.92	0.00	2.04	3.16	1.32
time (sec)	N/A	0.008	0.006	0.006	0.000	0.627	0.000	0.604	0.368	0.079

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	306	0	241	0	70	-1	103
N.S.	1	1.00	1.00	6.24	0.00	4.92	0.00	1.43	-0.02	2.10
time (sec)	N/A	0.020	0.015	0.010	0.000	1.197	0.000	0.602	0.000	0.134

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	44	31	25	27	29
N.S.	1	1.00	1.00	0.93	0.87	2.93	2.07	1.67	1.80	1.93
time (sec)	N/A	0.004	0.013	0.006	2.782	1.211	4.681	0.574	0.039	0.048

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	204	204	162	0	0	0	0	0	-1	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.031	0.047	0.310	0.000	0.000	0.000	0.000	0.000	4.661

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	24	24	33	42	0	0	27	0
N.S.	1	1.00	0.55	0.55	0.75	0.95	0.00	0.00	0.61	0.00
time (sec)	N/A	0.017	5.027	0.005	1.890	1.375	0.000	0.000	4.785	21.556

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	252	252	163	0	0	0	0	0	-1	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.073	0.138	0.322	0.000	0.000	0.000	0.000	0.000	5.082

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	202	202	166	0	0	0	0	0	-1	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.029	0.156	0.319	0.000	0.000	0.000	0.000	0.000	4.598

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	204	204	153	0	0	0	0	0	-1	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.042	0.151	0.324	0.000	0.000	0.000	0.000	0.000	4.660

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	204	204	162	0	0	0	0	0	-1	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.029	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.001

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	204	204	156	0	0	0	0	0	-1	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.029	0.156	0.333	0.000	0.000	0.000	0.000	0.000	4.631

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	113	113	118	938	0	1943	0	0	-1	0
N.S.	1	1.00	1.04	8.30	0.00	17.19	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.021	0.042	16.872	0.000	4.454	0.000	0.000	0.000	3.936

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F(-1)	F	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	109	109	124	0	0	1685	0	0	-1	0
N.S.	1	1.00	1.14	0.00	0.00	15.46	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.014	0.039	180.000	0.000	4.160	0.000	0.000	0.000	4.124

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	96	96	143	1033	0	285	0	0	-1	166
N.S.	1	1.00	1.49	10.76	0.00	2.97	0.00	0.00	-0.01	1.73
time (sec)	N/A	0.018	0.154	7.834	0.000	16.353	0.000	0.000	0.000	0.237

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	95	95	143	1553	0	315	0	0	-1	168
N.S.	1	1.00	1.51	16.35	0.00	3.32	0.00	0.00	-0.01	1.77
time (sec)	N/A	0.017	0.109	7.621	0.000	15.632	0.000	0.000	0.000	0.230

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	151	151	169	0	0	0	0	0	-1	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.027	0.174	0.322	0.000	0.000	0.000	0.000	0.000	5.419

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	153	153	167	0	0	0	0	0	-1	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.026	0.178	0.321	0.000	0.000	0.000	0.000	0.000	5.401

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	151	151	168	0	0	0	0	0	-1	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.028	0.146	0.314	0.000	0.000	0.000	0.000	0.000	5.434

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	153	153	172	0	0	0	0	0	-1	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.029	0.164	0.325	0.000	0.000	0.000	0.000	0.000	5.445

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	151	151	148	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.034	0.150	0.317	0.000	0.000	0.000	0.000	0.000	5.080

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	147	147	148	0	0	0	0	0	-1	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.024	0.168	0.322	0.000	0.000	0.000	0.000	0.000	5.074

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	123	123	136	549	0	0	0	0	-1	0
N.S.	1	1.00	1.11	4.46	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.020	0.120	58.096	0.000	0.000	0.000	0.000	0.000	4.564

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	123	123	136	1061	0	0	0	0	-1	0
N.S.	1	1.00	1.11	8.63	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	0.125	59.072	0.000	0.000	0.000	0.000	0.000	4.619

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	119	119	136	1063	0	0	0	0	-1	0
N.S.	1	1.00	1.14	8.93	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	0.104	58.821	0.000	0.000	0.000	0.000	0.000	4.583

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	119	119	136	725	0	0	0	0	-1	0
N.S.	1	1.00	1.14	6.09	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	0.109	77.110	0.000	0.000	0.000	0.000	0.000	4.629

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	70	70	124	512	0	1395	0	0	-1	0
N.S.	1	1.00	1.77	7.31	0.00	19.93	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.009	0.095	10.587	0.000	8.020	0.000	0.000	0.000	3.895

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	104	104	137	0	0	0	0	0	-1	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	0.111	0.316	0.000	0.000	0.000	0.000	0.000	4.405

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	74	74	125	539	0	269	0	0	-1	0
N.S.	1	1.00	1.69	7.28	0.00	3.64	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.011	0.046	2.611	0.000	3.721	0.000	0.000	0.000	8.168

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	91	57	94	70	315	0	0	-1	79
N.S.	1	1.15	0.72	1.19	0.89	3.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.019	0.027	0.026	1.493	2.248	0.000	0.000	0.000	0.112

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	91	54	94	0	303	0	0	-1	79
N.S.	1	1.23	0.73	1.27	0.00	4.09	0.00	0.00	-0.01	1.07
time (sec)	N/A	0.018	0.019	0.022	0.000	1.605	0.000	0.000	0.000	0.084

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	91	57	94	0	314	0	0	-1	79
N.S.	1	1.20	0.75	1.24	0.00	4.13	0.00	0.00	-0.01	1.04
time (sec)	N/A	0.017	0.029	0.022	0.000	1.765	0.000	0.000	0.000	0.101

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	15	38	0	38	17	17	-1	0
N.S.	1	1.00	0.75	1.90	0.00	1.90	0.85	0.85	-0.05	0.00
time (sec)	N/A	0.004	0.002	0.003	0.000	0.911	3.433	0.573	0.000	0.043

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	26	8	0	68	22	19	-1	0
N.S.	1	1.00	3.25	1.00	0.00	8.50	2.75	2.38	-0.12	0.00
time (sec)	N/A	0.002	0.005	0.317	0.000	1.298	2.321	0.575	0.000	0.001

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	8	0	34	8	26	-1	0
N.S.	1	1.00	1.00	1.00	0.00	4.25	1.00	3.25	-0.12	0.00
time (sec)	N/A	0.002	0.003	0.307	0.000	1.293	2.494	0.572	0.000	0.001

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	40	24	0	34	0	0	-1	0
N.S.	1	1.00	1.38	0.83	0.00	1.17	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.003	0.011	0.010	0.000	1.168	0.000	0.000	0.000	0.063

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	24	0	104	0	0	-1	0
N.S.	1	1.00	0.93	0.86	0.00	3.71	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.004	0.011	0.010	0.000	1.153	0.000	0.000	0.000	0.059

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	129	129	135	186	0	553	0	0	-1	137
N.S.	1	1.00	1.05	1.44	0.00	4.29	0.00	0.00	-0.01	1.06
time (sec)	N/A	0.019	0.113	1.667	0.000	9.143	0.000	0.000	0.000	0.334

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	120	120	135	187	0	553	0	0	-1	137
N.S.	1	1.00	1.12	1.56	0.00	4.61	0.00	0.00	-0.01	1.14
time (sec)	N/A	0.016	0.128	1.596	0.000	9.051	0.000	0.000	0.000	0.321

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	129	129	144	0	0	755	0	0	-1	137
N.S.	1	1.00	1.12	0.00	0.00	5.85	0.00	0.00	-0.01	1.06
time (sec)	N/A	0.023	0.138	0.331	0.000	28.018	0.000	0.000	0.000	0.274

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	124	124	145	0	0	776	0	0	-1	141
N.S.	1	1.00	1.17	0.00	0.00	6.26	0.00	0.00	-0.01	1.14
time (sec)	N/A	0.020	0.133	0.335	0.000	27.759	0.000	0.000	0.000	0.275

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	120	120	155	0	0	286	0	0	-1	136
N.S.	1	1.00	1.29	0.00	0.00	2.38	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.020	0.145	0.344	0.000	24.630	0.000	0.000	0.000	0.253

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	120	120	155	0	0	286	0	0	-1	136
N.S.	1	1.00	1.29	0.00	0.00	2.38	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.017	0.162	0.340	0.000	23.722	0.000	0.000	0.000	0.252

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	120	120	165	0	0	337	0	0	-1	136
N.S.	1	1.00	1.38	0.00	0.00	2.81	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.021	0.155	0.334	0.000	137.832	0.000	0.000	0.000	0.261

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	124	124	162	0	0	343	0	0	-1	140
N.S.	1	1.00	1.31	0.00	0.00	2.77	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.021	0.164	0.336	0.000	129.691	0.000	0.000	0.000	0.264

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	61	61	127	138	0	104	0	0	-1	63
N.S.	1	1.00	2.08	2.26	0.00	1.70	0.00	0.00	-0.02	1.03
time (sec)	N/A	0.009	0.148	1.139	0.000	9.421	0.000	0.000	0.000	0.124

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	61	61	127	138	0	243	0	0	-1	72
N.S.	1	1.00	2.08	2.26	0.00	3.98	0.00	0.00	-0.02	1.18
time (sec)	N/A	0.010	0.129	1.112	0.000	9.293	0.000	0.000	0.000	0.121

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	77	77	132	0	0	274	0	0	-1	79
N.S.	1	1.00	1.71	0.00	0.00	3.56	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.013	0.166	0.335	0.000	29.664	0.000	0.000	0.000	0.134

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	79	79	137	0	0	273	0	0	-1	90
N.S.	1	1.00	1.73	0.00	0.00	3.46	0.00	0.00	-0.01	1.14
time (sec)	N/A	0.014	0.148	0.334	0.000	29.244	0.000	0.000	0.000	0.145

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	85	85	157	0	0	276	0	0	-1	89
N.S.	1	1.00	1.85	0.00	0.00	3.25	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.018	0.171	0.351	0.000	20.455	0.000	0.000	0.000	0.160

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	85	85	157	0	0	278	0	0	-1	89
N.S.	1	1.00	1.85	0.00	0.00	3.27	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.014	0.148	0.363	0.000	20.762	0.000	0.000	0.000	0.156

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	101	101	163	0	0	338	0	0	-1	105
N.S.	1	1.00	1.61	0.00	0.00	3.35	0.00	0.00	-0.01	1.04
time (sec)	N/A	0.019	0.164	0.342	0.000	111.056	0.000	0.000	0.000	0.173

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	103	103	168	0	0	350	0	0	-1	107
N.S.	1	1.00	1.63	0.00	0.00	3.40	0.00	0.00	-0.01	1.04
time (sec)	N/A	0.021	0.163	0.337	0.000	110.161	0.000	0.000	0.000	0.172

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	53	53	115	121	0	91	0	0	-1	55
N.S.	1	1.00	2.17	2.28	0.00	1.72	0.00	0.00	-0.02	1.04
time (sec)	N/A	0.008	0.141	1.220	0.000	8.366	0.000	0.000	0.000	0.101

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	52	71	0	91	0	0	131	0
N.S.	1	1.00	0.98	1.34	0.00	1.72	0.00	0.00	2.47	0.00
time (sec)	N/A	0.020	0.032	0.005	0.000	0.997	0.000	0.000	5.736	0.541

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [66] had the largest ratio of [.3333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	17	0.059
2	A	2	1	1.00	17	0.059
3	A	2	1	1.00	17	0.059
4	A	2	1	1.00	15	0.067
5	A	2	2	1.00	17	0.118
6	A	2	2	1.00	17	0.118
7	A	3	3	1.00	17	0.176
8	A	2	1	1.00	19	0.053
9	A	2	1	1.00	19	0.053
10	A	2	1	1.00	17	0.059
11	A	3	2	1.00	19	0.105
12	A	4	3	1.00	19	0.158
13	A	3	3	1.00	19	0.158
14	A	2	1	1.00	19	0.053
15	A	2	1	1.00	19	0.053
16	A	2	1	1.00	17	0.059
17	A	3	2	1.00	19	0.105
18	A	4	3	1.00	19	0.158
19	A	5	4	1.00	19	0.210
20	A	3	2	1.00	19	0.105
21	A	3	2	1.00	19	0.105
22	A	3	2	1.00	19	0.105
23	A	2	2	1.00	17	0.118
24	A	3	2	1.00	19	0.105
25	A	4	3	1.00	19	0.158
26	A	5	4	1.00	19	0.210
27	A	4	3	1.00	19	0.158
28	A	4	3	1.00	19	0.158
29	A	4	3	1.00	19	0.158
30	A	4	3	1.00	19	0.158
31	A	2	2	1.00	17	0.118
32	A	4	3	1.00	19	0.158
33	A	5	4	1.00	19	0.210
34	A	6	4	1.00	19	0.210
35	A	5	4	1.00	19	0.210
36	A	5	4	1.00	19	0.210
37	A	5	4	1.00	19	0.210
38	A	3	3	1.00	19	0.158
39	A	3	3	1.00	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
40	A	5	4	1.00	19	0.210
41	A	6	4	1.00	19	0.210
42	A	7	4	1.00	19	0.210
43	A	3	3	1.00	15	0.200
44	A	5	3	1.00	15	0.200
45	A	6	6	1.00	21	0.286
46	A	5	5	1.00	21	0.238
47	A	4	4	1.00	19	0.210
48	A	3	3	1.00	11	0.273
49	A	5	5	1.00	21	0.238
50	A	3	3	1.00	21	0.143
51	A	4	4	1.00	21	0.190
52	A	6	5	1.00	21	0.238
53	A	7	6	1.00	21	0.286
54	A	6	5	1.00	21	0.238
55	A	5	4	1.00	19	0.210
56	A	4	3	1.00	11	0.273
57	A	6	6	1.00	21	0.286
58	A	6	6	1.00	21	0.286
59	A	4	3	1.00	21	0.143
60	A	5	4	1.00	21	0.190
61	A	7	5	1.00	21	0.238
62	A	8	6	1.00	21	0.286
63	A	7	5	1.00	21	0.238
64	A	6	4	1.00	19	0.210
65	A	5	3	1.00	11	0.273
66	A	7	7	1.00	21	0.333
67	A	7	7	1.00	21	0.333
68	A	7	7	1.00	21	0.333
69	A	5	3	1.00	21	0.143
70	A	6	4	1.00	21	0.190
71	A	4	4	1.00	19	0.210
72	A	4	4	1.00	17	0.235
73	A	4	4	1.00	21	0.190
74	A	5	5	1.00	21	0.238
75	A	4	4	1.00	21	0.190
76	A	3	3	1.00	19	0.158
77	A	2	2	1.00	11	0.182
78	A	2	2	1.00	21	0.095
79	A	3	3	1.00	21	0.143
80	A	5	5	1.00	21	0.238
81	A	6	5	1.00	21	0.238
82	A	5	5	1.00	21	0.238
83	A	4	4	1.17	21	0.190
84	A	3	3	1.00	19	0.158
85	A	1	1	1.00	11	0.091
86	A	3	3	1.00	21	0.143
87	A	5	5	1.00	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
88	A	6	5	1.00	21	0.238
89	A	6	6	1.00	21	0.286
90	A	5	5	1.00	21	0.238
91	A	4	4	1.00	21	0.190
92	A	2	2	1.00	19	0.105
93	A	2	2	1.00	11	0.182
94	A	5	5	1.00	21	0.238
95	A	6	5	1.00	21	0.238
96	A	7	5	1.00	21	0.238
97	A	5	3	1.00	21	0.143
98	A	4	3	1.00	21	0.143
99	A	3	3	1.00	19	0.158
100	A	2	2	1.00	11	0.182
101	A	3	3	1.00	21	0.143
102	A	3	3	1.00	21	0.143
103	A	4	4	1.00	21	0.190
104	A	5	4	1.00	21	0.190
105	A	2	2	1.00	26	0.077
106	A	2	2	1.00	19	0.105
107	A	2	2	1.00	21	0.095
108	A	2	2	1.00	15	0.133
109	A	1	1	1.00	24	0.042
110	A	2	2	1.00	24	0.083
111	A	1	1	1.00	26	0.038
112	A	1	1	1.00	24	0.042
113	A	1	1	1.00	23	0.043
114	A	1	1	1.00	24	0.042
115	A	1	1	1.00	22	0.045
116	A	1	1	1.00	19	0.053
117	A	1	1	1.00	19	0.053
118	A	1	1	1.00	24	0.042
119	A	1	1	1.00	22	0.045
120	A	1	1	1.00	27	0.037
121	A	1	1	1.00	28	0.036
122	A	1	1	1.00	29	0.034
123	A	1	1	1.00	30	0.033
124	A	1	1	1.00	26	0.038
125	A	1	1	1.00	26	0.038
126	A	1	1	1.00	23	0.043
127	A	1	1	1.00	23	0.043
128	A	1	1	1.00	23	0.043
129	A	1	1	1.00	23	0.043
130	A	1	1	1.00	17	0.059
131	A	1	1	1.00	21	0.048
132	A	1	1	1.00	21	0.048
133	A	3	3	1.15	29	0.103
134	A	3	3	1.23	29	0.103
135	A	3	3	1.20	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	2	1	1.00	23	0.043
137	A	2	2	1.00	23	0.087
138	A	2	2	1.00	21	0.095
139	A	2	2	1.00	21	0.095
140	A	2	2	1.00	23	0.087
141	A	1	1	1.00	21	0.048
142	A	1	1	1.00	21	0.048
143	A	1	1	1.00	21	0.048
144	A	1	1	1.00	23	0.043
145	A	1	1	1.00	23	0.043
146	A	1	1	1.00	23	0.043
147	A	1	1	1.00	23	0.043
148	A	1	1	1.00	25	0.040
149	A	1	1	1.00	21	0.048
150	A	1	1	1.00	21	0.048
151	A	1	1	1.00	21	0.048
152	A	1	1	1.00	23	0.043
153	A	1	1	1.00	25	0.040
154	A	1	1	1.00	25	0.040
155	A	1	1	1.00	25	0.040
156	A	1	1	1.00	27	0.037
157	A	1	1	1.00	19	0.053
158	A	1	1	1.00	50	0.020

Chapter 3

Listing of integrals

Local contents

3.1 $\int \left(a+bx^2\right)\left(c+dx^2\right)^4\,dx$

Optimal. Leaf size=94

$$\frac{1}{3}c^3x^3(4ad+bc)+\frac{2}{5}c^2dx^5(3ad+2bc)+\frac{1}{9}d^3x^9(ad+4bc)+\frac{2}{7}cd^2x^7(2ad+3bc)+ac^4x+\frac{1}{11}bd^4x^{11}$$

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{2}{5}c^2dx^5(3ad+2bc)+\frac{1}{3}c^3x^3(4ad+bc)+\frac{1}{9}d^3x^9(ad+4bc)+\frac{2}{7}cd^2x^7(2ad+3bc)+ac^4x+\frac{1}{11}bd^4x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^5)/5 + (2*
c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/9 + (b*d^4*x^11)/11
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^4\,dx=\int \left(ac^4+c^3(bc+4ad)x^2+2c^2d(2bc+3ad)x^4+2cd^2(3bc+2ad)x^6+d^3(4bc+a\right.$$
$$=ac^4x+\frac{1}{3}c^3(bc+4ad)x^3+\frac{2}{5}c^2d(2bc+3ad)x^5+\frac{2}{7}cd^2(3bc+2ad)x^7+\frac{1}{9}d^3(4bc+$$

Mathematica [A] time = 0.02, size = 94, normalized size = 1.00

$$\frac{1}{3}c^3x^3(4ad+bc)+\frac{2}{5}c^2dx^5(3ad+2bc)+\frac{1}{9}d^3x^9(ad+4bc)+\frac{2}{7}cd^2x^7(2ad+3bc)+ac^4x+\frac{1}{11}bd^4x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^5)/5 + (2*
c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/9 + (b*d^4*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^4\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)*(c + d*x^2)^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)*(c + d*x^2)^4, x]
```

fricas [A] time = 0.49, size = 98, normalized size = 1.04

$$\frac{1}{11}x^{11}d^4b+\frac{4}{9}x^9d^3cb+\frac{1}{9}x^9d^4a+\frac{6}{7}x^7d^2c^2b+\frac{4}{7}x^7d^3ca+\frac{4}{5}x^5dc^3b+\frac{6}{5}x^5d^2c^2a+\frac{1}{3}x^3c^4b+\frac{4}{3}x^3dc^3a+xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^4*b + 4/9*x^9*d^3*c*b + 1/9*x^9*d^4*a + 6/7*x^7*d^2*c^2*b + 4/7
*x^7*d^3*c*a + 4/5*x^5*d*c^3*b + 6/5*x^5*d^2*c^2*a + 1/3*x^3*c^4*b + 4/3*x^
3*d*c^3*a + x*c^4*a
```

giac [A] time = 0.57, size = 98, normalized size = 1.04

$$\frac{1}{11}\,bd^4x^{11}+\frac{4}{9}\,bcd^3x^9+\frac{1}{9}\,ad^4x^9+\frac{6}{7}\,bc^2d^2x^7+\frac{4}{7}\,acd^3x^7+\frac{4}{5}\,bc^3dx^5+\frac{6}{5}\,ac^2d^2x^5+\frac{1}{3}\,bc^4x^3+\frac{4}{3}\,ac^3dx^3+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/11*b*d^4*x^11 + 4/9*b*c*d^3*x^9 + 1/9*a*d^4*x^9 + 6/7*b*c^2*d^2*x^7 + 4/7
*a*c*d^3*x^7 + 4/5*b*c^3*d*x^5 + 6/5*a*c^2*d^2*x^5 + 1/3*b*c^4*x^3 + 4/3*a*
c^3*d*x^3 + a*c^4*x
```

maple [A] time = 0.00, size = 97, normalized size = 1.03

$$\frac{b\,d^4x^{11}}{11}+\frac{\left(a\,d^4+4bc\,d^3\right)x^9}{9}+\frac{\left(4ac\,d^3+6b\,c^2d^2\right)x^7}{7}+a\,c^4x+\frac{\left(6a\,c^2d^2+4b\,c^3d\right)x^5}{5}+\frac{\left(4a\,c^3d+b\,c^4\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^4,x)
```

```
[Out] 1/11*b*d^4*x^11+1/9*(a*d^4+4*b*c*d^3)*x^9+1/7*(4*a*c*d^3+6*b*c^2*d^2)*x^7+1
/5*(6*a*c^2*d^2+4*b*c^3*d)*x^5+1/3*(4*a*c^3*d+b*c^4)*x^3+a*c^4*x
```

maxima [A] time = 1.36, size = 96, normalized size = 1.02

$$\frac{1}{11}\,bd^4x^{11}+\frac{1}{9}\left(4\,bcd^3+ad^4\right)x^9+\frac{2}{7}\left(3\,bc^2d^2+2\,acd^3\right)x^7+ac^4x+\frac{2}{5}\left(2\,bc^3d+3\,ac^2d^2\right)x^5+\frac{1}{3}\left(bc^4+4\,ac^3d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/11*b*d^4*x^11 + 1/9*(4*b*c*d^3 + a*d^4)*x^9 + 2/7*(3*b*c^2*d^2 + 2*a*c*d^
3)*x^7 + a*c^4*x + 2/5*(2*b*c^3*d + 3*a*c^2*d^2)*x^5 + 1/3*(b*c^4 + 4*a*c^3
*d)*x^3
```

mupad [B] time = 4.77, size = 88, normalized size = 0.94

$$x^3\left(\frac{b\,c^4}{3}+\frac{4\,a\,d\,c^3}{3}\right)+x^9\left(\frac{a\,d^4}{9}+\frac{4\,b\,c\,d^3}{9}\right)+\frac{b\,d^4\,x^{11}}{11}+a\,c^4\,x+\frac{2\,c^2\,d\,x^5\,\left(3\,a\,d+2\,b\,c\right)}{5}+\frac{2\,c\,d^2\,x^7\,\left(2\,a\,d+3\,b\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^4,x)
```

```
[Out] x^3*((b*c^4)/3 + (4*a*c^3*d)/3) + x^9*((a*d^4)/9 + (4*b*c*d^3)/9) + (b*d^4*
x^11)/11 + a*c^4*x + (2*c^2*d*x^5*(3*a*d + 2*b*c))/5 + (2*c*d^2*x^7*(2*a*d
+ 3*b*c))/7
```

sympy [A] time = 0.09, size = 107, normalized size = 1.14

$$ac^4x+\frac{bd^4x^{11}}{11}+x^9\left(\frac{ad^4}{9}+\frac{4bcd^3}{9}\right)+x^7\left(\frac{4acd^3}{7}+\frac{6bc^2d^2}{7}\right)+x^5\left(\frac{6ac^2d^2}{5}+\frac{4bc^3d}{5}\right)+x^3\left(\frac{4ac^3d}{3}+\frac{bc^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**11/11 + x**9*(a*d**4/9 + 4*b*c*d**3/9) + x**7*(4*a*c*d
**3/7 + 6*b*c**2*d**2/7) + x**5*(6*a*c**2*d**2/5 + 4*b*c**3*d/5) + x**3*(4*
a*c**3*d/3 + b*c**4/3)
```

3.2 $\int \left(a+bx^2\right)\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=70

$$\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3,x]

[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*b
*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)^3\,dx &= \int \left(ac^3+c^2(bc+3ad)x^2+3cd(bc+ad)x^4+d^2(3bc+ad)x^6+bd^3x^8\right)dx\\ &= ac^3x+\frac{1}{3}c^2(bc+3ad)x^3+\frac{3}{5}cd(bc+ad)x^5+\frac{1}{7}d^2(3bc+ad)x^7+\frac{1}{9}bd^3x^9\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3,x]

[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*b
*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)*(c + d*x^2)^3,x]

[Out] IntegrateAlgebraic[(a + b*x^2)*(c + d*x^2)^3, x]
```

fricas [A] time = 0.80, size = 73, normalized size = 1.04

$$\frac{1}{9}x^9d^3b+\frac{3}{7}x^7d^2cb+\frac{1}{7}x^7d^3a+\frac{3}{5}x^5dc^2b+\frac{3}{5}x^5d^2ca+\frac{1}{3}x^3c^3b+x^3dc^2a+xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^3*b + 3/7*x^7*d^2*c*b + 1/7*x^7*d^3*a + 3/5*x^5*d*c^2*b + 3/5*x^5
*d^2*c*a + 1/3*x^3*c^3*b + x^3*d*c^2*a + x*c^3*a
```

giac [A] time = 0.56, size = 73, normalized size = 1.04

$$\frac{1}{9}bd^3x^9 + \frac{3}{7}bcd^2x^7 + \frac{1}{7}ad^3x^7 + \frac{3}{5}bc^2dx^5 + \frac{3}{5}acd^2x^5 + \frac{1}{3}bc^3x^3 + ac^2dx^3 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b*d^3*x^9 + 3/7*b*c*d^2*x^7 + 1/7*a*d^3*x^7 + 3/5*b*c^2*d*x^5 + 3/5*a*c
*d^2*x^5 + 1/3*b*c^3*x^3 + a*c^2*d*x^3 + a*c^3*x
```

maple [A] time = 0.00, size = 73, normalized size = 1.04

$$\frac{b\,d^3x^9}{9} + \frac{\left(a\,d^3 + 3bc\,d^2\right)x^7}{7} + a\,c^3x + \frac{\left(3ac\,d^2 + 3b\,c^2d\right)x^5}{5} + \frac{\left(3a\,c^2d + b\,c^3\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3,x)
```

```
[Out] 1/9*b*d^3*x^9+1/7*(a*d^3+3*b*c*d^2)*x^7+1/5*(3*a*c*d^2+3*b*c^2*d)*x^5+1/3*(
3*a*c^2*d+b*c^3)*x^3+a*c^3*x
```

maxima [A] time = 1.37, size = 70, normalized size = 1.00

$$\frac{1}{9}bd^3x^9 + \frac{1}{7}\left(3\,bcd^2 + ad^3\right)x^7 + \frac{3}{5}\left(bc^2d + acd^2\right)x^5 + ac^3x + \frac{1}{3}\left(bc^3 + 3\,ac^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^3*x^9 + 1/7*(3*b*c*d^2 + a*d^3)*x^7 + 3/5*(b*c^2*d + a*c*d^2)*x^5 +
 a*c^3*x + 1/3*(b*c^3 + 3*a*c^2*d)*x^3
```

mupad [B] time = 4.75, size = 65, normalized size = 0.93

$$x^3\left(\frac{b\,c^3}{3} + a\,d\,c^2\right) + x^7\left(\frac{a\,d^3}{7} + \frac{3\,b\,c\,d^2}{7}\right) + \frac{b\,d^3\,x^9}{9} + a\,c^3\,x + \frac{3\,c\,d\,x^5\,\left(a\,d + b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^3,x)
```

```
[Out] x^3*((b*c^3)/3 + a*c^2*d) + x^7*((a*d^3)/7 + (3*b*c*d^2)/7) + (b*d^3*x^9)/9
 + a*c^3*x + (3*c*d*x^5*(a*d + b*c))/5
```

sympy [A] time = 0.08, size = 76, normalized size = 1.09

$$ac^3x + \frac{bd^3x^9}{9} + x^7\left(\frac{ad^3}{7} + \frac{3bcd^2}{7}\right) + x^5\left(\frac{3acd^2}{5} + \frac{3bc^2d}{5}\right) + x^3\left(ac^2d + \frac{bc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**9/9 + x**7*(a*d**3/7 + 3*b*c*d**2/7) + x**5*(3*a*c*d**
2/5 + 3*b*c**2*d/5) + x**3*(a*c**2*d + b*c**3/3)
```

3.3 $\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx &= \int \left(ac^2+c(bc+2ad)x^2+d(2bc+ad)x^4+bd^2x^6\right)dx\\ &= ac^2x+\frac{1}{3}c(bc+2ad)x^3+\frac{1}{5}d(2bc+ad)x^5+\frac{1}{7}bd^2x^7\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)*(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)*(c + d*x^2)^2, x]
```

fricas [A] time = 0.40, size = 50, normalized size = 1.00

$$\frac{1}{7}x^7d^2b+\frac{2}{5}x^5dcb+\frac{1}{5}x^5d^2a+\frac{1}{3}x^3c^2b+\frac{2}{3}x^3dca+xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^2*b + 2/5*x^5*d*c*b + 1/5*x^5*d^2*a + 1/3*x^3*c^2*b + 2/3*x^3*d*c
*a + x*c^2*a
```

giac [A] time = 0.57, size = 50, normalized size = 1.00

$$\frac{1}{7}bd^2x^7 + \frac{2}{5}bcdx^5 + \frac{1}{5}ad^2x^5 + \frac{1}{3}bc^2x^3 + \frac{2}{3}acdx^3 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/7*b*d^2*x^7 + 2/5*b*c*d*x^5 + 1/5*a*d^2*x^5 + 1/3*b*c^2*x^3 + 2/3*a*c*d*x
^3 + a*c^2*x
```

maple [A] time = 0.00, size = 49, normalized size = 0.98

$$\frac{b\,d^2x^7}{7} + \frac{\left(a\,d^2 + 2bcd\right)x^5}{5} + a\,c^2x + \frac{\left(2acd + b\,c^2\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2,x)
```

```
[Out] 1/7*b*d^2*x^7+1/5*(a*d^2+2*b*c*d)*x^5+1/3*(2*a*c*d+b*c^2)*x^3+a*c^2*x
```

maxima [A] time = 1.35, size = 48, normalized size = 0.96

$$\frac{1}{7}bd^2x^7 + \frac{1}{5}\left(2\,bcd + ad^2\right)x^5 + ac^2x + \frac{1}{3}\left(bc^2 + 2\,acd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b*d^2*x^7 + 1/5*(2*b*c*d + a*d^2)*x^5 + a*c^2*x + 1/3*(b*c^2 + 2*a*c*d)
*x^3
```

mupad [B] time = 0.05, size = 48, normalized size = 0.96

$$x^3\left(\frac{b\,c^2}{3} + \frac{2\,a\,d\,c}{3}\right) + x^5\left(\frac{a\,d^2}{5} + \frac{2\,b\,c\,d}{5}\right) + \frac{b\,d^2\,x^7}{7} + a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^2,x)
```

```
[Out] x^3*((b*c^2)/3 + (2*a*c*d)/3) + x^5*((a*d^2)/5 + (2*b*c*d)/5) + (b*d^2*x^7)
/7 + a*c^2*x
```

sympy [A] time = 0.07, size = 53, normalized size = 1.06

$$ac^2x + \frac{bd^2x^7}{7} + x^5\left(\frac{ad^2}{5} + \frac{2bcd}{5}\right) + x^3\left(\frac{2acd}{3} + \frac{bc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**7/7 + x**5*(a*d**2/5 + 2*b*c*d/5) + x**3*(2*a*c*d/3 +
b*c**2/3)
```

3.4 $\int \left(a+bx^2\right)\left(c+dx^2\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)\,dx &= \int \left(ac+(bc+ad)x^2+bdx^4\right)\,dx\\ &= acx+\frac{1}{3}(bc+ad)x^3+\frac{1}{5}bdx^5\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)*(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)*(c + d*x^2), x]
```

fricas [A] time = 0.77, size = 26, normalized size = 0.93

$$\frac{1}{5}x^5db+\frac{1}{3}x^3cb+\frac{1}{3}x^3da+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="fricas")
[Out] 1/5*x^5*d*b + 1/3*x^3*c*b + 1/3*x^3*d*a + x*c*a
```

giac [A] time = 0.56, size = 26, normalized size = 0.93

$$\frac{1}{5}\,bdx^5 + \frac{1}{3}\,bcx^3 + \frac{1}{3}\,adx^3 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="giac")
[Out] 1/5*b*d*x^5 + 1/3*b*c*x^3 + 1/3*a*d*x^3 + a*c*x
```

maple [A] time = 0.00, size = 25, normalized size = 0.89

$$\frac{bd\,x^5}{5} + acx + \frac{(ad + bc)\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c),x)
[Out] a*c*x+1/3*(a*d+b*c)*x^3+1/5*b*d*x^5
```

maxima [A] time = 1.35, size = 24, normalized size = 0.86

$$\frac{1}{5}\,bdx^5 + \frac{1}{3}\,(bc + ad)x^3 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="maxima")
[Out] 1/5*b*d*x^5 + 1/3*(b*c + a*d)*x^3 + a*c*x
```

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^5}{5} + \left(\frac{a\,d}{3} + \frac{b\,c}{3}\right)\,x^3 + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2),x)
[Out] x^3*((a*d)/3 + (b*c)/3) + a*c*x + (b*d*x^5)/5
```

sympy [A] time = 0.06, size = 26, normalized size = 0.93

$$acx + \frac{bdx^5}{5} + x^3\left(\frac{ad}{3} + \frac{bc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c),x)
[Out] a*c*x + b*d*x**5/5 + x**3*(a*d/3 + b*c/3)
```

3.5 $\int \frac{a+bx^2}{c+dx^2} \, dx$

Optimal. Leaf size=40

$$\frac{bx}{d} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {388, 205}

$$\frac{bx}{d} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2),x]
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{c+dx^2} \, dx &= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^2} \, dx}{d} \\ &= \frac{bx}{d} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$\frac{bx}{d} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2),x]
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{c+dx^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)/(c + d*x^2), x]
```

fricas [A] time = 0.80, size = 99, normalized size = 2.48

$$\left[\frac{2\,bcdx + (bc - ad)\sqrt{-cd}\,\log\left(\frac{dx^2 - 2\,\sqrt{-cd}\,x - c}{dx^2 + c}\right)}{2\,cd^2}, \frac{bcdx - (bc - ad)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{cd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(
d*x^2 + c)))/(c*d^2), (b*c*d*x - (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x/c
))/(c*d^2)]
```

giac [A] time = 0.57, size = 34, normalized size = 0.85

$$\frac{bx}{d} - \frac{(bc - ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b*x/d - (b*c - a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d)
```

maple [A] time = 0.01, size = 45, normalized size = 1.12

$$\frac{a\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}} - \frac{bc\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d} + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c),x)
```

```
[Out] b*x/d+1/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-1/d/(c*d)^(1/2)*arctan(x*d/(c
*d)^(1/2))*b*c
```

maxima [A] time = 3.00, size = 34, normalized size = 0.85

$$\frac{bx}{d} - \frac{(bc - ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b*x/d - (b*c - a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d)
```

mupad [B] time = 0.06, size = 31, normalized size = 0.78

$$\frac{b\,x}{d} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)(a\,d - b\,c)}{\sqrt{c}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(c + d*x^2),x)
```

```
[Out] (b*x)/d + (atan((d^(1/2)*x)/c^(1/2))*(a*d - b*c))/(c^(1/2)*d^(3/2))
```

sympy [B] time = 0.28, size = 82, normalized size = 2.05

$$\frac{bx}{d} - \frac{\sqrt{-\frac{1}{cd^3}}\,(ad-bc)\log\left(-cd\sqrt{-\frac{1}{cd^3}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^3}}\,(ad-bc)\log\left(cd\sqrt{-\frac{1}{cd^3}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c),x)
```

```
[Out] b*x/d - sqrt(-1/(c*d**3))*(a*d - b*c)*log(-c*d*sqrt(-1/(c*d**3)) + x)/2 + s
qrt(-1/(c*d**3))*(a*d - b*c)*log(c*d*sqrt(-1/(c*d**3)) + x)/2
```

3.6 $\int \frac{a+bx^2}{\left(c+dx^2\right)^2} \, dx$

Optimal. Leaf size=63

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}-\frac{x(bc-ad)}{2cd\left(c+dx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 205}

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}-\frac{x(bc-ad)}{2cd\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(2*c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(2*c^(3/2)*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\left(c+dx^2\right)^2} \, dx &= -\frac{(bc-ad)x}{2cd\left(c+dx^2\right)}+\frac{(bc+ad)\int \frac{1}{c+dx^2} \, dx}{2cd}\\ &= -\frac{(bc-ad)x}{2cd\left(c+dx^2\right)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 1.00

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}-\frac{x(bc-ad)}{2cd\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*((b*c - a*d)*x)/(c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/(2*c^(3/2)*d^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)/(c + d*x^2)^2, x]
```

fricas [A] time = 0.82, size = 182, normalized size = 2.89

$$\left[-\frac{\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\left(bc^2d-acd^2\right)x}{4\left(c^2d^3x^2+c^3d^2\right)}, \frac{\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-\left(bc^2d-acd^2\right)x}{2\left(c^2d^3x^2+c^3d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) + 2*(b*c^2*d - a*c*d^2)*x)/(c^2*d^3*x^2 + c^3*d^
2), 1/2*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x
/c) - (b*c^2*d - a*c*d^2)*x)/(c^2*d^3*x^2 + c^3*d^2)]
```

giac [A] time = 0.58, size = 57, normalized size = 0.90

$$\frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,cd}-\frac{bcx-adx}{2\left(dx^2+c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d) - 1/2*(b*c*x - a*d*x)
/((d*x^2 + c)*c*d)
```

maple [A] time = 0.01, size = 68, normalized size = 1.08

$$\frac{a\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,c}+\frac{b\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,d}+\frac{(ad-bc)\,x}{2\left(d\,x^2+c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*(a*d-b*c)/c/d*x/(d*x^2+c)+1/2/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a
+1/2/d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b
```

maxima [A] time = 3.11, size = 57, normalized size = 0.90

$$-\frac{(bc-ad)x}{2\left(cd^2x^2+c^2d\right)}+\frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*c - a*d)*x/(c*d^2*x^2 + c^2*d) + 1/2*(b*c + a*d)*arctan(d*x/sqrt(c*
d))/(sqrt(c*d)*c*d)
```

mupad [B] time = 5.01, size = 51, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)(a\,d+b\,c)}{2\,c^{3/2}\,d^{3/2}}+\frac{x\,(a\,d-b\,c)}{2\,c\,d\,\left(d\,x^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(c + d*x^2)^2,x)
```

```
[Out] (atan((d^(1/2)*x)/c^(1/2))*(a*d + b*c))/(2*c^(3/2)*d^(3/2)) + (x*(a*d - b*c
))/(2*c*d*(c + d*x^2))
```

sympy [B] time = 0.39, size = 112, normalized size = 1.78

$$\frac{x(ad-bc)}{2c^2d+2cd^2x^2}-\frac{\sqrt{-\frac{1}{c^3d^3}}\,(ad+bc)\log\left(-c^2d\sqrt{-\frac{1}{c^3d^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^3d^3}}\,(ad+bc)\log\left(c^2d\sqrt{-\frac{1}{c^3d^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(2*c**2*d + 2*c*d**2*x**2) - sqrt(-1/(c**3*d**3))*(a*d + b*c)
*log(-c**2*d*sqrt(-1/(c**3*d**3)) + x)/4 + sqrt(-1/(c**3*d**3))*(a*d + b*c)
*log(c**2*d*sqrt(-1/(c**3*d**3)) + x)/4
```

3.7 $\int \frac{a+bx^2}{\left(c+dx^2\right)^3} \, dx$

Optimal. Leaf size=92

$$\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}+\frac{x(3ad+bc)}{8c^2d\left(c+dx^2\right)}-\frac{x(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {385, 199, 205}

$$\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}+\frac{x(3ad+bc)}{8c^2d\left(c+dx^2\right)}-\frac{x(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(4*c*d*(c + d*x^2)^2) + ((b*c + 3*a*d)*x)/(8*c^2*d*(c + d*
x^2)) + ((b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{\left(c+dx^2\right)^3} \, dx &= -\frac{(bc-ad)x}{4cd\left(c+dx^2\right)^2}+\frac{(bc+3ad)\int \frac{1}{\left(c+dx^2\right)^2} \, dx}{4cd}\\
&= -\frac{(bc-ad)x}{4cd\left(c+dx^2\right)^2}+\frac{(bc+3ad)x}{8c^2d\left(c+dx^2\right)}+\frac{(bc+3ad)\int \frac{1}{c+dx^2} \, dx}{8c^2d}\\
&= -\frac{(bc-ad)x}{4cd\left(c+dx^2\right)^2}+\frac{(bc+3ad)x}{8c^2d\left(c+dx^2\right)}+\frac{(bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.89

$$\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}+\frac{x\left(ad\left(5c+3dx^2\right)+bc\left(dx^2-c\right)\right)}{8c^2d\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(b*c*(-c + d*x^2) + a*d*(5*c + 3*d*x^2)))/(8*c^2*d*(c + d*x^2)^2) + ((b*
c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)/(c + d*x^2)^3, x]
```

fricas [A] time = 0.82, size = 300, normalized size = 3.26

$$\left[\frac{2\left(bc^2d^2+3\,acd^3\right)x^3-\left(\left(bcd^2+3\,ad^3\right)x^4+bc^3+3\,ac^2d+2\left(bc^2d+3\,acd^2\right)x^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)-2\left(bc^3d-5\,ac^2d^2\right)x}{16\left(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2\right)},\ \frac{\left(bc^2d^2+3\,acd^3\right)x^3+\left(\left(bcd^2+3\,ad^3\right)x^4+bc^3+3\,ac^2d+2\left(bc^2d+3\,acd^2\right)x^2\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-\left(bc^3d-5\,ac^2d^2\right)x}{8\left(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(b*c^2*d^2 + 3*a*c*d^3)*x^3 - ((b*c*d^2 + 3*a*d^3)*x^4 + b*c^3 + 3
*a*c^2*d + 2*(b*c^2*d + 3*a*c*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d
)*x - c)/(d*x^2 + c)) - 2*(b*c^3*d - 5*a*c^2*d^2)*x)/(c^3*d^4*x^4 + 2*c^4*d
^3*x^2 + c^5*d^2), 1/8*((b*c^2*d^2 + 3*a*c*d^3)*x^3 + ((b*c*d^2 + 3*a*d^3)*
x^4 + b*c^3 + 3*a*c^2*d + 2*(b*c^2*d + 3*a*c*d^2)*x^2)*sqrt(c*d)*arctan(sqr
t(c*d)*x/c) - (b*c^3*d - 5*a*c^2*d^2)*x)/(c^3*d^4*x^4 + 2*c^4*d^3*x^2 + c^5
*d^2)]
```

giac [A] time = 0.59, size = 78, normalized size = 0.85

$$\frac{(bc+3\,ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,c^2d}+\frac{bcdx^3+3\,ad^2x^3-bc^2x+5\,acdx}{8\left(dx^2+c\right)^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b*c + 3*a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d) + 1/8*(b*c*d*x^3
+ 3*a*d^2*x^3 - b*c^2*x + 5*a*c*d*x)/((d*x^2 + c)^2*c^2*d)
```

maple [A] time = 0.01, size = 90, normalized size = 0.98

$$\frac{3a\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,c^2}+\frac{b\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,cd}+\frac{\frac{(3ad+bc)x^3}{8c^2}+\frac{(5ad-bc)x}{8cd}}{\left(d\,x^2+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] (1/8*(3*a*d+b*c)/c^2*x^3+1/8*(5*a*d-b*c)/c/d*x)/(d*x^2+c)^2+3/8/c^2/(c*d)^(
1/2)*arctan(1/(c*d)^(1/2)*d*x)*a+1/8/c/d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d
*x)*b
```

maxima [A] time = 2.96, size = 92, normalized size = 1.00

$$\frac{\left(bcd + 3\,ad^2\right)x^3 - \left(bc^2 - 5\,acd\right)x}{8\left(c^2d^3x^4 + 2\,c^3d^2x^2 + c^4d\right)} + \frac{(bc + 3\,ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b*c*d + 3*a*d^2)*x^3 - (b*c^2 - 5*a*c*d)*x)/(c^2*d^3*x^4 + 2*c^3*d^2*
x^2 + c^4*d) + 1/8*(b*c + 3*a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d)
```

mupad [B] time = 5.06, size = 82, normalized size = 0.89

$$\frac{\frac{x^3\,(3\,a\,d+b\,c)}{8\,c^2} + \frac{x\,(5\,a\,d-b\,c)}{8\,c\,d}}{c^2 + 2\,c\,d\,x^2 + d^2\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,(3\,a\,d + b\,c)}{8\,c^{5/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(c + d*x^2)^3,x)
```

```
[Out] ((x^3*(3*a*d + b*c))/(8*c^2) + (x*(5*a*d - b*c))/(8*c*d))/(c^2 + d^2*x^4 +
2*c*d*x^2) + (atan((d^(1/2)*x)/c^(1/2))*(3*a*d + b*c))/(8*c^(5/2)*d^(3/2))
```

sympy [A] time = 0.54, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{c^5d^3}}\,(3ad + bc)\log\left(-c^3d\sqrt{-\frac{1}{c^5d^3}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{c^5d^3}}\,(3ad + bc)\log\left(c^3d\sqrt{-\frac{1}{c^5d^3}} + x\right)}{16} + \frac{x^3\left(3ad^2 + bcd\right) + x\left(5acd - bc^2\right)}{8c^4d + 16c^3d^2x^2 + 8c^2d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(-c**3*d*sqrt(-1/(c**5*d**3)) + x)/1
6 + sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(c**3*d*sqrt(-1/(c**5*d**3)) + x)
/16 + (x**3*(3*a*d**2 + b*c*d) + x*(5*a*c*d - b*c**2))/(8*c**4*d + 16*c**3*
d**2*x**2 + 8*c**2*d**3*x**4)
```

3.8 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}$$

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3+ac^2(2bc+3ad)x^2+c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+d\left(3b^2c^2+6abcd+\right.\right.\\ &= a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5+\frac{1}{7}d\left(3b^2c^2+6abc\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 122, normalized size = 1.00

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2)^3, x]
```

fricas [A] time = 0.74, size = 131, normalized size = 1.07

$$\frac{1}{11}x^{11}d^3b^2+\frac{1}{3}x^9d^2cb^2+\frac{2}{9}x^9d^3ba+\frac{3}{7}x^7dc^2b^2+\frac{6}{7}x^7d^2cba+\frac{1}{7}x^7d^3a^2+\frac{1}{5}x^5c^3b^2+\frac{6}{5}x^5dc^2ba+\frac{3}{5}x^5d^2ca^2+\frac{2}{3}x^3c^3ba+x^3dc^2a^2+xc^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^3*b^2 + 1/3*x^9*d^2*c*b^2 + 2/9*x^9*d^3*b*a + 3/7*x^7*d*c^2*b^2
 + 6/7*x^7*d^2*c*b*a + 1/7*x^7*d^3*a^2 + 1/5*x^5*c^3*b^2 + 6/5*x^5*d*c^2*b*
a + 3/5*x^5*d^2*c*a^2 + 2/3*x^3*c^3*b*a + x^3*d*c^2*a^2 + x*c^3*a^2
```

giac [A] time = 0.56, size = 131, normalized size = 1.07

$$\frac{1}{11}b^2d^3x^{11}+\frac{1}{3}b^2cd^2x^9+\frac{2}{9}abd^3x^9+\frac{3}{7}b^2c^2dx^7+\frac{6}{7}abcd^2x^7+\frac{1}{7}a^2d^3x^7+\frac{1}{5}b^2c^3x^5+\frac{6}{5}abc^2dx^5+\frac{3}{5}a^2cd^2x^5+\frac{2}{3}abc^3x^3+a^2c^2dx^3+a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/3*b^2*c*d^2*x^9 + 2/9*a*b*d^3*x^9 + 3/7*b^2*c^2*d*x^7
 + 6/7*a*b*c*d^2*x^7 + 1/7*a^2*d^3*x^7 + 1/5*b^2*c^3*x^5 + 6/5*a*b*c^2*d*x^
5 + 3/5*a^2*c*d^2*x^5 + 2/3*a*b*c^3*x^3 + a^2*c^2*d*x^3 + a^2*c^3*x
```

maple [A] time = 0.00, size = 125, normalized size = 1.02

$$\frac{b^2d^3x^{11}}{11}+\frac{\left(2abd^3+3b^2cd^2\right)x^9}{9}+\frac{\left(a^2d^3+6abcd^2+3b^2c^2d\right)x^7}{7}+a^2c^3x+\frac{\left(3a^2cd^2+6abc^2d+b^2c^3\right)x^5}{5}+\frac{\left(3a^2c^2d+2abc^3\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/11*b^2*d^3*x^11+1/9*(2*a*b*d^3+3*b^2*c*d^2)*x^9+1/7*(a^2*d^3+6*a*b*c*d^2+
3*b^2*c^2*d)*x^7+1/5*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^5+1/3*(3*a^2*c^2*d
+2*a*b*c^3)*x^3+a^2*c^3*x
```

maxima [A] time = 1.39, size = 124, normalized size = 1.02

$$\frac{1}{11}b^2d^3x^{11}+\frac{1}{9}\left(3b^2cd^2+2abd^3\right)x^9+\frac{1}{7}\left(3b^2c^2d+6abcd^2+a^2d^3\right)x^7+a^2c^3x+\frac{1}{5}\left(b^2c^3+6abc^2d+3a^2cd^2\right)x^5+\frac{1}{3}\left(2abc^3+3a^2c^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

mupad [B] time = 4.95, size = 116, normalized size = 0.95

$$x^5\left(\frac{3a^2cd^2}{5}+\frac{6abc^2d}{5}+\frac{b^2c^3}{5}\right)+x^7\left(\frac{a^2d^3}{7}+\frac{6abcd^2}{7}+\frac{3b^2c^2d}{7}\right)+a^2c^3x+\frac{b^2d^3x^{11}}{11}+\frac{ac^2x^3\,(3ad+2bc)}{3}+\frac{bd^2x^9\,(2ad+3bc)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^5*((b^2*c^3)/5 + (3*a^2*c*d^2)/5 + (6*a*b*c^2*d)/5) + x^7*((a^2*d^3)/7 +
(3*b^2*c^2*d)/7 + (6*a*b*c*d^2)/7) + a^2*c^3*x + (b^2*d^3*x^11)/11 + (a*c^2
*x^3*(3*a*d + 2*b*c))/3 + (b*d^2*x^9*(2*a*d + 3*b*c))/9
```

sympy [A] time = 0.09, size = 136, normalized size = 1.11

$$a^2c^3x+\frac{b^2d^3x^{11}}{11}+x^9\left(\frac{2abd^3}{9}+\frac{b^2cd^2}{3}\right)+x^7\left(\frac{a^2d^3}{7}+\frac{6abcd^2}{7}+\frac{3b^2c^2d}{7}\right)+x^5\left(\frac{3a^2cd^2}{5}+\frac{6abc^2d}{5}+\frac{b^2c^3}{5}\right)+x^3\left(a^2c^2d+\frac{2abc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**11/11 + x**9*(2*a*b*d**3/9 + b**2*c*d**2/3) + x*
*7*(a**2*d**3/7 + 6*a*b*c*d**2/7 + 3*b**2*c**2*d/7) + x**5*(3*a**2*c*d**2/5
 + 6*a*b*c**2*d/5 + b**2*c**3/5) + x**3*(a**2*c**2*d + 2*a*b*c**3/3)
```

3.9 $\int (a+bx^2)^2 (c+dx^2)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)^2 (c+dx^2)^2 \, dx &= \int \left(a^2c^2+2ac(bc+ad)x^2+\left(b^2c^2+4abcd+a^2d^2\right)x^4+2bd(bc+ad)x^6+b^2d^2x^8\right) dx\\ &= a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5+\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2d^2x^9\end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 1.00

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2)^2, x]
```

fricas [A] time = 0.66, size = 91, normalized size = 1.11

$$\frac{1}{9}x^9d^2b^2+\frac{2}{7}x^7dcb^2+\frac{2}{7}x^7d^2ba+\frac{1}{5}x^5c^2b^2+\frac{4}{5}x^5dcba+\frac{1}{5}x^5d^2a^2+\frac{2}{3}x^3c^2ba+\frac{2}{3}x^3dca^2+xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^2*b^2 + 2/7*x^7*d*c*b^2 + 2/7*x^7*d^2*b*a + 1/5*x^5*c^2*b^2 + 4/5
*x^5*d*c*b*a + 1/5*x^5*d^2*a^2 + 2/3*x^3*c^2*b*a + 2/3*x^3*d*c*a^2 + x*c^2*
a^2
```

giac [A] time = 0.57, size = 91, normalized size = 1.11

$$\frac{1}{9}b^2d^2x^9 + \frac{2}{7}b^2cdx^7 + \frac{2}{7}abd^2x^7 + \frac{1}{5}b^2c^2x^5 + \frac{4}{5}abcdx^5 + \frac{1}{5}a^2d^2x^5 + \frac{2}{3}abc^2x^3 + \frac{2}{3}a^2cdx^3 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*b^2*c*d*x^7 + 2/7*a*b*d^2*x^7 + 1/5*b^2*c^2*x^5 + 4/5
*a*b*c*d*x^5 + 1/5*a^2*d^2*x^5 + 2/3*a*b*c^2*x^3 + 2/3*a^2*c*d*x^3 + a^2*c^
2*x
```

maple [A] time = 0.00, size = 87, normalized size = 1.06

$$\frac{b^2d^2x^9}{9} + \frac{\left(2ab\,d^2 + 2b^2cd\right)x^7}{7} + a^2c^2x + \frac{\left(a^2d^2 + 4abcd + b^2c^2\right)x^5}{5} + \frac{\left(2a^2cd + 2ab\,c^2\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/9*b^2*d^2*x^9+1/7*(2*a*b*d^2+2*b^2*c*d)*x^7+1/5*(a^2*d^2+4*a*b*c*d+b^2*c^
2)*x^5+1/3*(2*a^2*c*d+2*a*b*c^2)*x^3+a^2*c^2*x
```

maxima [A] time = 1.33, size = 82, normalized size = 1.00

$$\frac{1}{9}b^2d^2x^9 + \frac{2}{7}\left(b^2cd + abd^2\right)x^7 + \frac{1}{5}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^5 + a^2c^2x + \frac{2}{3}\left(abc^2 + a^2cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

mupad [B] time = 0.05, size = 75, normalized size = 0.91

$$x^5\left(\frac{a^2\,d^2}{5} + \frac{4\,a\,b\,c\,d}{5} + \frac{b^2\,c^2}{5}\right) + a^2\,c^2\,x + \frac{b^2\,d^2\,x^9}{9} + \frac{2\,a\,c\,x^3\,\left(a\,d + b\,c\right)}{3} + \frac{2\,b\,d\,x^7\,\left(a\,d + b\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^5*((a^2*d^2)/5 + (b^2*c^2)/5 + (4*a*b*c*d)/5) + a^2*c^2*x + (b^2*d^2*x^9)
/9 + (2*a*c*x^3*(a*d + b*c))/3 + (2*b*d*x^7*(a*d + b*c))/7
```

sympy [A] time = 0.08, size = 97, normalized size = 1.18

$$a^2c^2x + \frac{b^2d^2x^9}{9} + x^7\left(\frac{2abd^2}{7} + \frac{2b^2cd}{7}\right) + x^5\left(\frac{a^2d^2}{5} + \frac{4abcd}{5} + \frac{b^2c^2}{5}\right) + x^3\left(\frac{2a^2cd}{3} + \frac{2abc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**9/9 + x**7*(2*a*b*d**2/7 + 2*b**2*c*d/7) + x**5*
(a**2*d**2/5 + 4*a*b*c*d/5 + b**2*c**2/5) + x**3*(2*a**2*c*d/3 + 2*a*b*c**2
/3)
```

3.10 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\,dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\,dx &= \int \left(a^2c+a(2bc+ad)x^2+b(bc+2ad)x^4+b^2dx^6\right)\,dx\\ &= a^2cx+\frac{1}{3}a(2bc+ad)x^3+\frac{1}{5}b(bc+2ad)x^5+\frac{1}{7}b^2dx^7\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2), x]
```

fricas [A] time = 0.48, size = 50, normalized size = 1.00

$$\frac{1}{7}x^7db^2+\frac{1}{5}x^5cb^2+\frac{2}{5}x^5dba+\frac{2}{3}x^3cba+\frac{1}{3}x^3da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d*b^2 + 1/5*x^5*c*b^2 + 2/5*x^5*d*b*a + 2/3*x^3*c*b*a + 1/3*x^3*d*a
^2 + x*c*a^2
```

giac [A] time = 0.57, size = 50, normalized size = 1.00

$$\frac{1}{7}b^2dx^7 + \frac{1}{5}b^2cx^5 + \frac{2}{5}abdx^5 + \frac{2}{3}abcx^3 + \frac{1}{3}a^2dx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*b^2*c*x^5 + 2/5*a*b*d*x^5 + 2/3*a*b*c*x^3 + 1/3*a^2*d*x
^3 + a^2*c*x
```

maple [A] time = 0.00, size = 49, normalized size = 0.98

$$\frac{b^2d\,x^7}{7} + \frac{\left(2abd + b^2c\right)x^5}{5} + a^2cx + \frac{\left(a^2d + 2abc\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/7*b^2*d*x^7+1/5*(2*a*b*d+b^2*c)*x^5+1/3*(a^2*d+2*a*b*c)*x^3+a^2*c*x
```

maxima [A] time = 1.32, size = 48, normalized size = 0.96

$$\frac{1}{7}b^2dx^7 + \frac{1}{5}\left(b^2c + 2\,abd\right)x^5 + a^2cx + \frac{1}{3}\left(2\,abc + a^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d)
*x^3
```

mupad [B] time = 0.05, size = 48, normalized size = 0.96

$$x^3\left(\frac{d\,a^2}{3} + \frac{2\,b\,c\,a}{3}\right) + x^5\left(\frac{c\,b^2}{5} + \frac{2\,a\,d\,b}{5}\right) + \frac{b^2\,d\,x^7}{7} + a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^3*((a^2*d)/3 + (2*a*b*c)/3) + x^5*((b^2*c)/5 + (2*a*b*d)/5) + (b^2*d*x^7)
/7 + a^2*c*x
```

sympy [A] time = 0.07, size = 53, normalized size = 1.06

$$a^2cx + \frac{b^2dx^7}{7} + x^5\left(\frac{2abd}{5} + \frac{b^2c}{5}\right) + x^3\left(\frac{a^2d}{3} + \frac{2abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**7/7 + x**5*(2*a*b*d/5 + b**2*c/5) + x**3*(a**2*d/3 + 2
*a*b*c/3)
```

3.11 $\int \frac{(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=63

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^3}{3d}$$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {390, 205}

$$-\frac{bx(bc-2ad)}{d^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}+\frac{b^2x^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{c+dx^2}\,dx &= \int \left(-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^2}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2(c+dx^2)}\right)dx\\ &= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\int \frac{1}{c+dx^2}\,dx}{d^2}\\ &= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.94

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}+\frac{bx(6ad-3bc+bdx^2)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] (b*x*(-3*b*c + 6*a*d + b*d*x^2))/(3*d^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{c+dx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2), x]
```

fricas [A] time = 0.87, size = 179, normalized size = 2.84

$$\left[\frac{2\,b^2cd^2x^3-3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)-6\left(b^2c^2d-2\,abcd^2\right)x}{6\,cd^3},\ \frac{b^2cd^2x^3+3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-3\left(b^2c^2d-2\,abcd^2\right)x}{3\,cd^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b^2*c*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*log((d
*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c
*d^3), 1/3*(b^2*c*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*arc
tan(sqrt(c*d)*x/c) - 3*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c*d^3)]
```

giac [A] time = 0.59, size = 72, normalized size = 1.14

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^2}+\frac{b^2d^2x^3-3\,b^2cdx+6\,abd^2x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d^2*x^3 - 3*b^2*c*d*x + 6*a*b*d^2*x)/d^3
```

maple [A] time = 0.00, size = 95, normalized size = 1.51

$$\frac{b^2x^3}{3d}+\frac{a^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}-\frac{2abc\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d}+\frac{b^2c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^2}+\frac{2abx}{d}-\frac{b^2cx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/3*b^2*x^3/d+2*b/d*a*x-b^2/d^2*x*c+1/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)
*a^2-2/d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b*c+1/d^2/(c*d)^(1/2)*arct
an(1/(c*d)^(1/2)*d*x)*b^2*c^2
```

maxima [A] time = 2.89, size = 68, normalized size = 1.08

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^2}+\frac{b^2dx^3-3\left(b^2c-2\,abd\right)x}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d*x^3 - 3*(b^2*c - 2*a*b*d)*x)/d^2
```

mupad [B] time = 0.09, size = 90, normalized size = 1.43

$$\frac{b^2\,x^3}{3\,d}-x\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,{\left(a\,d-b\,c\right)}^2}{\sqrt{c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{\sqrt{c}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2),x)
```

```
[Out] (b^2*x^3)/(3*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((d^(1/2)*x*(a*d - b*c
)^2)/(c^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(c^(1/2)*d^(
5/2))
```

sympy [B] time = 0.42, size = 172, normalized size = 2.73

$$\frac{b^2x^3}{3d}+x\left(\frac{2ab}{d}-\frac{b^2c}{d^2}\right)-\frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(-\frac{cd^2\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(\frac{cd^2\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**3/(3*d) + x*(2*a*b/d - b**2*c/d**2) - sqrt(-1/(c*d**5))*(a*d - b*c)
**2*log(-c*d**2*sqrt(-1/(c*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b
**2*c**2) + x)/2 + sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(c*d**2*sqrt(-1/(c*d
**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
```

3.12 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2} \, dx$

Optimal. Leaf size=82

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x}{d^2}$$

Rubi [A] time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(3*b*c
 + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \int \left(\frac{b^2}{d^2} - \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{d^2\left(c+dx^2\right)^2}\right) dx$$

$$= \frac{b^2x}{d^2} - \frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{\left(c+dx^2\right)^2}\,dx}{d^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(3bc+ad))\int \frac{1}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)} - \frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

Mathematica [**A**] time = 0.06, size = 89, normalized size = 1.09

$$-\frac{\left(-a^2d^2-2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}} + \frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)} + \frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((3*b^2*c^2 - 2*a*b
*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2)^2, x]
```

fricas [**B**] time = 0.87, size = 302, normalized size = 3.68

$$\left[\frac{4\,b^2c^2d^2x^3 + \left(3\,b^2c^3 - 2\,abc^2d - a^2cd^2 + \left(3\,b^2c^2d - 2\,abcd^2 - a^2d^3\right)x^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2 - 2\,\sqrt{-cd}\,x - c}{dx^2+c}\right) + 2\left(3\,b^2c^3d - 2\,abc^2d^2 + a^2cd^3\right)x}{4\left(c^2d^4x^2 + c^3d^3\right)},\ \frac{2\,b^2c^2d^2x^3 - \left(3\,b^2c^3 - 2\,abc^2d - a^2cd^2 + \left(3\,b^2c^2d - 2\,abcd^2 - a^2d^3\right)x^2\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right) + \left(3\,b^2c^3d - 2\,abc^2d^2 + a^2cd^3\right)x}{2\left(c^2d^4x^2 + c^3d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b^2*c^2*d^2*x^3 + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2
*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c
)/(d*x^2 + c)) + 2*(3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^
2 + c^3*d^3), 1/2*(2*b^2*c^2*d^2*x^3 - (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/
c) + (3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^2 + c^3*d^3)]
```

giac [**A**] time = 0.57, size = 95, normalized size = 1.16

$$\frac{b^2x}{d^2} - \frac{\left(3\,b^2c^2 - 2\,abcd - a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,cd^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\left(dx^2+c\right)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sq
rt(c*d)*c*d^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c*d
^2)
```

maple [A] time = 0.01, size = 129, normalized size = 1.57

$$\frac{a^2 x}{2\left(d x^2+c\right) c}+\frac{a^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\, c}-\frac{abx}{\left(d x^2+c\right) d}+\frac{ab \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\, d}+\frac{b^2 c x}{2\left(d x^2+c\right) d^2}-\frac{3b^2 c \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\, d^2}+\frac{b^2 x}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] b^2*x/d^2+1/2/c*x/(d*x^2+c)*a^2-1/d*x/(d*x^2+c)*a*b+1/2/d^2*c*x/(d*x^2+c)*b
^2+1/2/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2+1/d/(c*d)^(1/2)*arctan(1
/(c*d)^(1/2)*d*x)*a*b-3/2/d^2*c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2
```

maxima [A] time = 2.84, size = 96, normalized size = 1.17

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)x}{2\left(cd^3x^2+c^2d^2\right)}+\frac{b^2x}{d^2}-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^2 + c^2*d^2) + b^2*x/d^2 - 1
/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d^2
)
```

mupad [B] time = 5.02, size = 124, normalized size = 1.51

$$\frac{b^2\,x}{d^2}+\frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c\,\left(d^3\,x^2+c\,d^2\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,\left(a\,d-b\,c\right)\,\left(a\,d+3\,b\,c\right)}{\sqrt{c}\,\left(a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+3\,b\,c\right)}{2\,c^{3/2}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c*d^2 + d^3*x^2)) +
 (atan((d^(1/2)*x*(a*d - b*c)*(a*d + 3*b*c))/(c^(1/2)*(a^2*d^2 - 3*b^2*c^2
+ 2*a*b*c*d)))*(a*d - b*c)*(a*d + 3*b*c))/(2*c^(3/2)*d^(5/2))
```

sympy [B] time = 0.70, size = 236, normalized size = 2.88

$$\frac{b^2x}{d^2}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{2c^2d^2+2cd^3x^2}-\frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)\,(ad+3bc)\log\left(-\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)\,(ad+3bc)\log\left(\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d**3
*x**2) - sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(-c**2*d**2*sqrt
(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*
c**2) + x)/4 + sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(c**2*d**2
*sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*
b**2*c**2) + x)/4
```

3.13 $\int \frac{(a+bx^2)^2}{(c+dx^2)^3} \, dx$

Optimal. Leaf size=116

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 385, 205}

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(4*c*d*(c + d*x^2)^2) + (3*(a^2/c^2 - b^2/d^2)
*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx &= -\frac{(bc-ad)x\left(a+bx^2\right)}{4cd\left(c+dx^2\right)^2}+\frac{\int \frac{a(bc+3ad)+b(3bc+ad)x^2}{\left(c+dx^2\right)^2}\,dx}{4cd} \\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{4cd\left(c+dx^2\right)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\left(c+dx^2\right)}+\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\int \frac{1}{c+dx^2}\,dx}{8c^2d^2} \\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{4cd\left(c+dx^2\right)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\left(c+dx^2\right)}+\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 121, normalized size = 1.04

$$
\frac{x\left(a^2d^2\left(5c+3dx^2\right)-2abcd\left(c-dx^2\right)-b^2c^2\left(3c+5dx^2\right)\right)}{8c^2d^2\left(c+dx^2\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] (x*(-2*a*b*c*d*(c - d*x^2) + a^2*d^2*(5*c + 3*d*x^2) - b^2*c^2*(3*c + 5*d*x
^2)))/(8*c^2*d^2*(c + d*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2)^3, x]
```

fricas [B] time = 0.75, size = 449, normalized size = 3.87

$$
\left[-\frac{2\left(5b^2c^3d^2-2abc^2d^3-3a^2cd^4\right)x^3+\left(3b^2c^4+2abc^3d+3a^2c^2d^2+\left(3b^2c^2d^2+2abcd^3+3a^2d^4\right)x^4+2\left(3b^2c^3d+2abc^2d^2+3a^2cd^3\right)x^2\right)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\left(3b^2c^4d+2abc^3d^2-5a^2c^2d^3\right)x}{16\left(c^3d^5x^4+2c^4d^4x^2+c^5d^3\right)},\ -\frac{\left(5b^2c^3d^2-2abc^2d^3-3a^2cd^4\right)x^3-\left(3b^2c^4+2abc^3d+3a^2c^2d^2+\left(3b^2c^2d^2+2abcd^3+3a^2d^4\right)x^4+2\left(3b^2c^3d+2abc^2d^2+3a^2cd^3\right)x^2\right)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)+\left(3b^2c^4d+2abc^3d^2-5a^2c^2d^3\right)x}{8\left(c^3d^5x^4+2c^4d^4x^2+c^5d^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 + (3*b^2*c^4 +
2*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4
 + 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2
 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(3*b^2*c^4*d + 2*a*b*c^3*d^2 - 5*a^
2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^3), -1/8*((5*b^2*c^3*d^2
 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 - (3*b^2*c^4 + 2*a*b*c^3*d + 3*a^2*c^2*
d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 2*a*
b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (3*b^2*c^4*
d + 2*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^
3)]
```

giac [A] time = 0.59, size = 126, normalized size = 1.09

$$\frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d^2}-\frac{5\,b^2c^2dx^3-2\,abcd^2x^3-3\,a^2d^3x^3+3\,b^2c^3x+2\,abc^2dx-5\,a^2cd^2x}{8\,\left(dx^2+c\right)^2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^
2*d^2) - 1/8*(5*b^2*c^2*d*x^3 - 2*a*b*c*d^2*x^3 - 3*a^2*d^3*x^3 + 3*b^2*c^3
*x + 2*a*b*c^2*d*x - 5*a^2*c*d^2*x)/((d*x^2 + c)^2*c^2*d^2)
```

maple [A] time = 0.01, size = 147, normalized size = 1.27

$$\frac{3a^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\;c^2}+\frac{ab\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\sqrt{cd}\;cd}+\frac{3b^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\;d^2}+\frac{\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8c\,d^2}}{\left(d\,x^2+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] (1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2/d*x^3+1/8*(5*a^2*d^2-2*a*b*c*d-3*b
^2*c^2)/d^2/c*x)/(d*x^2+c)^2+3/8/c^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*
a^2+1/4/c/d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b+3/8/d^2/(c*d)^(1/2)*a
rctan(1/(c*d)^(1/2)*d*x)*b^2
```

maxima [A] time = 3.08, size = 138, normalized size = 1.19

$$-\frac{\left(5\,b^2c^2d-2\,abcd^2-3\,a^2d^3\right)x^3+\left(3\,b^2c^3+2\,abc^2d-5\,a^2cd^2\right)x}{8\,\left(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2\right)}+\frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (3*b^2*c^3 + 2*a*b*c^2*
d - 5*a^2*c*d^2)*x)/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) + 1/8*(3*b^2*c^
2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^2)
```

mupad [B] time = 5.03, size = 130, normalized size = 1.12

$$\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c^{5/2}\,d^{5/2}}-\frac{\frac{x\,\left(-5\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c\,d^2}-\frac{x^3\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d-5\,b^2\,c^2\right)}{8\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^3,x)
```

```
[Out] (atan((d^(1/2)*x)/c^(1/2))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(8*c^(5/2)*
d^(5/2)) - ((x*(3*b^2*c^2 - 5*a^2*d^2 + 2*a*b*c*d))/(8*c*d^2) - (x^3*(3*a^2
*d^2 - 5*b^2*c^2 + 2*a*b*c*d))/(8*c^2*d))/(c^2 + d^2*x^4 + 2*c*d*x^2)
```

sympy [B] time = 0.98, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{c^5d^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(-c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^5d^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{x^3\left(3a^2d^3+2abcd^2-5b^2c^2d\right)+x\left(5a^2cd^2-2abc^2d-3b^2c^3\right)}{8c^4d^2+16c^3d^3x^2+8c^2d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-c**3*d**
2*sqrt(-1/(c**5*d**5)) + x)/16 + sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*
c*d + 3*b**2*c**2)*log(c**3*d**2*sqrt(-1/(c**5*d**5)) + x)/16 + (x**3*(3*a*
*2*d**3 + 2*a*b*c*d**2 - 5*b**2*c**2*d) + x*(5*a**2*c*d**2 - 2*a*b*c**2*d -
 3*b**2*c**3))/(8*c**4*d**2 + 16*c**3*d**3*x**2 + 8*c**2*d**4*x**4)
```

3.14 $\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=154

$$a^3c^3x+\frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{7}x^7(ad+bc)\left(a^2d^2+8abcd+b^2c^2\right)+\frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right)+a^2c^2x$$

Rubi [A] time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{7}x^7(ad+bc)\left(a^2d^2+8abcd+b^2c^2\right)+\frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right)+a^2c^2x^3(ad+bc)+a^3c^3x+\frac{3}{11}b^2d^2x^{11}(ad+bc)+\frac{1}{13}b^3d^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2
)*x^5)/5 + ((b*c + a*d)*(b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d*(b^2*
c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^2*(b*c + a*d)*x^11)/11 + (b^3*
d^3*x^13)/13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^3\left(c+dx^2\right)^3\,dx=\int \left(a^3c^3+3a^2c^2(bc+ad)x^2+3ac\left(b^2c^2+3abcd+a^2d^2\right)x^4+(bc+ad)\left(b^2c^2+\right.\right.$$
$$=a^3c^3x+a^2c^2(bc+ad)x^3+\frac{3}{5}ac\left(b^2c^2+3abcd+a^2d^2\right)x^5+\frac{1}{7}(bc+ad)\left(b^2c^2+8a\right.$$

Mathematica [A] time = 0.03, size = 161, normalized size = 1.05

$$a^3c^3x+\frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right)+\frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right)+a^2c^2x^3(ad+bc)+\frac{1}{7}x^7\left(a^3d^3+9a^2bcd^2+9ab^2c^2d+b^3c^3\right)+\frac{3}{11}b^2d^2x^{11}(ad+bc)+\frac{1}{13}b^3d^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2
)*x^5)/5 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^7)/7 + (b
*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^2*(b*c + a*d)*x^11)/11
 + (b^3*d^3*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^3\left(c+dx^2\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^3*(c + d*x^2)^3, x]
```

fricas [A] time = 0.66, size = 187, normalized size = 1.21

$$\frac{1}{13}x^{13}d^3b^3+\frac{3}{11}x^{11}d^2cb^3+\frac{3}{11}x^{11}d^3b^2a+\frac{1}{3}x^9dc^2b^3+x^9d^2cb^2a+\frac{1}{3}x^9d^3ba^2+\frac{1}{7}x^7c^3b^3+\frac{9}{7}x^7dc^2b^2a+\frac{9}{7}x^7d^2cba^2+\frac{1}{7}x^7d^3a^3+\frac{3}{5}x^5c^3b^2a+\frac{9}{5}x^5dc^2ba^2+\frac{3}{5}x^5d^2ca^3+x^3c^3ba^2+x^3dc^2a^3+xc^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^3*b^3 + 3/11*x^11*d^2*c*b^3 + 3/11*x^11*d^3*b^2*a + 1/3*x^9*d*c
^2*b^3 + x^9*d^2*c*b^2*a + 1/3*x^9*d^3*b*a^2 + 1/7*x^7*c^3*b^3 + 9/7*x^7*d*
c^2*b^2*a + 9/7*x^7*d^2*c*b*a^2 + 1/7*x^7*d^3*a^3 + 3/5*x^5*c^3*b^2*a + 9/5
*x^5*d*c^2*b*a^2 + 3/5*x^5*d^2*c*a^3 + x^3*c^3*b*a^2 + x^3*d*c^2*a^3 + x*c^
3*a^3
```

giac [A] time = 0.57, size = 187, normalized size = 1.21

$$\frac{1}{13}b^3d^3x^{13}+\frac{3}{11}b^3cd^2x^{11}+\frac{3}{11}ab^2d^3x^{11}+\frac{1}{3}b^3c^2dx^9+ab^2cd^2x^9+\frac{1}{3}a^2bd^3x^9+\frac{1}{7}b^3c^3x^7+\frac{9}{7}ab^2c^2dx^7+\frac{9}{7}a^2bcd^2x^7+\frac{1}{7}a^3d^3x^7+\frac{3}{5}ab^2c^3x^5+\frac{9}{5}a^2bc^2dx^5+\frac{3}{5}a^3cd^2x^5+a^2bc^3x^3+a^3c^2dx^3+a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*b^3*c*d^2*x^11 + 3/11*a*b^2*d^3*x^11 + 1/3*b^3*c^2
*d*x^9 + a*b^2*c*d^2*x^9 + 1/3*a^2*b*d^3*x^9 + 1/7*b^3*c^3*x^7 + 9/7*a*b^2*
c^2*d*x^7 + 9/7*a^2*b*c*d^2*x^7 + 1/7*a^3*d^3*x^7 + 3/5*a*b^2*c^3*x^5 + 9/5
*a^2*b*c^2*d*x^5 + 3/5*a^3*c*d^2*x^5 + a^2*b*c^3*x^3 + a^3*c^2*d*x^3 + a^3*
c^3*x
```

maple [A] time = 0.00, size = 177, normalized size = 1.15

$$\frac{b^3d^3x^{13}}{13}+\frac{\left(3a\,b^2d^3+3b^3c\,d^2\right)x^{11}}{11}+\frac{\left(3a^2b\,d^3+9a\,b^2c\,d^2+3b^3c^2d\right)x^9}{9}+a^3c^3x+\frac{\left(a^3d^3+9a^2bc\,d^2+9a\,b^2c^2d+b^3c^3\right)x^7}{7}+\frac{\left(3a^3c\,d^2+9a^2b\,c^2d+3a\,b^2c^3\right)x^5}{5}+\frac{\left(3a^3c^2d+3a^2b\,c^3\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(d*x^2+c)^3,x)
```

```
[Out] 1/13*b^3*d^3*x^13+1/11*(3*a*b^2*d^3+3*b^3*c*d^2)*x^11+1/9*(3*a^2*b*d^3+9*a*
b^2*c*d^2+3*b^3*c^2*d)*x^9+1/7*(a^3*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3
)*x^7+1/5*(3*a^3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*c^3)*x^5+1/3*(3*a^3*c^2*d+3*a^
2*b*c^3)*x^3+a^3*c^3*x
```

maxima [A] time = 1.38, size = 167, normalized size = 1.08

$$\frac{1}{13}b^3d^3x^{13}+\frac{3}{11}\left(b^3cd^2+ab^2d^3\right)x^{11}+\frac{1}{3}\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x^9+\frac{1}{7}\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)x^7+a^3c^3x+\frac{3}{5}\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)x^5+\left(a^2bc^3+a^3c^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*(b^3*c*d^2 + a*b^2*d^3)*x^11 + 1/3*(b^3*c^2*d + 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^9 + 1/7*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2
 + a^3*d^3)*x^7 + a^3*c^3*x + 3/5*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x
^5 + (a^2*b*c^3 + a^3*c^2*d)*x^3
```

mupad [B] time = 4.90, size = 152, normalized size = 0.99

$$x^7\left(\frac{a^3\,d^3}{7}+\frac{9\,a^2\,b\,c\,d^2}{7}+\frac{9\,a\,b^2\,c^2\,d}{7}+\frac{b^3\,c^3}{7}\right)+a^3\,c^3\,x+\frac{b^3\,d^3\,x^{13}}{13}+\frac{3\,a\,c\,x^5\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{5}+\frac{b\,d\,x^9\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{3}+a^2\,c^2\,x^3\,\left(a\,d+b\,c\right)+\frac{3\,b^2\,d^2\,x^{11}\,\left(a\,d+b\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3*(c + d*x^2)^3,x)
```

```
[Out] x^7*((a^3*d^3)/7 + (b^3*c^3)/7 + (9*a*b^2*c^2*d)/7 + (9*a^2*b*c*d^2)/7) + a
^3*c^3*x + (b^3*d^3*x^13)/13 + (3*a*c*x^5*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/
```

```
5 + (b*d*x^9*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/3 + a^2*c^2*x^3*(a*d + b*c) +
 (3*b^2*d^2*x^11*(a*d + b*c))/11
```

sympy [A]   time = 0.10, size = 189, normalized size = 1.23

$$a^3c^3x + \frac{b^3d^3x^{13}}{13} + x^{11}\left(\frac{3ab^2d^3}{11} + \frac{3b^3cd^2}{11}\right) + x^9\left(\frac{a^2bd^3}{3} + ab^2cd^2 + \frac{b^3c^2d}{3}\right) + x^7\left(\frac{a^3d^3}{7} + \frac{9a^2bcd^2}{7} + \frac{9ab^2c^2d}{7} + \frac{b^3c^3}{7}\right) + x^5\left(\frac{3a^3cd^2}{5} + \frac{9a^2bc^2d}{5} + \frac{3ab^2c^3}{5}\right) + x^3\left(a^3c^2d + a^2bc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(d*x**2+c)**3,x)
```

```
[Out] a**3*c**3*x + b**3*d**3*x**13/13 + x**11*(3*a*b**2*d**3/11 + 3*b**3*c*d**2/
11) + x**9*(a**2*b*d**3/3 + a*b**2*c*d**2 + b**3*c**2*d/3) + x**7*(a**3*d**
3/7 + 9*a**2*b*c*d**2/7 + 9*a*b**2*c**2*d/7 + b**3*c**3/7) + x**5*(3*a**3*c
*d**2/5 + 9*a**2*b*c**2*d/5 + 3*a*b**2*c**3/5) + x**3*(a**3*c**2*d + a**2*b
*c**3)
```

3.15 $\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=122

$$a^3c^2x+\frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}a^2cx^3(2ad+3bc)+\frac{1}{9}b^2dx^9(3ad+2bc)+\frac{1}{11}b^3d$$

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}a^2cx^3(2ad+3bc)+a^3c^2x+\frac{1}{9}b^2dx^9(3ad+2bc)+\frac{1}{11}b^3d^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (b^2*d*(2*b*c
+ 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^3\left(c+dx^2\right)^2\,dx &= \int \left(a^3c^2+a^2c(3bc+2ad)x^2+a\left(3b^2c^2+6abcd+a^2d^2\right)x^4+b\left(b^2c^2+6abcd+3a^2\right.\right.\\ &= a^3c^2x+\frac{1}{3}a^2c(3bc+2ad)x^3+\frac{1}{5}a\left(3b^2c^2+6abcd+a^2d^2\right)x^5+\frac{1}{7}b\left(b^2c^2+6abcd+\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 122, normalized size = 1.00

$$a^3c^2x+\frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}a^2cx^3(2ad+3bc)+\frac{1}{9}b^2dx^9(3ad+2bc)+\frac{1}{11}b^3d^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (b^2*d*(2*b*c
+ 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^3\left(c+dx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^3*(c + d*x^2)^2, x]
```

fricas [A] time = 0.74, size = 131, normalized size = 1.07

$$\frac{1}{11}x^{11}d^2b^3+\frac{2}{9}x^9dcb^3+\frac{1}{3}x^9d^2b^2a+\frac{1}{7}x^7c^2b^3+\frac{6}{7}x^7dcb^2a+\frac{3}{7}x^7d^2ba^2+\frac{3}{5}x^5c^2b^2a+\frac{6}{5}x^5dcba^2+\frac{1}{5}x^5d^2a^3+x^3c^2ba^2+\frac{2}{3}x^3dca^3+xc^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^2*b^3 + 2/9*x^9*d*c*b^3 + 1/3*x^9*d^2*b^2*a + 1/7*x^7*c^2*b^3 +
 6/7*x^7*d*c*b^2*a + 3/7*x^7*d^2*b*a^2 + 3/5*x^5*c^2*b^2*a + 6/5*x^5*d*c*b*
a^2 + 1/5*x^5*d^2*a^3 + x^3*c^2*b*a^2 + 2/3*x^3*d*c*a^3 + x*c^2*a^3
```

giac [A] time = 0.59, size = 131, normalized size = 1.07

$$\frac{1}{11}b^3d^2x^{11}+\frac{2}{9}b^3cdx^9+\frac{1}{3}ab^2d^2x^9+\frac{1}{7}b^3c^2x^7+\frac{6}{7}ab^2cdx^7+\frac{3}{7}a^2bd^2x^7+\frac{3}{5}ab^2c^2x^5+\frac{6}{5}a^2bcdx^5+\frac{1}{5}a^3d^2x^5+a^2bc^2x^3+\frac{2}{3}a^3cdx^3+a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^3*d^2*x^11 + 2/9*b^3*c*d*x^9 + 1/3*a*b^2*d^2*x^9 + 1/7*b^3*c^2*x^7 +
 6/7*a*b^2*c*d*x^7 + 3/7*a^2*b*d^2*x^7 + 3/5*a*b^2*c^2*x^5 + 6/5*a^2*b*c*d*
x^5 + 1/5*a^3*d^2*x^5 + a^2*b*c^2*x^3 + 2/3*a^3*c*d*x^3 + a^3*c^2*x
```

maple [A] time = 0.00, size = 125, normalized size = 1.02

$$\frac{b^3d^2x^{11}}{11}+\frac{\left(3a\,b^2d^2+2b^3cd\right)x^9}{9}+\frac{\left(3a^2b\,d^2+6a\,b^2cd+b^3c^2\right)x^7}{7}+a^3c^2x+\frac{\left(a^3d^2+6a^2bcd+3a\,b^2c^2\right)x^5}{5}+\frac{\left(2a^3cd+3a^2b\,c^2\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(d*x^2+c)^2,x)
```

```
[Out] 1/11*b^3*d^2*x^11+1/9*(3*a*b^2*d^2+2*b^3*c*d)*x^9+1/7*(3*a^2*b*d^2+6*a*b^2*
c*d+b^3*c^2)*x^7+1/5*(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2)*x^5+1/3*(2*a^3*c*d+3
*a^2*b*c^2)*x^3+a^3*c^2*x
```

maxima [A] time = 1.36, size = 124, normalized size = 1.02

$$\frac{1}{11}b^3d^2x^{11}+\frac{1}{9}\left(2\,b^3cd+3\,ab^2d^2\right)x^9+\frac{1}{7}\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^7+a^3c^2x+\frac{1}{5}\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^5+\frac{1}{3}\left(3\,a^2bc^2+2\,a^3cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d^2*x^11 + 1/9*(2*b^3*c*d + 3*a*b^2*d^2)*x^9 + 1/7*(b^3*c^2 + 6*a*
b^2*c*d + 3*a^2*b*d^2)*x^7 + a^3*c^2*x + 1/5*(3*a*b^2*c^2 + 6*a^2*b*c*d + a
^3*d^2)*x^5 + 1/3*(3*a^2*b*c^2 + 2*a^3*c*d)*x^3
```

mupad [B] time = 4.87, size = 116, normalized size = 0.95

$$x^5\left(\frac{a^3\,d^2}{5}+\frac{6\,a^2\,b\,c\,d}{5}+\frac{3\,a\,b^2\,c^2}{5}\right)+x^7\left(\frac{3\,a^2\,b\,d^2}{7}+\frac{6\,a\,b^2\,c\,d}{7}+\frac{b^3\,c^2}{7}\right)+a^3\,c^2\,x+\frac{b^3\,d^2\,x^{11}}{11}+\frac{a^2\,c\,x^3\,\left(2\,a\,d+3\,b\,c\right)}{3}+\frac{b^2\,d\,x^9\,\left(3\,a\,d+2\,b\,c\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3*(c + d*x^2)^2,x)
```

```
[Out] x^5*((a^3*d^2)/5 + (3*a*b^2*c^2)/5 + (6*a^2*b*c*d)/5) + x^7*((b^3*c^2)/7 +
(3*a^2*b*d^2)/7 + (6*a*b^2*c*d)/7) + a^3*c^2*x + (b^3*d^2*x^11)/11 + (a^2*c
*x^3*(2*a*d + 3*b*c))/3 + (b^2*d*x^9*(3*a*d + 2*b*c))/9
```

sympy [A] time = 0.09, size = 136, normalized size = 1.11

$$a^3c^2x+\frac{b^3d^2x^{11}}{11}+x^9\left(\frac{ab^2d^2}{3}+\frac{2b^3cd}{9}\right)+x^7\left(\frac{3a^2bd^2}{7}+\frac{6ab^2cd}{7}+\frac{b^3c^2}{7}\right)+x^5\left(\frac{a^3d^2}{5}+\frac{6a^2bcd}{5}+\frac{3ab^2c^2}{5}\right)+x^3\left(\frac{2a^3cd}{3}+a^2bc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(d*x**2+c)**2,x)
```

```
[Out] a**3*c**2*x + b**3*d**2*x**11/11 + x**9*(a*b**2*d**2/3 + 2*b**3*c*d/9) + x*
*7*(3*a**2*b*d**2/7 + 6*a*b**2*c*d/7 + b**3*c**2/7) + x**5*(a**3*d**2/5 + 6
*a**2*b*c*d/5 + 3*a*b**2*c**2/5) + x**3*(2*a**3*c*d/3 + a**2*b*c**2)
```

3.16 $\int \left(a+bx^2\right)^3 \left(c+dx^2\right)\, dx$

Optimal. Leaf size=70

$$a^3cx+\frac{1}{3}a^2x^3(ad+3bc)+\frac{1}{7}b^2x^7(3ad+bc)+\frac{3}{5}abx^5(ad+bc)+\frac{1}{9}b^3dx^9$$

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{3}a^2x^3(ad+3bc)+a^3cx+\frac{1}{7}b^2x^7(3ad+bc)+\frac{3}{5}abx^5(ad+bc)+\frac{1}{9}b^3dx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(c + d*x^2),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 + (b^2*(b*c
 + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^3 \left(c+dx^2\right) dx &= \int \left(a^3c+a^2(3bc+ad)x^2+3ab(bc+ad)x^4+b^2(bc+3ad)x^6+b^3dx^8\right) dx\\ &= a^3cx+\frac{1}{3}a^2(3bc+ad)x^3+\frac{3}{5}ab(bc+ad)x^5+\frac{1}{7}b^2(bc+3ad)x^7+\frac{1}{9}b^3dx^9\end{aligned}$$

Mathematica [A] time = 0.01, size = 70, normalized size = 1.00

$$a^3cx+\frac{1}{3}a^2x^3(ad+3bc)+\frac{1}{7}b^2x^7(3ad+bc)+\frac{3}{5}abx^5(ad+bc)+\frac{1}{9}b^3dx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 + (b^2*(b*c
 + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3*(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^3*(c + d*x^2), x]
```

fricas [A] time = 0.49, size = 73, normalized size = 1.04

$$\frac{1}{9}x^9db^3+\frac{1}{7}x^7cb^3+\frac{3}{7}x^7db^2a+\frac{3}{5}x^5cb^2a+\frac{3}{5}x^5dba^2+x^3cba^2+\frac{1}{3}x^3da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d*b^3 + 1/7*x^7*c*b^3 + 3/7*x^7*d*b^2*a + 3/5*x^5*c*b^2*a + 3/5*x^5
*d*b*a^2 + x^3*c*b*a^2 + 1/3*x^3*d*a^3 + x*c*a^3
```

giac [A] time = 0.56, size = 73, normalized size = 1.04

$$\frac{1}{9}b^3dx^9 + \frac{1}{7}b^3cx^7 + \frac{3}{7}ab^2dx^7 + \frac{3}{5}ab^2cx^5 + \frac{3}{5}a^2bdx^5 + a^2bcx^3 + \frac{1}{3}a^3dx^3 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/9*b^3*d*x^9 + 1/7*b^3*c*x^7 + 3/7*a*b^2*d*x^7 + 3/5*a*b^2*c*x^5 + 3/5*a^2
*b*d*x^5 + a^2*b*c*x^3 + 1/3*a^3*d*x^3 + a^3*c*x
```

maple [A] time = 0.00, size = 73, normalized size = 1.04

$$\frac{b^3d\,x^9}{9} + \frac{\left(3a\,b^2d + b^3c\right)x^7}{7} + a^3cx + \frac{\left(3a^2bd + 3a\,b^2c\right)x^5}{5} + \frac{\left(a^3d + 3a^2bc\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(d*x^2+c),x)
```

```
[Out] 1/9*b^3*d*x^9+1/7*(3*a*b^2*d+b^3*c)*x^7+1/5*(3*a^2*b*d+3*a*b^2*c)*x^5+1/3*(
a^3*d+3*a^2*b*c)*x^3+c*a^3*x
```

maxima [A] time = 1.29, size = 70, normalized size = 1.00

$$\frac{1}{9}b^3dx^9 + \frac{1}{7}\left(b^3c + 3\,ab^2d\right)x^7 + \frac{3}{5}\left(ab^2c + a^2bd\right)x^5 + a^3cx + \frac{1}{3}\left(3\,a^2bc + a^3d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*d*x^9 + 1/7*(b^3*c + 3*a*b^2*d)*x^7 + 3/5*(a*b^2*c + a^2*b*d)*x^5 +
 a^3*c*x + 1/3*(3*a^2*b*c + a^3*d)*x^3
```

mupad [B] time = 0.03, size = 65, normalized size = 0.93

$$x^7\left(\frac{c\,b^3}{7} + \frac{3\,a\,d\,b^2}{7}\right) + x^3\left(\frac{d\,a^3}{3} + b\,c\,a^2\right) + \frac{b^3\,d\,x^9}{9} + a^3\,c\,x + \frac{3\,a\,b\,x^5\,\left(a\,d + b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3*(c + d*x^2),x)
```

```
[Out] x^7*((b^3*c)/7 + (3*a*b^2*d)/7) + x^3*((a^3*d)/3 + a^2*b*c) + (b^3*d*x^9)/9
 + a^3*c*x + (3*a*b*x^5*(a*d + b*c))/5
```

sympy [A] time = 0.08, size = 76, normalized size = 1.09

$$a^3cx + \frac{b^3dx^9}{9} + x^7\left(\frac{3ab^2d}{7} + \frac{b^3c}{7}\right) + x^5\left(\frac{3a^2bd}{5} + \frac{3ab^2c}{5}\right) + x^3\left(\frac{a^3d}{3} + a^2bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(d*x**2+c),x)
```

```
[Out] a**3*c*x + b**3*d*x**9/9 + x**7*(3*a*b**2*d/7 + b**3*c/7) + x**5*(3*a**2*b*
d/5 + 3*a*b**2*c/5) + x**3*(a**3*d/3 + a**2*b*c)
```

3.17 $\int \frac{(a+bx^2)^3}{c+dx^2}\,dx$

Optimal. Leaf size=98

$$\frac{bx\left(3a^2d^2-3abcd+b^2c^2\right)}{d^3}-\frac{b^2x^3(bc-3ad)}{3d^2}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{7/2}}+\frac{b^3x^5}{5d}$$

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$\frac{bx\left(3a^2d^2-3abcd+b^2c^2\right)}{d^3}-\frac{b^2x^3(bc-3ad)}{3d^2}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{7/2}}+\frac{b^3x^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2),x]
```

```
[Out] (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x)/d^3 - (b^2*(b*c - 3*a*d)*x^3)/(3*d^
2) + (b^3*x^5)/(5*d) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]
*d^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^3}{c+dx^2}\,dx &= \int \left(\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)}{d^3}-\frac{b^2(bc-3ad)x^2}{d^2}+\frac{b^3x^4}{d}+\frac{-b^3c^3+3ab^2c^2d-3a^2bcd^2+a^3}{d^3\left(c+dx^2\right)}\right.\\ &=\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x}{d^3}-\frac{b^2(bc-3ad)x^3}{3d^2}+\frac{b^3x^5}{5d}-\frac{(bc-ad)^3\int\frac{1}{c+dx^2}\,dx}{d^3}\\ &=\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x}{d^3}-\frac{b^2(bc-3ad)x^3}{3d^2}+\frac{b^3x^5}{5d}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 93, normalized size = 0.95

$$\frac{bx\left(45a^2d^2+15abd\left(dx^2-3c\right)+b^2\left(15c^2-5cdx^2+3d^2x^4\right)\right)}{15d^3}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2),x]
```

```
[Out] (b*x*(45*a^2*d^2 + 15*a*b*d*(-3*c + d*x^2) + b^2*(15*c^2 - 5*c*d*x^2 + 3*d^
2*x^4)))/(15*d^3) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^
(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3}{c+dx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^3/(c + d*x^2), x]
```

fricas [A] time = 0.57, size = 290, normalized size = 2.96

$$\left[\frac{6\,b^3cd^3x^5-10\left(b^3c^2d^2-3\,ab^2cd^3\right)x^3+15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)+30\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3\right)x}{30\,cd^4},\ \frac{3\,b^3cd^3x^5-5\left(b^3c^2d^2-3\,ab^2cd^3\right)x^3-15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)+15\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3\right)x}{15\,cd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^3*c*d^3*x^5 - 10*(b^3*c^2*d^2 - 3*a*b^2*c*d^3)*x^3 + 15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(
-c*d)*x - c)/(d*x^2 + c)) + 30*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
)*x)/(c*d^4), 1/15*(3*b^3*c*d^3*x^5 - 5*(b^3*c^2*d^2 - 3*a*b^2*c*d^3)*x^3 -
 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)*arctan(sq
rt(c*d)*x/c) + 15*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3)*x)/(c*d^4)]
```

giac [A] time = 0.58, size = 130, normalized size = 1.33

$$-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^3}+\frac{3\,b^3d^4x^5-5\,b^3cd^3x^3+15\,ab^2d^4x^3+15\,b^3c^2d^2x-45\,ab^2cd^3x+45\,a^2bd^4x}{15\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))/
(sqrt(c*d)*d^3) + 1/15*(3*b^3*d^4*x^5 - 5*b^3*c*d^3*x^3 + 15*a*b^2*d^4*x^3
+ 15*b^3*c^2*d^2*x - 45*a*b^2*c*d^3*x + 45*a^2*b*d^4*x)/d^5
```

maple [A] time = 0.00, size = 161, normalized size = 1.64

$$\frac{b^3x^5}{5d}+\frac{a\,b^2x^3}{d}-\frac{b^3c\,x^3}{3d^2}+\frac{a^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}-\frac{3a^2bc\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d}+\frac{3a\,b^2c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d^2}-\frac{b^3c^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d^3}+\frac{3a^2bx}{d}-\frac{3a\,b^2cx}{d^2}+\frac{b^3c^2x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] 1/5*b^3*x^5/d+b^2/d*x^3*a-1/3*b^3/d^2*x^3*c+3*b/d*a^2*x-3*b^2/d^2*a*c*x+b^3
/d^3*c^2*x+1/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^3-3/d/(c*d)^(1/2)*arct
an(1/(c*d)^(1/2)*d*x)*a^2*b*c+3/d^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a
*b^2*c^2-1/d^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^3*c^3
```

maxima [A] time = 3.02, size = 122, normalized size = 1.24

$$-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^3}+\frac{3\,b^3d^2x^5-5\left(b^3cd-3\,ab^2d^2\right)x^3+15\left(b^3c^2-3\,ab^2cd+3\,a^2bd^2\right)x}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))/
(sqrt(c*d)*d^3) + 1/15*(3*b^3*d^2*x^5 - 5*(b^3*c*d - 3*a*b^2*d^2)*x^3 + 15*
(b^3*c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*x)/d^3
```

mupad [B] time = 4.87, size = 145, normalized size = 1.48

$$x^3\left(\frac{a\,b^2}{d}-\frac{b^3\,c}{3\,d^2}\right)+x\left(\frac{3\,a^2\,b}{d}-\frac{c\left(\frac{3\,a\,b^2}{d}-\frac{b^3\,c}{d^2}\right)}{d}\right)+\frac{b^3\,x^5}{5\,d}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)^3}{\sqrt{c}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)(a\,d-b\,c)^3}{\sqrt{c}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/(c + d*x^2),x)
```

```
[Out] x^3*((a*b^2)/d - (b^3*c)/(3*d^2)) + x*((3*a^2*b)/d - (c*((3*a*b^2)/d - (b^3
*c)/d^2))/d) + (b^3*x^5)/(5*d) + (atan((d^(1/2)*x*(a*d - b*c)^3)/(c^(1/2)*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(c^(1/2
)*d^(7/2))
```

sympy [B] time = 0.57, size = 238, normalized size = 2.43

$$\frac{b^3x^5}{5d}+x^3\left(\frac{ab^2}{d}-\frac{b^3c}{3d^2}\right)+x\left(\frac{3a^2b}{d}-\frac{3ab^2c}{d^2}+\frac{b^3c^2}{d^3}\right)-\frac{\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3\log\left(-\frac{cd^3\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3\log\left(\frac{cd^3\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] b**3*x**5/(5*d) + x**3*(a*b**2/d - b**3*c/(3*d**2)) + x*(3*a**2*b/d - 3*a*b
**2*c/d**2 + b**3*c**2/d**3) - sqrt(-1/(c*d**7))*(a*d - b*c)**3*log(-c*d**3
*sqrt(-1/(c*d**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c
**2*d - b**3*c**3) + x)/2 + sqrt(-1/(c*d**7))*(a*d - b*c)**3*log(c*d**3*sqr
t(-1/(c*d**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*
d - b**3*c**3) + x)/2
```

3.18 $\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(ad+5bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}-\frac{x(bc-ad)^3}{2cd^3\left(c+dx^2\right)}+\frac{b^3x^3}{3d^2}$$

Rubi [A] time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(ad+5bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}-\frac{x(bc-ad)^3}{2cd^3\left(c+dx^2\right)}+\frac{b^3x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)^3*x)/(2*c
*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*c^(3/2)*d^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^2}\,dx = \int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{b^3x^2}{d^2}+\frac{(bc-ad)^2(2bc+ad)+3bd(bc-ad)^2x^2}{d^3\left(c+dx^2\right)^2}\right)dx$$

$$= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}+\frac{\int \frac{(bc-ad)^2(2bc+ad)+3bd(bc-ad)^2x^2}{\left(c+dx^2\right)^2}\,dx}{d^3}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3\left(c+dx^2\right)}+\frac{\left((bc-ad)^2(5bc+ad)\right)\int \frac{1}{c+dx^2}\,dx}{2cd^3}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3\left(c+dx^2\right)}+\frac{(bc-ad)^2(5bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}$$

Mathematica [A] time = 0.06, size = 107, normalized size = 1.00

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(ad+5bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}-\frac{x(bc-ad)^3}{2cd^3\left(c+dx^2\right)}+\frac{b^3x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)^3*x)/(2*c
*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*c^(3/2)*d^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^3/(c + d*x^2)^2, x]
```

fricas [B] time = 0.82, size = 444, normalized size = 4.15

$$\left[\frac{4\,b^3c^2d^3x^5-4\left(5\,b^3c^3d^2-9\,ab^2c^2d^3\right)x^3-3\left(5\,b^3c^4-9\,ab^2c^3d+3\,a^2bc^2d^2+a^3cd^3+\left(5\,b^3c^3d-9\,ab^2c^2d^2+3\,a^2bcd^3+a^3d^4\right)x^2\right)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)-6\left(5\,b^3c^4d-9\,ab^2c^3d^2+3\,a^2bc^2d^3-a^3cd^4\right)x}{12\left(c^2d^5x^2+c^3d^4\right)},\ \frac{2\,b^3c^2d^3x^5-2\left(5\,b^3c^3d^2-9\,ab^2c^2d^3\right)x^3+3\left(5\,b^3c^4-9\,ab^2c^3d+3\,a^2bc^2d^2+a^3cd^3+\left(5\,b^3c^3d-9\,ab^2c^2d^2+3\,a^2bcd^3+a^3d^4\right)x^2\right)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-3\left(5\,b^3c^4d-9\,ab^2c^3d^2+3\,a^2bc^2d^3-a^3cd^4\right)x}{6\left(c^2d^5x^2+c^3d^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^3*c^2*d^3*x^5 - 4*(5*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3)*x^3 - 3*(5*b
^3*c^4 - 9*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 + a^3*c*d^3 + (5*b^3*c^3*d - 9*a*b
^2*c^2*d^2 + 3*a^2*b*c*d^3 + a^3*d^4)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-
c*d)*x - c)/(d*x^2 + c)) - 6*(5*b^3*c^4*d - 9*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d
^3 - a^3*c*d^4)*x)/(c^2*d^5*x^2 + c^3*d^4), 1/6*(2*b^3*c^2*d^3*x^5 - 2*(5*b
^3*c^3*d^2 - 9*a*b^2*c^2*d^3)*x^3 + 3*(5*b^3*c^4 - 9*a*b^2*c^3*d + 3*a^2*b*
c^2*d^2 + a^3*c*d^3 + (5*b^3*c^3*d - 9*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + a^3*
d^4)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - 3*(5*b^3*c^4*d - 9*a*b^2*c^3*d^
2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)/(c^2*d^5*x^2 + c^3*d^4)]
```

giac [A] time = 0.58, size = 152, normalized size = 1.42

$$\frac{\left(5\,b^3c^3 - 9\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,cd^3} - \frac{b^3c^3x - 3\,ab^2c^2dx + 3\,a^2bcd^2x - a^3d^3x}{2\left(dx^2+c\right)cd^3} + \frac{b^3d^4x^3 - 6\,b^3cd^3x + 9\,ab^2d^4x}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(d*x/sqrt(c
*d))/(sqrt(c*d)*c*d^3) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((d*x^2 + c)*c*d^3) + 1/3*(b^3*d^4*x^3 - 6*b^3*c*d^3*x + 9*a*
b^2*d^4*x)/d^6
```

maple [B] time = 0.01, size = 205, normalized size = 1.92

$$\frac{b^3x^3}{3d^2} + \frac{a^3x}{2\left(d\,x^2+c\right)c} + \frac{a^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\;c} - \frac{3a^2bx}{2\left(d\,x^2+c\right)d} + \frac{3a^2b\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\;d} + \frac{3a\,b^2cx}{2\left(d\,x^2+c\right)d^2} - \frac{9a\,b^2c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\;d^2} - \frac{b^3c^2x}{2\left(d\,x^2+c\right)d^3} + \frac{5b^3c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\;d^3} + \frac{3a\,b^2x}{d^2} - \frac{2b^3cx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c)^2,x)
```

```
[Out] 1/3*b^3*x^3/d^2+3*b^2/d^2*a*x-2*b^3/d^3*x*c+1/2/c*x/(d*x^2+c)*a^3-3/2/d*x/(
d*x^2+c)*a^2*b+3/2/d^2*c*x/(d*x^2+c)*a*b^2-1/2/d^3*c^2*x/(d*x^2+c)*b^3+1/2/
c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^3+3/2/d/(c*d)^(1/2)*arctan(1/(c*d
)^(1/2)*d*x)*a^2*b-9/2/d^2*c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b^2+5/
2/d^3*c^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^3
```

maxima [A] time = 2.99, size = 147, normalized size = 1.37

$$-\frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)x}{2\left(cd^4x^2 + c^2d^3\right)} + \frac{b^3dx^3 - 3\left(2\,b^3c - 3\,ab^2d\right)x}{3\,d^3} + \frac{\left(5\,b^3c^3 - 9\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(c*d^4*x^2 + c^2
*d^3) + 1/3*(b^3*d*x^3 - 3*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 1/2*(5*b^3*c^3 -
9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c
*d^3)
```

mupad [B] time = 0.10, size = 181, normalized size = 1.69

$$x\left(\frac{3\,a\,b^2}{d^2} - \frac{2\,b^3\,c}{d^3}\right) + \frac{b^3\,x^3}{3\,d^2} + \frac{x\left(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3\right)}{2\,c\left(d^4\,x^2 + c\,d^3\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)^2\,(a\,d+5\,b\,c)}{\sqrt{c}\left(a^3\,d^3+3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+5\,b^3\,c^3\right)}\right)(a\,d-b\,c)^2\,(a\,d+5\,b\,c)}{2\,c^{3/2}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/(c + d*x^2)^2,x)
```

```
[Out] x*((3*a*b^2)/d^2 - (2*b^3*c)/d^3) + (b^3*x^3)/(3*d^2) + (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*c*(c*d^3 + d^4*x^2)) + (atan((d^(1/
2)*x*(a*d - b*c)^2*(a*d + 5*b*c))/(c^(1/2)*(a^3*d^3 + 5*b^3*c^3 - 9*a*b^2*c
^2*d + 3*a^2*b*c*d^2)))*(a*d - b*c)^2*(a*d + 5*b*c))/(2*c^(3/2)*d^(7/2))
```

sympy [B] time = 1.05, size = 314, normalized size = 2.93

$$\frac{b^3x^3}{3d^2} + x\left(\frac{3ab^2}{d^2} - \frac{2b^3c}{d^3}\right) + \frac{x\left(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3\right)}{2c^2d^3 + 2cd^4x^2} - \frac{\sqrt{-\frac{1}{c^3d^7}}\,(ad-bc)^2\,(ad+5bc)\log\left(-\frac{c^2d^3\sqrt{-\frac{1}{c^3d^7}}\,(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3} + x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^7}}\,(ad-bc)^2\,(ad+5bc)\log\left(\frac{c^2d^3\sqrt{-\frac{1}{c^3d^7}}\,(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] b**3*x**3/(3*d**2) + x*(3*a*b**2/d**2 - 2*b**3*c/d**3) + x*(a**3*d**3 - 3*a
**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*c**2*d**3 + 2*c*d**4*x**2) -
 sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)*log(-c**2*d**3*sqrt(-1/(
c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a
*b**2*c**2*d + 5*b**3*c**3) + x)/4 + sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a
*d + 5*b*c)*log(c**2*d**3*sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)
/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4
```

3.19 $\int \frac{(a+bx^2)^3}{(c+dx^2)^3} \, dx$

Optimal. Leaf size=130

$$-\frac{3(bc-ad)\left((ad+bc)^2+4b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}+\frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3\left(c+dx^2\right)}-\frac{x(bc-ad)^3}{4cd^3\left(c+dx^2\right)^2}+\frac{b^3x}{d^3}$$

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {390, 1157, 385, 205}

$$-\frac{3(bc-ad)\left((ad+bc)^2+4b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}+\frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3\left(c+dx^2\right)}-\frac{x(bc-ad)^3}{4cd^3\left(c+dx^2\right)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2*
(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(b*c - a*d)*(4*b^2*c^2 + (b*c
 + a*d)^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^3}\,dx = \int \left(\frac{b^3}{d^3} - \frac{b^3c^3 - a^3d^3 + 3bd(bc-ad)(bc+ad)x^2 + 3b^2d^2(bc-ad)x^4}{d^3\left(c+dx^2\right)^3}\right)dx$$

$$= \frac{b^3x}{d^3} - \frac{\int \frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{\left(c+dx^2\right)^3}\,dx}{d^3}$$

$$= \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3\left(c+dx^2\right)^2} + \frac{\int \frac{-3(bc-ad)(bc+ad)^2-12b^2cd(bc-ad)x^2}{\left(c+dx^2\right)^2}\,dx}{4cd^3}$$

$$= \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3\left(c+dx^2\right)^2} + \frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3\left(c+dx^2\right)} - \frac{\left(3(bc-ad)\left(4b^2c^2+(bc+ad)^2\right)\right)\int \frac{1}{c+d\ldots}}{8c^2d^3}$$

$$= \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3\left(c+dx^2\right)^2} + \frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3\left(c+dx^2\right)} - \frac{3(bc-ad)\left(4b^2c^2+(bc+ad)^2\right)\tan^{-1}\left(\ldots\right.}{8c^{5/2}d^{7/2}}$$

Mathematica [A] time = 0.08, size = 141, normalized size = 1.08

$$-\frac{3\left(-a^3d^3-a^2bcd^2-3ab^2c^2d+5b^3c^3\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}} + \frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3\left(c+dx^2\right)} - \frac{x(bc-ad)^3}{4cd^3\left(c+dx^2\right)^2} + \frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2*
(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(5*b^3*c^3 - 3*a*b^2*c^2*d -
a^2*b*c*d^2 - a^3*d^3)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^3/(c + d*x^2)^3, x]
```

fricas [B] time = 0.79, size = 618, normalized size = 4.75

$$\left[\frac{16\,b^3c^3d^3x^5 + 2\left(25\,b^3c^4d^2 - 15\,ab^2c^3d^3 + 3\,a^2bc^2d^4 + 3\,a^3cd^5\right)x^3 + 3\left(5\,b^3c^5 - 3\,ab^2c^4d - a^2bc^3d^2 - a^3c^2d^3 + \left(5\,b^3c^3d^2 - 3\,ab^2c^2d^3 - a^2bcd^4 - a^3d^5\right)x^4 + 2\left(5\,b^3c^4d - 3\,ab^2c^3d^2 - a^2bc^2d^3 - a^3cd^4\right)x^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right) + 2\left(15\,b^3c^5d - 9\,ab^2c^4d^2 - 3\,a^2bc^3d^3 + 5\,a^3c^2d^4\right)x}{16\left(c^3d^6x^4 + 2\,c^4d^5x^2 + c^5d^4\right)}, \frac{8\,b^3c^3d^3x^5 + \left(25\,b^3c^4d^2 - 15\,ab^2c^3d^3 + 3\,a^2bc^2d^4 + 3\,a^3cd^5\right)x^3 - 3\left(5\,b^3c^5 - 3\,ab^2c^4d - a^2bc^3d^2 - a^3c^2d^3 + \left(5\,b^3c^3d^2 - 3\,ab^2c^2d^3 - a^2bcd^4 - a^3d^5\right)x^4 + 2\left(5\,b^3c^4d - 3\,ab^2c^3d^2 - a^2bc^2d^3 - a^3cd^4\right)x^2\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right) + \left(15\,b^3c^5d - 9\,ab^2c^4d^2 - 3\,a^2bc^3d^3 + 5\,a^3c^2d^4\right)x}{8\left(c^3d^6x^4 + 2\,c^4d^5x^2 + c^5d^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b^3*c^3*d^3*x^5 + 2*(25*b^3*c^4*d^2 - 15*a*b^2*c^3*d^3 + 3*a^2*b*
c^2*d^4 + 3*a^3*c*d^5)*x^3 + 3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^2*b*c^3*d^2 -
 a^3*c^2*d^3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2*b*c*d^4 - a^3*d^5)*x^
4 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2*d^3 - a^3*c*d^4)*x^2)*sqrt
(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(15*b^3*c^5*d - 9*
a*b^2*c^4*d^2 - 3*a^2*b*c^3*d^3 + 5*a^3*c^2*d^4)*x)/(c^3*d^6*x^4 + 2*c^4*d^
```

```
5*x^2 + c^5*d^4), 1/8*(8*b^3*c^3*d^3*x^5 + (25*b^3*c^4*d^2 - 15*a*b^2*c^3*d
^3 + 3*a^2*b*c^2*d^4 + 3*a^3*c*d^5)*x^3 - 3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^
2*b*c^3*d^2 - a^3*c^2*d^3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2*b*c*d^4
- a^3*d^5)*x^4 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2*d^3 - a^3*c*d
^4)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (15*b^3*c^5*d - 9*a*b^2*c^4*d^2
- 3*a^2*b*c^3*d^3 + 5*a^3*c^2*d^4)*x)/(c^3*d^6*x^4 + 2*c^4*d^5*x^2 + c^5*d^
4)]
```

giac [A] time = 0.59, size = 180, normalized size = 1.38

$$\frac{b^3x}{d^3} - \frac{3\left(5\,b^3c^3 - 3\,ab^2c^2d - a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d^3} + \frac{9\,b^3c^3dx^3 - 15\,ab^2c^2d^2x^3 + 3\,a^2bcd^3x^3 + 3\,a^3d^4x^3 + 7\,b^3c^4x - 9\,ab^2c^3dx - 3\,a^2bc^2d^2x + 5\,a^3cd^3x}{8\left(dx^2+c\right)^2c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3 - 3/8*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*arctan(
d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^3) + 1/8*(9*b^3*c^3*d*x^3 - 15*a*b^2*c^2*d^
2*x^3 + 3*a^2*b*c*d^3*x^3 + 3*a^3*d^4*x^3 + 7*b^3*c^4*x - 9*a*b^2*c^3*d*x -
 3*a^2*b*c^2*d^2*x + 5*a^3*c*d^3*x)/((d*x^2 + c)^2*c^2*d^3)
```

maple [B] time = 0.01, size = 266, normalized size = 2.05

$$\frac{3a^3d\,x^3}{8\left(d\,x^2+c\right)^2c^2} + \frac{3a^2b\,x^3}{8\left(d\,x^2+c\right)^2c} - \frac{15a\,b^2x^3}{8\left(d\,x^2+c\right)^2d} + \frac{9b^3c\,x^3}{8\left(d\,x^2+c\right)^2d^2} + \frac{5a^3x}{8\left(d\,x^2+c\right)^2c} - \frac{3a^2bx}{8\left(d\,x^2+c\right)^2d} - \frac{9a\,b^2cx}{8\left(d\,x^2+c\right)^2d^2} + \frac{7b^3c^2x}{8\left(d\,x^2+c\right)^2d^3} + \frac{3a^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,c^2} + \frac{3a^2b\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,cd} + \frac{9a\,b^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,d^2} - \frac{15b^3c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,d^3} + \frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c)^3,x)
```

```
[Out] b^3*x/d^3+3/8*d/(d*x^2+c)^2/c^2*x^3*a^3+3/8/(d*x^2+c)^2/c*x^3*a^2*b-15/8/d/
(d*x^2+c)^2*x^3*a*b^2+9/8/d^2/(d*x^2+c)^2*c*x^3*b^3+5/8/(d*x^2+c)^2/c*x*a^3
-3/8/d/(d*x^2+c)^2*x*a^2*b-9/8/d^2/(d*x^2+c)^2*c*x*a*b^2+7/8/d^3/(d*x^2+c)^
2*c^2*x*b^3+3/8/c^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^3+3/8/d/c/(c*d)
^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2*b+9/8/d^2/(c*d)^(1/2)*arctan(1/(c*d)^(
1/2)*d*x)*a*b^2-15/8/d^3*c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^3
```

maxima [A] time = 2.98, size = 187, normalized size = 1.44

$$\frac{b^3x}{d^3} + \frac{3\left(3\,b^3c^3d - 5\,ab^2c^2d^2 + a^2bcd^3 + a^3d^4\right)x^3 + \left(7\,b^3c^4 - 9\,ab^2c^3d - 3\,a^2bc^2d^2 + 5\,a^3cd^3\right)x}{8\left(c^2d^5x^4 + 2\,c^3d^4x^2 + c^4d^3\right)} - \frac{3\left(5\,b^3c^3 - 3\,ab^2c^2d - a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3 + 1/8*(3*(3*b^3*c^3*d - 5*a*b^2*c^2*d^2 + a^2*b*c*d^3 + a^3*d^4)*
x^3 + (7*b^3*c^4 - 9*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x)/(c^2*d
^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3) - 3/8*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*
c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^3)
```

mupad [B] time = 4.96, size = 240, normalized size = 1.85

$$\frac{\frac{x\left(5\,a^3\,d^3-3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+7\,b^3\,c^3\right)}{8\,c} + \frac{3\,x^3\left(a^3\,d^4+a^2\,b\,c\,d^3-5\,a\,b^2\,c^2\,d^2+3\,b^3\,c^3\,d\right)}{8\,c^2}}{c^2\,d^3 + 2\,c\,d^4\,x^2 + d^5\,x^4} + \frac{b^3\,x}{d^3} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\left(a^2\,d^2+2\,a\,b\,c\,d+5\,b^2\,c^2\right)}{\sqrt{c}\,\left(a^3\,d^3+a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-5\,b^3\,c^3\right)}\right)(a\,d-b\,c)\,\left(a^2\,d^2+2\,a\,b\,c\,d+5\,b^2\,c^2\right)}{8\,c^{5/2}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/(c + d*x^2)^3,x)
```

```
[Out] ((x*(5*a^3*d^3 + 7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(8*c) + (3*x^3
*(a^3*d^4 + 3*b^3*c^3*d - 5*a*b^2*c^2*d^2 + a^2*b*c*d^3))/(8*c^2))/(c^2*d^3
 + d^5*x^4 + 2*c*d^4*x^2) + (b^3*x)/d^3 + (3*atan((d^(1/2)*x*(a*d - b*c)*(a
```

```
^2*d^2 + 5*b^2*c^2 + 2*a*b*c*d))/(c^(1/2)*(a^3*d^3 - 5*b^3*c^3 + 3*a*b^2*c^
2*d + a^2*b*c*d^2)))*(a*d - b*c)*(a^2*d^2 + 5*b^2*c^2 + 2*a*b*c*d))/(8*c^(5
/2)*d^(7/2))
```

sympy [B] time = 1.81, size = 422, normalized size = 3.25

$$\frac{b^3x}{d^3} - \frac{3\sqrt{-\frac{1}{c^5d^7}}\,(ad-bc)\left(a^2d^2+2abcd+5b^2c^2\right)\log\left(-\frac{3c^3d^3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)\left(a^2d^2+2abcd+5b^2c^2\right)}{3a^3d^3+3a^2bcd^2+9ab^2c^2d-15b^3c^3}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{c^5d^7}}\,(ad-bc)\left(a^2d^2+2abcd+5b^2c^2\right)\log\left(\frac{3c^3d^3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)\left(a^2d^2+2abcd+5b^2c^2\right)}{3a^3d^3+3a^2bcd^2+9ab^2c^2d-15b^3c^3}+x\right)}{16} + \frac{x^3\left(3a^3d^4+3a^2bcd^3-15ab^2c^2d^2+9b^3c^3d\right)+x\left(5a^3cd^3-3a^2bc^2d^2-9ab^2c^3d+7b^3c^4\right)}{8c^4d^3+16c^3d^4x^2+8c^2d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] b**3*x/d**3 - 3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*a*b*c*d + 5
*b**2*c**2)*log(-3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 +
2*a*b*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**2*b*c*d**2 + 9*a*b**2*c**2*d -
 15*b**3*c**3) + x)/16 + 3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*
a*b*c*d + 5*b**2*c**2)*log(3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a*
*2*d**2 + 2*a*b*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**2*b*c*d**2 + 9*a*b**
2*c**2*d - 15*b**3*c**3) + x)/16 + (x**3*(3*a**3*d**4 + 3*a**2*b*c*d**3 - 1
5*a*b**2*c**2*d**2 + 9*b**3*c**3*d) + x*(5*a**3*c*d**3 - 3*a**2*b*c**2*d**2
 - 9*a*b**2*c**3*d + 7*b**3*c**4))/(8*c**4*d**3 + 16*c**3*d**4*x**2 + 8*c**
2*d**5*x**4)
```

3.20 $\int \frac{(c+dx^2)^4}{a+bx^2}\, dx$

Optimal. Leaf size=142

$$\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{3b^3}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{9/2}}+\frac{d^3x^5(4bc-ad)}{5b^2}+\frac{d^4 \ldots}{7 \ldots}$$

Rubi [A] time = 0.09, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$\frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{3b^3}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^3x^5(4bc-ad)}{5b^2}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{9/2}}+\frac{d^4x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) + (d^3*(4*b*c - a*d)*x^5)/(5*b^2) + (d
^4*x^7)/(7*b) + ((b*c - a*d)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(9/2
))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^2}{b^3}+\frac{d^3(4bc-ad)x^4}{b^2}+\ldots\right.$$
$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{3b^3}+\frac{d^3(4bc-ad)x^5}{5b^2}+\frac{d^4x \ldots}{7b \ldots}$$
$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{3b^3}+\frac{d^3(4bc-ad)x^5}{5b^2}+\frac{d^4x \ldots}{7b \ldots}$$

Mathematica [A] time = 0.09, size = 136, normalized size = 0.96

$$\frac{dx\left(-105a^3d^3+35a^2bd^2\left(12c+dx^2\right)-7ab^2d\left(90c^2+20cdx^2+3d^2x^4\right)+3b^3\left(140c^3+70c^2dx^2+28cd^2x^4+5d^3x^6\right)\right)}{105b^4}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2),x]
```

```
[Out] (d*x*(-105*a^3*d^3 + 35*a^2*b*d^2*(12*c + d*x^2) - 7*a*b^2*d*(90*c^2 + 20*c
*d*x^2 + 3*d^2*x^4) + 3*b^3*(140*c^3 + 70*c^2*d*x^2 + 28*c*d^2*x^4 + 5*d^3*
x^6)))/(105*b^4) + ((b*c - a*d)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(
9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^4}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^4/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^4/(a + b*x^2), x]
```

fricas [A] time = 0.63, size = 428, normalized size = 3.01

$$\left[\frac{30\,ab^4d^4x^7+42\left(4\,ab^4cd^3-a^2b^3d^4\right)x^5+70\left(6\,ab^4c^2d^2-4\,a^2b^3cd^3+a^3b^2d^4\right)x^3-105\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+210\left(4\,ab^4c^3d-6\,a^2b^3c^2d^2+4\,a^3b^2cd^3-a^4bd^4\right)x}{210\,ab^5},\ \frac{15\,ab^4d^4x^7+21\left(4\,ab^4cd^3-a^2b^3d^4\right)x^5+35\left(6\,ab^4c^2d^2-4\,a^2b^3cd^3+a^3b^2d^4\right)x^3+105\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+105\left(4\,ab^4c^3d-6\,a^2b^3c^2d^2+4\,a^3b^2cd^3-a^4bd^4\right)x}{105\,ab^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*a*b^4*d^4*x^7 + 42*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^5 + 70*(6*a*b
^4*c^2*d^2 - 4*a^2*b^3*c*d^3 + a^3*b^2*d^4)*x^3 - 105*(b^4*c^4 - 4*a*b^3*c^
3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 210*(4*a*b^4*c^3*d - 6*a^2*b^3*c^2*d^2 +
 4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)/(a*b^5), 1/105*(15*a*b^4*d^4*x^7 + 21*(4*a
*b^4*c*d^3 - a^2*b^3*d^4)*x^5 + 35*(6*a*b^4*c^2*d^2 - 4*a^2*b^3*c*d^3 + a^3
*b^2*d^4)*x^3 + 105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 105*(4*a*b^4*c^3*d - 6*a
^2*b^3*c^2*d^2 + 4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)/(a*b^5)]
```

giac [A] time = 0.58, size = 198, normalized size = 1.39

$$\frac{\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{15\,b^6d^4x^7+84\,b^6cd^3x^5-21\,ab^5d^4x^5+210\,b^6c^2d^2x^3-140\,ab^5cd^3x^3+35\,a^2b^4d^4x^3+420\,b^6c^3dx-630\,ab^5c^2d^2x+420\,a^2b^4cd^3x-105\,a^3b^3d^4x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*d^4*x^7 + 84*b^6*c*d^3*x
^5 - 21*a*b^5*d^4*x^5 + 210*b^6*c^2*d^2*x^3 - 140*a*b^5*c*d^3*x^3 + 35*a^2*
b^4*d^4*x^3 + 420*b^6*c^3*d*x - 630*a*b^5*c^2*d^2*x + 420*a^2*b^4*c*d^3*x -
 105*a^3*b^3*d^4*x)/b^7
```

maple [A] time = 0.01, size = 246, normalized size = 1.73

$$\frac{d^4x^7}{7b}-\frac{a\,d^4x^5}{5b^2}+\frac{4c\,d^3x^5}{5b}+\frac{a^2d^4x^3}{3b^3}-\frac{4ac\,d^3x^3}{3b^2}+\frac{2c^2d^2x^3}{b}+\frac{a^4d^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}-\frac{4a^3c\,d^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{6a^2c^2d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}-\frac{4a\,c^3d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{c^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{a^3d^4x}{b^4}+\frac{4a^2c\,d^3x}{b^3}-\frac{6a\,c^2d^2x}{b^2}+\frac{4c^3dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a),x)
```

```
[Out] 1/7*d^4*x^7/b-1/5*d^4/b^2*x^5*a+4/5*d^3/b*x^5*c+1/3*d^4/b^3*x^3*a^2-4/3*d^3
/b^2*x^3*a*c+2*d^2/b*x^3*c^2-d^4/b^4*a^3*x+4*d^3/b^3*a^2*c*x-6*d^2/b^2*a*c^
2*x+4*d/b*c^3*x+1/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^4*d^4-4/b^3/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^3*c*d^3+6/b^2/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)*a^2*c^2*d^2-4/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*c^3
*d+1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^4
```

maxima [A] time = 3.03, size = 187, normalized size = 1.32

$$\frac{\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{15\,b^3d^4x^7 + 21\left(4\,b^3cd^3 - ab^2d^4\right)x^5 + 35\left(6\,b^3c^2d^2 - 4\,ab^2cd^3 + a^2bd^4\right)x^3 + 105\left(4\,b^3c^3d - 6\,ab^2c^2d^2 + 4\,a^2bcd^3 - a^3d^4\right)x}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^3*d^4*x^7 + 21*(4*b^3*c*d^
3 - a*b^2*d^4)*x^5 + 35*(6*b^3*c^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^3 + 1
05*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4
```

mupad [B] time = 4.86, size = 216, normalized size = 1.52

$$x\left(\frac{4\,c^3\,d}{b} - \frac{a\left(\frac{a\left(\frac{a\,d^4}{b^2} - \frac{4\,c\,d^3}{b}\right)}{b} + \frac{6\,c^2\,d^2}{b}\right)}{b}\right) - x^5\left(\frac{a\,d^4}{5\,b^2} - \frac{4\,c\,d^3}{5\,b}\right) + x^3\left(\frac{a\left(\frac{a\,d^4}{b^2} - \frac{4\,c\,d^3}{b}\right)}{3\,b} + \frac{2\,c^2\,d^2}{b}\right) + \frac{d^4\,x^7}{7\,b} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^4}{\sqrt{a}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}\right)\,(a\,d-b\,c)^4}{\sqrt{a}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^4/(a + b*x^2),x)
```

```
[Out] x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b)
 - x^5*((a*d^4)/(5*b^2) - (4*c*d^3)/(5*b)) + x^3*((a*((a*d^4)/b^2 - (4*c*d^
3)/b))/(3*b) + (2*c^2*d^2)/b) + (d^4*x^7)/(7*b) + (atan((b^(1/2)*x*(a*d - b
*c)^4)/(a^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*
a^3*b*c*d^3)))*(a*d - b*c)^4)/(a^(1/2)*b^(9/2))
```

sympy [B] time = 0.76, size = 326, normalized size = 2.30

$$x^5\left(-\frac{ad^4}{5b^2} + \frac{4cd^3}{5b}\right) + x^3\left(\frac{a^2d^4}{3b^3} - \frac{4acd^3}{3b^2} + \frac{2c^2d^2}{b}\right) + x\left(-\frac{a^3d^4}{b^4} + \frac{4a^2cd^3}{b^3} - \frac{6ac^2d^2}{b^2} + \frac{4c^3d}{b}\right) - \frac{\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^4\log\left(-\frac{ab^4\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^4\log\left(\frac{ab^4\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4} + x\right)}{2} + \frac{d^4x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a),x)
```

```
[Out] x**5*(-a*d**4/(5*b**2) + 4*c*d**3/(5*b)) + x**3*(a**2*d**4/(3*b**3) - 4*a*c
*d**3/(3*b**2) + 2*c**2*d**2/b) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b**3 -
 6*a*c**2*d**2/b**2 + 4*c**3*d/b) - sqrt(-1/(a*b**9))*(a*d - b*c)**4*log(-a
*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**
2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x)/2 + sqrt(-1/(a*b**9))*
(a*d - b*c)**4*log(a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**4/(a**4*d**4 - 4*a
**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x)/2
+ d**4*x**7/(7*b)
```

3.21 $\int \frac{(c+dx^2)^3}{a+bx^2} dx$

Optimal. Leaf size=98

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{d^3x^5}{5b}$$

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}+\frac{d^3x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^3)/(3*b^
2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]
*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3}+\frac{d^2(3bc-ad)x^2}{b^2}+\frac{d^3x^4}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\left(a+bx^2\right)}\right)dx\\
&= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\int \frac{1}{a+bx^2}\,dx}{b^3}\\
&= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 92, normalized size = 0.94

$$\frac{dx\left(15a^2d^2-5abd\left(9c+dx^2\right)+3b^2\left(15c^2+5cdx^2+d^2x^4\right)\right)}{15b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*x*(15*a^2*d^2 - 5*a*b*d*(9*c + d*x^2) + 3*b^2*(15*c^2 + 5*c*d*x^2 + d^2*
x^4)))/(15*b^3) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(7
/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2), x]
```

fricas [A] time = 0.80, size = 292, normalized size = 2.98

$$\left[\frac{6\,ab^3d^3x^5+10\left(3\,ab^3cd^2-a^2b^2d^3\right)x^3+15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+30\left(3\,ab^3c^2d-3\,a^2b^2cd^2+a^3bd^3\right)x}{30\,ab^4},\ \frac{3\,ab^3d^3x^5+5\left(3\,ab^3cd^2-a^2b^2d^3\right)x^3+15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+15\left(3\,ab^3c^2d-3\,a^2b^2cd^2+a^3bd^3\right)x}{15\,ab^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*d^3*x^5 + 10*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 + 15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(
-a*b)*x - a)/(b*x^2 + a)) + 30*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3
)*x)/(a*b^4), 1/15*(3*a*b^3*d^3*x^5 + 5*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 +
 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a) + 15*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x)/(a*b^4)]
```

giac [A] time = 0.58, size = 129, normalized size = 1.32

$$\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{3\,b^4d^3x^5+15\,b^4cd^2x^3-5\,ab^3d^3x^3+45\,b^4c^2dx-45\,ab^3cd^2x+15\,a^2b^2d^3x}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^4*d^3*x^5 + 15*b^4*c*d^2*x^3 - 5*a*b^3*d^3*x^3 +
 45*b^4*c^2*d*x - 45*a*b^3*c*d^2*x + 15*a^2*b^2*d^3*x)/b^5
```

maple [A] time = 0.00, size = 161, normalized size = 1.64

$$\frac{d^3x^5}{5b}-\frac{a\,d^3x^3}{3b^2}+\frac{c\,d^2x^3}{b}-\frac{a^3d^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3}+\frac{3a^2c\,d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}-\frac{3a\,c^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{c^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{a^2d^3x}{b^3}-\frac{3ac\,d^2x}{b^2}+\frac{3c^2dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/5/b*d^3*x^5-1/3*d^3/b^2*x^3*a+d^2/b*x^3*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c*x+3
*d/b*c^2*x-1/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^3*d^3+3/b^2/(a*b)^
(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^2*c*d^2-3/b/(a*b)^(1/2)*arctan(1/(a*b)^(1
/2)*b*x)*a*c^2*d+1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^3
```

maxima [A] time = 2.91, size = 122, normalized size = 1.24

$$\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{3\,b^2d^3x^5+5\left(3\,b^2cd^2-abd^3\right)x^3+15\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 15*(
3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

mupad [B] time = 0.08, size = 146, normalized size = 1.49

$$x\left(\frac{3c^2d}{b}+\frac{a\left(\frac{ad^3}{b^2}-\frac{3cd^2}{b}\right)}{b}\right)-x^3\left(\frac{ad^3}{3b^2}-\frac{cd^2}{b}\right)+\frac{d^3x^5}{5b}-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x\,(ad-bc)^3}{\sqrt{a}\,\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}\right)(ad-bc)^3}{\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^3*((a*d^3)/(3*b^2)
- (c*d^2)/b) + (d^3*x^5)/(5*b) - (atan((b^(1/2)*x*(a*d - b*c)^3)/(a^(1/2)*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(a^(1/2
)*b^(7/2))
```

sympy [B] time = 0.59, size = 238, normalized size = 2.43

$$x^3\left(-\frac{ad^3}{3b^2}+\frac{cd^2}{b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)+\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] x**3*(-a*d**3/(3*b**2) + c*d**2/b) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b**2 +
3*c**2*d/b) + sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(-a*b**3*sqrt(-1/(a*b**7)
)*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3
) + x)/2 - sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(a*b**3*sqrt(-1/(a*b**7))*(a
*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) +
x)/2 + d**3*x**5/(5*b)
```

3.22 $\int \frac{(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=63

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^3}{3b}$$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$\frac{dx(2bc-ad)}{b^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{d^2x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{a+bx^2}\,dx &= \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^2}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2\left(a+bx^2\right)}\right)dx\\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\int \frac{1}{a+bx^2}\,dx}{b^2}\\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.94

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{dx\left(-3ad+6bc+bdx^2\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*x*(6*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^2\right)^2}{a + bx^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2), x]
```

fricas [A] time = 0.91, size = 181, normalized size = 2.87

$$\left[\frac{2\,ab^2d^2x^3 - 3\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 6\left(2\,ab^2cd - a^2bd^2\right)x}{6\,ab^3}, \frac{ab^2d^2x^3 + 3\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + 3\left(2\,ab^2cd - a^2bd^2\right)x}{3\,ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*b^2*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*log((b
*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a
*b^3), 1/3*(a*b^2*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + 3*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a*b^3)]
```

giac [A] time = 0.57, size = 72, normalized size = 1.14

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{b^2d^2x^3 + 6\,b^2cdx - 3\,abd^2x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b^2*d^2*x^3 + 6*b^2*c*d*x - 3*a*b*d^2*x)/b^3
```

maple [A] time = 0.00, size = 95, normalized size = 1.51

$$\frac{d^2x^3}{3b} + \frac{a^2d^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\; b^2} - \frac{2acd \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\; b} + \frac{c^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{a\,d^2x}{b^2} + \frac{2cdx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/3/b*d^2*x^3-d^2/b^2*a*x+2*d/b*x*c+1/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x)*a^2*d^2-2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*c*d+1/(a*b)^(1/2)*
arctan(1/(a*b)^(1/2)*b*x)*c^2
```

maxima [A] time = 2.93, size = 69, normalized size = 1.10

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{bd^2x^3 + 3\left(2\,bcd - ad^2\right)x}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b*d^2*x^3 + 3*(2*b*c*d - a*d^2)*x)/b^2
```

mupad [B] time = 4.90, size = 90, normalized size = 1.43

$$\frac{d^2\,x^3}{3\,b}-x\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2}{\sqrt{a}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2),x)
```

```
[Out] (d^2*x^3)/(3*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((b^(1/2)*x*(a*d - b*c
)^2)/(a^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(a^(1/2)*b^(
5/2))
```

sympy [B] time = 0.44, size = 172, normalized size = 2.73

$$x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)-\frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(-\frac{ab^2\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(\frac{ab^2\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{d^2x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) - sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(-a*b**2*s
qrt(-1/(a*b**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
+ sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(a*b**2*sqrt(-1/(a*b**5))*(a*d - b*c)
**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x**3/(3*b)
```

3.23 $\int \frac{c+dx^2}{a+bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{dx}{b}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 205}

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{a+bx^2}\,dx &= \frac{dx}{b}-\frac{(-bc+ad)\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{dx}{b}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.03

$$\frac{dx}{b}-\frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b - ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2), x]
```

fricas [A] time = 1.05, size = 98, normalized size = 2.51

$$\left[\frac{2\,abdx + \sqrt{-ab}\,(bc - ad)\log\left(\frac{bx^2 + 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right)}{2\,ab^2}, \frac{abdx + \sqrt{ab}\,(bc - ad)\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*d*x + sqrt(-a*b)*(b*c - a*d)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*d*x + sqrt(a*b)*(b*c - a*d)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

giac [A] time = 0.57, size = 33, normalized size = 0.85

$$\frac{dx}{b} + \frac{(bc - ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

maple [A] time = 0.00, size = 45, normalized size = 1.15

$$-\frac{ad\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} + \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/b*d*x-1/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*d+1/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.99, size = 33, normalized size = 0.85

$$\frac{dx}{b} + \frac{(bc - ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 0.06, size = 32, normalized size = 0.82

$$\frac{d\,x}{b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d - b\,c)}{\sqrt{a}\;b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2),x)
```

```
[Out] (d*x)/b - (atan((b^(1/2)*x)/a^(1/2))*(a*d - b*c))/(a^(1/2)*b^(3/2))
```

sympy [B] time = 0.28, size = 82, normalized size = 2.10

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*d - b*c)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(
a*b**3))*(a*d - b*c)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + d*x/b
```

3.24 $\int \frac{1}{(a+bx^2)(c+dx^2)} \, dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {391, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+bx^2)(c+dx^2)} \, dx &= \frac{b\int \frac{1}{a+bx^2} \, dx}{bc-ad}-\frac{d\int \frac{1}{c+dx^2} \, dx}{bc-ad}\\ &=\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.87

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/Sqrt[c])/(b*c - a*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 0.92, size = 292, normalized size = 4.17

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\ -\frac{2\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2\,(bc-ad)},\ \frac{2\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\ \frac{\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)}{bc-ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2*s
qrt(d/c)*arctan(x*sqrt(d/c)) + sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(b*c - a*d), 1/2*(2*sqrt(b/a)*arctan(x*sqrt(b/a)) - sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), (sqrt(b/a
)*arctan(x*sqrt(b/a)) - sqrt(d/c)*arctan(x*sqrt(d/c)))/(b*c - a*d)]
```

giac [A] time = 0.59, size = 54, normalized size = 0.77

$$\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/(
(b*c - a*d)*sqrt(c*d))
```

maple [A] time = 0.01, size = 55, normalized size = 0.79

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)\sqrt{ab}}+\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] d/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)-b/(a*d-b*c)/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.98, size = 54, normalized size = 0.77

$$\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

[Out] b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))

mupad [B] time = 0.32, size = 135, normalized size = 1.93

$$\frac{\ln\left(b\,x-\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,a^2\,d-2\,a\,b\,c}-\frac{\ln\left(d\,x+\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,\left(b\,c^2-a\,c\,d\right)}-\frac{\ln\left(b\,x+\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,\left(a^2\,d-a\,b\,c\right)}+\frac{\ln\left(d\,x-\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,b\,c^2-2\,a\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x^2)*(c + d*x^2)),x)

[Out] (log(b*x - (-a*b)^(1/2))*(-a*b)^(1/2))/(2*a^2*d - 2*a*b*c) - (log(d*x + (-c*d)^(1/2))*(-c*d)^(1/2))/(2*(b*c^2 - a*c*d)) - (log(b*x + (-a*b)^(1/2))*(-a*b)^(1/2))/(2*(a^2*d - a*b*c)) + (log(d*x - (-c*d)^(1/2))*(-c*d)^(1/2))/(2*b*c^2 - 2*a*c*d)

sympy [B] time = 2.80, size = 712, normalized size = 10.17

$$\frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{-\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc}-\frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2\left(ad-bc\right)}-\frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc}+\frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2\left(ad-bc\right)}+\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{-\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc}-\frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2\left(ad-bc\right)}-\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc}+\frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a)/(d*x**2+c),x)

[Out] sqrt(-b/a)*log(x + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d - b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) - a*b**3*c**4*(-b/a)**(3/2)/(a*d - b*c)**3 - b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-b/a)*log(x + (a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(a*d - b*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) + sqrt(-d/c)*log(x + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2*d**2*(-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3 - a**2*d**2*sqrt(-d/c)/(a*d - b*c) - a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)**3 - b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-d/c)*log(x + (a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-d/c)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3 + a**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)**3 + b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c))

3.25 $\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=109

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

Rubi [A] time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx &= -\frac{dx}{2c(bc-ad)\left(c+dx^2\right)}+\frac{\int \frac{2bc-ad-bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2c(bc-ad)}\\
&= -\frac{dx}{2c(bc-ad)\left(c+dx^2\right)}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2}-\frac{(d(3bc-ad))\int \frac{1}{c+dx^2}\,dx}{2c(bc-ad)^2}\\
&= -\frac{dx}{2c(bc-ad)\left(c+dx^2\right)}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 95, normalized size = 0.87

$$\frac{\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{d}\,(ad-3bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}+\frac{dx(ad-bc)}{c\left(c+dx^2\right)}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((d*(-(b*c) + a*d)*x)/(c*(c + d*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/Sqrt[a] + (Sqrt[d]*(-3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2)
)/(2*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)^2), x]
```

fricas [A] time = 1.04, size = 711, normalized size = 6.52

$$\left[\frac{2\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-\left(3bc^2-acd+\left(3bcd-ad^2\right)x^2\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(bcd-ad^2\right)x}{4\left(b^2c^4-2abc^3d+a^2c^2d^2+\left(b^2c^3d-2abc^2d^2+a^2cd^3\right)x^2\right)},\ -\frac{\left(3bc^2-acd+\left(3bcd-ad^2\right)x^2\right)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\left(bcd-ad^2\right)x}{2\left(b^2c^4-2abc^3d+a^2c^2d^2+\left(b^2c^3d-2abc^2d^2+a^2cd^3\right)x^2\right)},\ \frac{4\left(bcdx^2+bc^2\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\left(3bc^2-acd+\left(3bcd-ad^2\right)x^2\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(bcd-ad^2\right)x}{4\left(b^2c^4-2abc^3d+a^2c^2d^2+\left(b^2c^3d-2abc^2d^2+a^2cd^3\right)x^2\right)},\ \frac{2\left(bcdx^2+bc^2\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\left(3bc^2-acd+\left(3bcd-ad^2\right)x^2\right)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-\left(bcd-ad^2\right)x}{2\left(b^2c^4-2abc^3d+a^2c^2d^2+\left(b^2c^3d-2abc^2d^2+a^2cd^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(
b*x^2 + a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*log((d*x
^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2
*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), -1
/2*((3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
 - (b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3
*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/4*(4*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*
arctan(x*sqrt(b/a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*
log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2
*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*
x^2), 1/2*(2*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - (3*b*c^2 -
 a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (b*c*d - a*
d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 +
 a^2*c*d^3)*x^2)]
```

giac [A] time = 0.57, size = 122, normalized size = 1.12

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}}-\frac{\left(3\,bcd-ad^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{cd}}-\frac{dx}{2\left(bc^2-acd\right)\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*(3*b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*sqrt(c*d)) - 1/2*d*x/((b*c^2 - a*c*d)*(d*x^2 + c))
```

maple [A] time = 0.01, size = 144, normalized size = 1.32

$$\frac{a\,d^{2}x}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)c}+\frac{a\,d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^{2}\sqrt{cd}\,c}+\frac{b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ad-bc\right)^{2}\sqrt{ab}}-\frac{bdx}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)}-\frac{3bd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^{2}\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^2/(a*d-b*c)^2/c*x/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x/(d*x^2+c)*b+1/2*d^2
/(a*d-b*c)^2/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a-3/2*d/(a*d-b*c)^2/(c
*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b+b^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(1/
(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.04, size = 133, normalized size = 1.22

$$\frac{b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{ab}}-\frac{dx}{2\left(bc^{3}-ac^{2}d+\left(bc^{2}d-acd^{2}\right)x^{2}\right)}-\frac{\left(3\,bcd-ad^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}\right)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) - 1/2*(3*b*c*d - a*d^2)*ar
ctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
```

mupad [B] time = 5.69, size = 3637, normalized size = 33.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (d*x)/(2*c*(c + d*x^2)*(a*d - b*c)) - (atan((((-c^3*d)^(1/2)*(a*d - 3*b*c)*
((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7
+ 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 -
a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*d - 3
*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*
c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a
*d - 3*b*c))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 + a
^2*c^3*d^2 - 2*a*b*c^4*d)) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*d^5
 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d
)) + (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4
*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 + 3*
a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7*c^
7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4
*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d
)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/(4*
(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a
*b*c^4*d)))/(((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5 - a^3*c^2*d^3 + 3*a
^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*
d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^
3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*
c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 +
 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7
*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*
a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^
```

```
3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/
(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a
*b*c^4*d)) - ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^
3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)) + (((4*b^7*c^
6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^
4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 -
3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6
*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 1
6*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2
*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/(4*(b^2*c^5 + a^2*c
^3*d^2 - 2*a*b*c^4*d))))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))))*(-c^3*
d)^(1/2)*(a*d - 3*b*c)*1i)/(2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)) - (ata
n((((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 +
 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(2*(b^3*c^5
- a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) - (x*(-a*b^3)^(1/2)*(16*b
^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 4
8*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*
c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^2 +
a*b^2*c^2 - 2*a^2*b*c*d)) - (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^
4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)))*1i)/(a^3*d^2 + a*b^2*c^2 - 2
*a^2*b*c*d) - ((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*
b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(
2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) + (x*(-a*b^3)^
(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*
c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*
(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6
*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)))*1i)/(a^3*d^2 + a*
b^2*c^2 - 2*a^2*b*c*d))/(((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d
^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3
*c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) - (
x*(-a*b^3)^(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 +
32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4
+ a^2*c^2*d^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)
^(1/2))/(2*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) - (x*(a^2*b^3*d^5 + 13*b^5*
c^2*d^3 - 6*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))))/(a^3*
d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - ((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5
 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-a*b^3)^(1/2)*((((4*b
^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a
^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^
3*d^2 - 3*a*b^2*c^4*d)) + (x*(-a*b^3)^(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*
d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5
*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^
2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))
+ (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*
d^2 - 2*a*b*c^3*d))))/(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2)*
1i)/(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
[Out] Timed out
```

3.26 $\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Rubi [A] time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {414, 527, 522, 205}

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8*c^2*(b*c
- a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c
 - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{dx}{4c(bc-ad)(c+dx^2)^2} + \frac{\int \frac{4bc-3ad-3bdx^2}{(a+bx^2)(c+dx^2)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{\int \frac{8b^2c^2-7abcd+3a^2d^2-bd(7bc-3ad)x^2}{(a+bx^2)(c+dx^2)}}{8c^2(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{b^3\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3} - \frac{(d(15b^2c^2-1\ldots}{\ldots}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3} - \frac{\sqrt{d}(15b^2c^{\ldots}}{\ldots}$$

Mathematica [A] time = 0.24, size = 158, normalized size = 0.99

$$\frac{1}{8}\left(-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3} - \frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(ad-bc)^3} + \frac{dx(3ad-7bc)}{c^2(c+dx^2)(bc-ad)^2} - \frac{2dx}{c(c+dx^2)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-2*d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) + (d*(-7*b*c + 3*a*d)*x)/(c^2*(b*c
- a*d)^2*(c + d*x^2)) - (8*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-
(b*c) + a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)^3), x]
```

fricas [B] time = 2.42, size = 1585, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 + 8*(b^2*c^2*d^2*x
^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*
d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*
a^2*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c))
+ 2*(9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*
d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*
b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4
*d^3 - a^3*c^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*
x^3 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
```

```
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 4*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 +
b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (9*b^
2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2
*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4
 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^
3*c^3*d^4)*x^2), -1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 -
16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/
a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(9*b^2*c
^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*
c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 -
a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c
^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 - 8*(b^2
*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (
15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c*d^3
+ 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqr
t(d/c)*arctan(x*sqrt(d/c)) + (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x
)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 -
 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*
b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2)]
```

giac [A] time = 0.58, size = 217, normalized size = 1.36

$$\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{ab}} - \frac{\left(15\,b^2c^2d - 10\,abcd^2 + 3\,a^2d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3\right)\sqrt{cd}} - \frac{7\,bcd^2x^3 - 3\,ad^3x^3 + 9\,bc^2dx - 5\,acd^2x}{8\left(b^2c^4 - 2\,abc^3d + a^2c^2d^2\right)\left(dx^2 + c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*(7*b*c*d^2*x^3 - 3*a*d^3*x^3 + 9*b*c^2*d*x - 5*a*c*d^2*x)/((b^2*
c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

maple [B] time = 0.01, size = 310, normalized size = 1.94

$$\frac{3a^2d^4x^3}{8\left(ad-bc\right)^3\left(dx^2+c\right)^2c^2} - \frac{5ab\,d^3x^3}{4\left(ad-bc\right)^3\left(dx^2+c\right)^2c} + \frac{7b^2d^2x^3}{8\left(ad-bc\right)^3\left(dx^2+c\right)^2} + \frac{5a^2d^3x}{8\left(ad-bc\right)^3\left(dx^2+c\right)^2c} - \frac{7ab\,d^2x}{4\left(ad-bc\right)^3\left(dx^2+c\right)^2} + \frac{9b^2cdx}{8\left(ad-bc\right)^3\left(dx^2+c\right)^2} + \frac{3a^2d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^3\sqrt{cd}\,c^2} - \frac{5ab\,d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\left(ad-bc\right)^3\sqrt{cd}\,c} - \frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ad-bc\right)^3\sqrt{ab}} + \frac{15b^2d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^3\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^4/(a*d-b*c)^3/(d*x^2+c)^2/c^2*x^3*a^2-5/4*d^3/(a*d-b*c)^3/(d*x^2+c)^2
/c*x^3*a*b+7/8*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2+5/8*d^3/(a*d-b*c)^3/(d*x
^2+c)^2/c*x*a^2-7/4*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x*a*b+9/8*d/(a*d-b*c)^3/(d*
x^2+c)^2*c*x*b^2+3/8*d^3/(a*d-b*c)^3/c^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d
*x)*a^2-5/4*d^2/(a*d-b*c)^3/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b+15/
8*d/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-b^3/(a*d-b*c)^3/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [B] time = 3.08, size = 277, normalized size = 1.73

$$\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{ab}} - \frac{\left(15\,b^2c^2d - 10\,abcd^2 + 3\,a^2d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3\right)\sqrt{cd}} - \frac{\left(7\,bcd^2 - 3\,ad^3\right)x^3 + \left(9\,bc^2d - 5\,acd^2\right)x}{8\left(b^2c^6 - 2\,abc^5d + a^2c^4d^2 + \left(b^2c^4d^2 - 2\,abc^3d^3 + a^2c^2d^4\right)x^4 + 2\left(b^2c^5d - 2\,abc^4d^2 + a^2c^3d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*((7*b*c*d^2 - 3*a*d^3)*x^3 + (9*b*c^2*d - 5*a*c*d^2)*x)/(b^2*c^6
 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*
x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)
```

mupad [B] time = 6.87, size = 6033, normalized size = 37.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] ((x^3*(3*a*d^3 - 7*b*c*d^2))/(8*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(
5*a*d^2 - 9*b*c*d))/(8*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4 +
 2*c*d*x^2) - (atan((((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 -
 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8
+ a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - ((-
a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^
4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 30
40*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^
10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^
3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256*b^9*c^11*
d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1
280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7
*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*
(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*
d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)))*1i)/(2*(
a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) + ((-a*b^5)^(1/2)*(
(x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6
+ 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^
2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 176
0*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^
6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d
^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 1
5*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d
)) + (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b
^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6
*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^
3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d
^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*
b^2*c^2*d - 3*a^3*b*c*d^2)))*1i)/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d
- 3*a^3*b*c*d^2)))/((9*a^3*b^5*d^6 - 105*b^8*c^3*d^3 + 115*a*b^7*c^2*d^4 -
51*a^2*b^6*c*d^5)/(32*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^
4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + ((-
a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*
a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*
c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - ((-a*b^5)^(1/2)*((256*b^10*
c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5
 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800
*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5
*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 -
 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^
3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 23
04*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*
d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 -
 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^
3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2
*b^2*c^2*d - 3*a^3*b*c*d^2)) - ((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7
```

*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32
*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7
*d)) + ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*
b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^
5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(
64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*
b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (x*(-a*b^5)^(1/2)*(256
*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c
^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8
 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3
*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*
a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))
))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))*(-a*b^5)^(
1/2)*1i)/(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2) - (atan(((
((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6
 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a
^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - (((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^
3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 68
16*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^
2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*
d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-c^5*
d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2 - 1280*a*b
^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^
7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))
/(512*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(b^4*c^8 +
a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)))*(-c^5*
d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3
+ 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2
 - 10*a*b*c*d)*1i)/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c
^7*d)) + (((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3
*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5
*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + (((256*b^10*c^10*d^2 - 1760*a*
b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^
6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 +
 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^
2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) +
 (x*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2
- 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*
a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2
*c^4*d^9))/(512*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(
b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d
)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^
3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 +
15*b^2*c^2 - 10*a*b*c*d)*1i)/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 -
 3*a*b^2*c^7*d)))/((9*a^3*b^5*d^6 - 105*b^8*c^3*d^3 + 115*a*b^7*c^2*d^4 - 5
1*a^2*b^6*c*d^5)/(32*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4
*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (((x
*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 +
190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*
b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - (((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 +
 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*
a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c
^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2
 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-c^5*d)^
(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2 - 1280*a*b^8*
c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d
^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(5
12*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(b^4*c^8 + a^4
*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)))*(-c^5*d)^

```
(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3 + 3
*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 -
10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d))
 - (((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c
*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 +
 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + (((256*b^10*c^10*d^2 - 1760*a*b^9*c^
9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6
- 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^
8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*
c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (x*(-
c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2 - 1280
*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^
5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d
^9))/(512*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(b^4*c^
8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)))*(-
c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*
d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2
*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*
c^7*d))))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*1i)/(8*(b^3*
c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.27 $\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=192

$$\frac{(bc-ad)^4(9ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}+\frac{d^3x^3\left(3a^2d^2-10abcd+10b^2c^2\right)}{3b^4}+\frac{d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10\right.}{b^5}$$

Rubi [A] time = 0.16, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{d^3x^3\left(3a^2d^2-10abcd+10b^2c^2\right)}{3b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{(bc-ad)^4(9ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}+\frac{d^4x^5(5bc-2ad)}{5b^3}+\frac{x(bc-ad)^5}{2ab^5\left(a+bx^2\right)}+\frac{d^5x^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) + (d^4*(5*b*c - 2*a*d
)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c - a*d)^5*x)/(2*a*b^5*(a + b*x^2)
) + ((b*c - a*d)^4*(b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^
(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^5}{\left(a+bx^2\right)^2}\,dx = \int \left(\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)}{b^5} + \frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^2}{b^4} + \frac{d^4(5bc\ \ldots}{\ldots}\right.$$

$$= \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5} + \frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^3}{3b^4} + \frac{d^4(5bc-\ldots}{5b^3\ldots}$$

$$= \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5} + \frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^3}{3b^4} + \frac{d^4(5bc-\ldots}{5b^3\ldots}$$

$$= \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5} + \frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^3}{3b^4} + \frac{d^4(5bc-\ldots}{5b^3\ldots}$$

Mathematica [A] time = 0.10, size = 192, normalized size = 1.00

$$\frac{(bc-ad)^4(9ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}} + \frac{d^3x^3\left(3a^2d^2-10abcd+10b^2c^2\right)}{3b^4} + \frac{d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)}{b^5} + \frac{x(bc-ad)^5}{2ab^5\left(a+bx^2\right)} + \frac{d^4x^5(5bc-2ad)}{5b^3} + \frac{d^5x^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) + (d^4*(5*b*c - 2*a*d
)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c - a*d)^5*x)/(2*a*b^5*(a + b*x^2)
) + ((b*c - a*d)^4*(b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^
(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^5}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^5/(a + b*x^2)^2, x]
```

fricas [B] time = 0.68, size = 810, normalized size = 4.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*a^2*b^5*d^5*x^9 + 12*(35*a^2*b^5*c*d^4 - 9*a^3*b^4*d^5)*x^7 + 28
*(50*a^2*b^5*c^2*d^3 - 35*a^3*b^4*c*d^4 + 9*a^4*b^3*d^5)*x^5 + 140*(30*a^2*
b^5*c^3*d^2 - 50*a^3*b^4*c^2*d^3 + 35*a^4*b^3*c*d^4 - 9*a^5*b^2*d^5)*x^3 -
105*(a*b^5*c^5 + 5*a^2*b^4*c^4*d - 30*a^3*b^3*c^3*d^2 + 50*a^4*b^2*c^2*d^3
- 35*a^5*b*c*d^4 + 9*a^6*d^5 + (b^6*c^5 + 5*a*b^5*c^4*d - 30*a^2*b^4*c^3*d^
2 + 50*a^3*b^3*c^2*d^3 - 35*a^4*b^2*c*d^4 + 9*a^5*b*d^5)*x^2)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 210*(a*b^6*c^5 - 5*a^2*b^5*c^
4*d + 30*a^3*b^4*c^3*d^2 - 50*a^4*b^3*c^2*d^3 + 35*a^5*b^2*c*d^4 - 9*a^6*b*
d^5)*x)/(a^2*b^7*x^2 + a^3*b^6), 1/210*(30*a^2*b^5*d^5*x^9 + 6*(35*a^2*b^5*
c*d^4 - 9*a^3*b^4*d^5)*x^7 + 14*(50*a^2*b^5*c^2*d^3 - 35*a^3*b^4*c*d^4 + 9*
```

```
a^4*b^3*d^5)*x^5 + 70*(30*a^2*b^5*c^3*d^2 - 50*a^3*b^4*c^2*d^3 + 35*a^4*b^3
*c*d^4 - 9*a^5*b^2*d^5)*x^3 + 105*(a*b^5*c^5 + 5*a^2*b^4*c^4*d - 30*a^3*b^3
*c^3*d^2 + 50*a^4*b^2*c^2*d^3 - 35*a^5*b*c*d^4 + 9*a^6*d^5 + (b^6*c^5 + 5*a
*b^5*c^4*d - 30*a^2*b^4*c^3*d^2 + 50*a^3*b^3*c^2*d^3 - 35*a^4*b^2*c*d^4 + 9
*a^5*b*d^5)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 105*(a*b^6*c^5 - 5*a^2*b
^5*c^4*d + 30*a^3*b^4*c^3*d^2 - 50*a^4*b^3*c^2*d^3 + 35*a^5*b^2*c*d^4 - 9*a
^6*b*d^5)*x)/(a^2*b^7*x^2 + a^3*b^6)]
```

giac [A]   time = 0.58, size = 306, normalized size = 1.59

$$\frac{\left(b^{5}c^{5}+5\,ab^{4}c^{4}d-30\,a^{2}b^{3}c^{3}d^{2}+50\,a^{3}b^{2}c^{2}d^{3}-35\,a^{4}bcd^{4}+9\,a^{5}d^{5}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^{5}}+\frac{b^{5}c^{5}x-5\,ab^{4}c^{4}dx+10\,a^{2}b^{3}c^{3}d^{2}x-10\,a^{3}b^{2}c^{2}d^{3}x+5\,a^{4}bcd^{4}x-a^{5}d^{5}x}{2\left(bx^{2}+a\right)ab^{5}}+\frac{15\,b^{12}d^{5}x^{7}+105\,b^{12}cd^{4}x^{5}-42\,ab^{11}d^{5}x^{5}+350\,b^{12}c^{2}d^{3}x^{3}-350\,ab^{11}cd^{4}x^{3}+105\,a^{2}b^{10}d^{5}x^{3}+1050\,b^{12}c^{3}d^{2}x-2100\,ab^{11}c^{2}d^{3}x+1575\,a^{2}b^{10}cd^{4}x-420\,a^{3}b^{9}d^{5}x}{105\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^5*c^5 + 5*a*b^4*c^4*d - 30*a^2*b^3*c^3*d^2 + 50*a^3*b^2*c^2*d^3 - 35
*a^4*b*c*d^4 + 9*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^5) + 1/2*(b^
5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x + 5
*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^2 + a)*a*b^5) + 1/105*(15*b^12*d^5*x^7 +
105*b^12*c*d^4*x^5 - 42*a*b^11*d^5*x^5 + 350*b^12*c^2*d^3*x^3 - 350*a*b^11*
c*d^4*x^3 + 105*a^2*b^10*d^5*x^3 + 1050*b^12*c^3*d^2*x - 2100*a*b^11*c^2*d^
3*x + 1575*a^2*b^10*c*d^4*x - 420*a^3*b^9*d^5*x)/b^14
```

maple [B]   time = 0.01, size = 402, normalized size = 2.09

$$\frac{d^{5}x^{7}}{7b^{2}}-\frac{2a\,d^{5}x^{5}}{5b^{3}}+\frac{c\,d^{4}x^{5}}{b^{2}}+\frac{a^{2}d^{5}x^{3}}{b^{4}}-\frac{10ac\,d^{4}x^{3}}{3b^{3}}+\frac{10c^{2}d^{3}x^{3}}{3b^{2}}-\frac{a^{4}d^{5}x}{2\left(b\,x^{2}+a\right)b^{5}}+\frac{9a^{4}d^{5}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{5}}+\frac{5a^{3}c\,d^{4}x}{2\left(b\,x^{2}+a\right)b^{4}}-\frac{35a^{3}c\,d^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{4}}-\frac{5a^{2}c^{2}d^{3}x}{\left(b\,x^{2}+a\right)b^{3}}+\frac{25a^{2}c^{2}d^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{3}}+\frac{5a\,c^{3}d^{2}x}{\left(b\,x^{2}+a\right)b^{2}}-\frac{15a\,c^{3}d^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{2}}+\frac{c^{5}x}{2\left(b\,x^{2}+a\right)a}+\frac{c^{5}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{5c^{4}dx}{2\left(b\,x^{2}+a\right)b}+\frac{5c^{4}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}-\frac{4a^{3}d^{5}x}{b^{5}}+\frac{15a^{2}c\,d^{4}x}{b^{4}}-\frac{20a\,c^{2}d^{3}x}{b^{3}}+\frac{10c^{3}d^{2}x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^5/(b*x^2+a)^2,x)
```

```
[Out] 1/7*d^5*x^7/b^2-2/5*d^5/b^3*x^5*a+d^4/b^2*x^5*c+d^5/b^4*x^3*a^2-10/3*d^4/b^
3*x^3*a*c+10/3*d^3/b^2*x^3*c^2-4*d^5/b^5*a^3*x+15*d^4/b^4*a^2*c*x-20*d^3/b^
3*a*c^2*x+10*d^2/b^2*c^3*x-1/2/b^5*a^4*x/(b*x^2+a)*d^5+5/2/b^4*a^3*x/(b*x^2
+a)*c*d^4-5/b^3*a^2*x/(b*x^2+a)*c^2*d^3+5/b^2*a*x/(b*x^2+a)*c^3*d^2-5/2/b*x
/(b*x^2+a)*c^4*d+1/2/a*x/(b*x^2+a)*c^5+9/2/b^5*a^4/(a*b)^(1/2)*arctan(1/(a*
b)^(1/2)*b*x)*d^5-35/2/b^4*a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d^4+
25/b^3*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2*d^3-15/b^2*a/(a*b)^(1/
2)*arctan(1/(a*b)^(1/2)*b*x)*c^3*d^2+5/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)
*b*x)*c^4*d+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^5
```

maxima [A]   time = 3.03, size = 294, normalized size = 1.53

$$\frac{\left(b^{5}c^{5}-5\,ab^{4}c^{4}d+10\,a^{2}b^{3}c^{3}d^{2}-10\,a^{3}b^{2}c^{2}d^{3}+5\,a^{4}bcd^{4}-a^{5}d^{5}\right)x}{2\left(ab^{6}x^{2}+a^{2}b^{5}\right)}+\frac{15\,b^{3}d^{5}x^{7}+21\left(5\,b^{3}cd^{4}-2\,ab^{2}d^{5}\right)x^{5}+35\left(10\,b^{3}c^{2}d^{3}-10\,ab^{2}cd^{4}+3\,a^{2}bd^{5}\right)x^{3}+105\left(10\,b^{3}c^{3}d^{2}-20\,ab^{2}c^{2}d^{3}+15\,a^{2}bcd^{4}-4\,a^{3}d^{5}\right)x}{105\,b^{5}}+\frac{\left(b^{5}c^{5}+5\,ab^{4}c^{4}d-30\,a^{2}b^{3}c^{3}d^{2}+50\,a^{3}b^{2}c^{2}d^{3}-35\,a^{4}bcd^{4}+9\,a^{5}d^{5}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^2 + a^2*b^5) + 1/105*(15*b^3*d^5*x^7 + 21
*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^5 + 35*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 + 3*
a^2*b*d^5)*x^3 + 105*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4 -
4*a^3*d^5)*x)/b^5 + 1/2*(b^5*c^5 + 5*a*b^4*c^4*d - 30*a^2*b^3*c^3*d^2 + 50*
a^3*b^2*c^2*d^3 - 35*a^4*b*c*d^4 + 9*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a*b^5)
```

mupad [B]   time = 5.02, size = 386, normalized size = 2.01

$$x\left(\frac{10\,c^{3}\,d^{2}}{b^{2}}-\frac{2\,a\left(\frac{2\,a\left(\frac{2\,a\,d^{5}}{b^{3}}-\frac{5\,c\,d^{4}}{b^{2}}\right)}{b}-\frac{a^{2}\,d^{5}}{b^{4}}+\frac{10\,c^{2}\,d^{3}}{b^{2}}\right)}{b}+\frac{a^{2}\left(\frac{2\,a\,d^{5}}{b^{3}}-\frac{5\,c\,d^{4}}{b^{2}}\right)}{b^{2}}\right)-x^{5}\left(\frac{2\,a\,d^{5}}{5\,b^{3}}-\frac{c\,d^{4}}{b^{2}}\right)+x^{3}\left(\frac{2\,a\left(\frac{2\,a\,d^{5}}{b^{3}}-\frac{5\,c\,d^{4}}{b^{2}}\right)}{3\,b}-\frac{a^{2}\,d^{5}}{3\,b^{4}}+\frac{10\,c^{2}\,d^{3}}{3\,b^{2}}\right)+\frac{d^{5}\,x^{7}}{7\,b^{2}}-\frac{x\left(a^{5}\,d^{5}-5\,a^{4}\,b\,c\,d^{4}+10\,a^{3}\,b^{2}\,c^{2}\,d^{3}-10\,a^{2}\,b^{3}\,c^{3}\,d^{2}+5\,a\,b^{4}\,c^{4}\,d-b^{5}\,c^{5}\right)}{2\,a\left(b^{6}\,x^{2}+a\,b^{5}\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^{4}\,(9\,a\,d+b\,c)}{\sqrt{a}\left(9\,a^{5}\,d^{5}-35\,a^{4}\,b\,c\,d^{4}+50\,a^{3}\,b^{2}\,c^{2}\,d^{3}-30\,a^{2}\,b^{3}\,c^{3}\,d^{2}+5\,a\,b^{4}\,c^{4}\,d+b^{5}\,c^{5}\right)}\right)(a\,d-b\,c)^{4}\,(9\,a\,d+b\,c)}{2\,a^{3/2}\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^5/(a + b*x^2)^2,x)
```

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2*
d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2
) - x^5*((2*a*d^5)/(5*b^3) - (c*d^4)/b^2) + x^3*((2*a*((2*a*d^5)/b^3 - (5*c
*d^4)/b^2))/(3*b) - (a^2*d^5)/(3*b^4) + (10*c^2*d^3)/(3*b^2)) + (d^5*x^7)/(
7*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 +
5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(2*a*(a*b^5 + b^6*x^2)) + (atan((b^(1/2)*x*
(a*d - b*c)^4*(9*a*d + b*c))/(a^(1/2)*(9*a^5*d^5 + b^5*c^5 - 30*a^2*b^3*c^3
*d^2 + 50*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 35*a^4*b*c*d^4)))*(a*d - b*c)^4
*(9*a*d + b*c))/(2*a^(3/2)*b^(11/2))
```

sympy [B] time = 1.91, size = 502, normalized size = 2.61

$$x^{5}\left(-\frac{2ad^{5}}{5b^{3}}+\frac{cd^{4}}{b^{2}}\right)+x^{3}\left(\frac{a^{2}d^{5}}{b^{4}}-\frac{10acd^{4}}{3b^{3}}+\frac{10c^{2}d^{3}}{3b^{2}}\right)+x\left(-\frac{4a^{3}d^{5}}{b^{5}}+\frac{15a^{2}cd^{4}}{b^{4}}-\frac{20ac^{2}d^{3}}{b^{3}}+\frac{10c^{3}d^{2}}{b^{2}}\right)+\frac{x\left(-a^{5}d^{5}+5a^{4}bcd^{4}-10a^{3}b^{2}c^{2}d^{3}+10a^{2}b^{3}c^{3}d^{2}-5ab^{4}c^{4}d+b^{5}c^{5}\right)}{2a^{2}b^{5}+2ab^{6}x^{2}}-\frac{\sqrt{-\frac{1}{a^{3}b^{11}}}(ad-bc)^{4}(9ad+bc)\log\left(-\frac{a^{2}b^{5}\sqrt{-\frac{1}{a^{3}b^{11}}}(ad-bc)^{4}(9ad+bc)}{9a^{5}d^{5}-35a^{4}bcd^{4}+50a^{3}b^{2}c^{2}d^{3}-30a^{2}b^{3}c^{3}d^{2}+5ab^{4}c^{4}d+b^{5}c^{5}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{3}b^{11}}}(ad-bc)^{4}(9ad+bc)\log\left(\frac{a^{2}b^{5}\sqrt{-\frac{1}{a^{3}b^{11}}}(ad-bc)^{4}(9ad+bc)}{9a^{5}d^{5}-35a^{4}bcd^{4}+50a^{3}b^{2}c^{2}d^{3}-30a^{2}b^{3}c^{3}d^{2}+5ab^{4}c^{4}d+b^{5}c^{5}}+x\right)}{4}+\frac{d^{5}x^{7}}{7b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**5/(b*x**2+a)**2,x)
```

```
[Out] x**5*(-2*a*d**5/(5*b**3) + c*d**4/b**2) + x**3*(a**2*d**5/b**4 - 10*a*c*d**
4/(3*b**3) + 10*c**2*d**3/(3*b**2)) + x*(-4*a**3*d**5/b**5 + 15*a**2*c*d**4
/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d**5 + 5*a**4*b
*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b**4*c**4*d
 + b**5*c**5)/(2*a**2*b**5 + 2*a*b**6*x**2) - sqrt(-1/(a**3*b**11))*(a*d -
b*c)**4*(9*a*d + b*c)*log(-a**2*b**5*sqrt(-1/(a**3*b**11))*(a*d - b*c)**4*(
9*a*d + b*c)/(9*a**5*d**5 - 35*a**4*b*c*d**4 + 50*a**3*b**2*c**2*d**3 - 30*
a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + b**5*c**5) + x)/4 + sqrt(-1/(a**3*b
**11))*(a*d - b*c)**4*(9*a*d + b*c)*log(a**2*b**5*sqrt(-1/(a**3*b**11))*(a*
d - b*c)**4*(9*a*d + b*c)/(9*a**5*d**5 - 35*a**4*b*c*d**4 + 50*a**3*b**2*c*
*2*d**3 - 30*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + b**5*c**5) + x)/4 + d*
*5*x**7/(7*b**2)
```

3.28 $\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=142

$$\frac{(bc-ad)^3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}+\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}+\frac{x(bc-ad)^4}{2ab^4\left(a+bx^2\right)}+\frac{2d^3x^3(2bc-ad)}{3b^3}+\frac{d^4x^5}{5b^2}$$

Rubi [A] time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}+\frac{(bc-ad)^3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}+\frac{2d^3x^3(2bc-ad)}{3b^3}+\frac{x(bc-ad)^4}{2ab^4\left(a+bx^2\right)}+\frac{d^4x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^3)
/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b^4*(a + b*x^2)) + ((
b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^2}\,dx = \int \left(\frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)}{b^4} + \frac{2d^3(2bc-ad)x^2}{b^3} + \frac{d^4x^4}{b^2} + \frac{(bc-ad)^3(bc+3ad)+4bd(bc-}{b^4\left(a+bx^2\right)^2}\right.$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4} + \frac{2d^3(2bc-ad)x^3}{3b^3} + \frac{d^4x^5}{5b^2} + \frac{\int \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^2}{\left(a+bx^2\right)^2}dx}{b^4}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4} + \frac{2d^3(2bc-ad)x^3}{3b^3} + \frac{d^4x^5}{5b^2} + \frac{(bc-ad)^4x}{2ab^4\left(a+bx^2\right)} + \frac{\left((bc-ad)^3(bc+\right.}{2a}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4} + \frac{2d^3(2bc-ad)x^3}{3b^3} + \frac{d^4x^5}{5b^2} + \frac{(bc-ad)^4x}{2ab^4\left(a+bx^2\right)} + \frac{(bc-ad)^3(bc+7}{2a^{3/}}$$

Mathematica [A] time = 0.09, size = 142, normalized size = 1.00

$$\frac{(bc-ad)^3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}} + \frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4} + \frac{x(bc-ad)^4}{2ab^4\left(a+bx^2\right)} + \frac{2d^3x^3(2bc-ad)}{3b^3} + \frac{d^4x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^3)
/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b^4*(a + b*x^2)) + ((
b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^4/(a + b*x^2)^2, x]
```

fricas [B] time = 0.86, size = 612, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a^2*b^4*d^4*x^7 + 4*(20*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*x^5 + 20*(
18*a^2*b^4*c^2*d^2 - 20*a^3*b^3*c*d^3 + 7*a^4*b^2*d^4)*x^3 + 15*(a*b^4*c^4
+ 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 20*a^4*b*c*d^3 - 7*a^5*d^4 + (b^5*
c^4 + 4*a*b^4*c^3*d - 18*a^2*b^3*c^2*d^2 + 20*a^3*b^2*c*d^3 - 7*a^4*b*d^4)*
x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(a*b^5*c
^4 - 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 20*a^4*b^2*c*d^3 + 7*a^5*b*d^4)
*x)/(a^2*b^6*x^2 + a^3*b^5), 1/30*(6*a^2*b^4*d^4*x^7 + 2*(20*a^2*b^4*c*d^3
- 7*a^3*b^3*d^4)*x^5 + 10*(18*a^2*b^4*c^2*d^2 - 20*a^3*b^3*c*d^3 + 7*a^4*b^
2*d^4)*x^3 + 15*(a*b^4*c^4 + 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 20*a^4*
b*c*d^3 - 7*a^5*d^4 + (b^5*c^4 + 4*a*b^4*c^3*d - 18*a^2*b^3*c^2*d^2 + 20*a^
3*b^2*c*d^3 - 7*a^4*b*d^4)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 15*(a*b^5
```

```
*c^4 - 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 20*a^4*b^2*c*d^3 + 7*a^5*b*d^
4)*x)/(a^2*b^6*x^2 + a^3*b^5)]
```

giac [A] time = 0.58, size = 220, normalized size = 1.55

$$\frac{\left(b^4c^4 + 4\,ab^3c^3d - 18\,a^2b^2c^2d^2 + 20\,a^3bcd^3 - 7\,a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^4} + \frac{b^4c^4x - 4\,ab^3c^3dx + 6\,a^2b^2c^2d^2x - 4\,a^3bcd^3x + a^4d^4x}{2\left(bx^2 + a\right)ab^4} + \frac{3\,b^8d^4x^5 + 20\,b^8cd^3x^3 - 10\,ab^7d^4x^3 + 90\,b^8c^2d^2x - 120\,ab^7cd^3x + 45\,a^2b^6d^4x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*c^4 + 4*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 7*a^4*
d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/2*(b^4*c^4*x - 4*a*b^3*c^3
*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*d^4*x)/((b*x^2 + a)*a*b^
4) + 1/15*(3*b^8*d^4*x^5 + 20*b^8*c*d^3*x^3 - 10*a*b^7*d^4*x^3 + 90*b^8*c^2
*d^2*x - 120*a*b^7*c*d^3*x + 45*a^2*b^6*d^4*x)/b^10
```

maple [B] time = 0.01, size = 296, normalized size = 2.08

$$\frac{d^4x^5}{5b^2} - \frac{2a\,d^4x^3}{3b^3} + \frac{4c\,d^3x^3}{3b^2} + \frac{a^3d^4x}{2\left(b\,x^2 + a\right)b^4} - \frac{7a^3d^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^4} - \frac{2a^2c\,d^3x}{\left(b\,x^2 + a\right)b^3} + \frac{10a^2c\,d^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3a\,c^2d^2x}{\left(b\,x^2 + a\right)b^2} - \frac{9a\,c^2d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{c^4x}{2\left(b\,x^2 + a\right)a} + \frac{c^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a} - \frac{2c^3dx}{\left(b\,x^2 + a\right)b} + \frac{2c^3d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b} + \frac{3a^2d^4x}{b^4} - \frac{8ac\,d^3x}{b^3} + \frac{6c^2d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a)^2,x)
```

```
[Out] 1/5*d^4*x^5/b^2-2/3*d^4/b^3*x^3*a+4/3*d^3/b^2*x^3*c+3*d^4/b^4*a^2*x-8*d^3/b
^3*a*c*x+6*d^2/b^2*c^2*x+1/2/b^4*a^3*x/(b*x^2+a)*d^4-2/b^3*a^2*x/(b*x^2+a)*
c*d^3+3/b^2*a*x/(b*x^2+a)*c^2*d^2-2/b*x/(b*x^2+a)*c^3*d+1/2/a*x/(b*x^2+a)*c
^4-7/2/b^4*a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^4+10/b^3*a^2/(a*b)^(
1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d^3-9/b^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/
2)*b*x)*c^2*d^2+2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^3*d+1/2/a/(a*b)
^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^4
```

maxima [A] time = 3.00, size = 213, normalized size = 1.50

$$\frac{\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)x}{2\left(ab^5x^2 + a^2b^4\right)} + \frac{3\,b^2d^4x^5 + 10\left(2\,b^2cd^3 - abd^4\right)x^3 + 15\left(6\,b^2c^2d^2 - 8\,abcd^3 + 3\,a^2d^4\right)x}{15\,b^4} + \frac{\left(b^4c^4 + 4\,ab^3c^3d - 18\,a^2b^2c^2d^2 + 20\,a^3bcd^3 - 7\,a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*x/(a*b^5*x^2 + a^2*b^4) + 1/15*(3*b^2*d^4*x^5 + 10*(2*b^2*c*d^3 - a*b*d^4)
*x^3 + 15*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 1/2*(b^4*c^4 +
 4*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 7*a^4*d^4)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

mupad [B] time = 5.05, size = 261, normalized size = 1.84

$$x\left(\frac{2\,a\,\left(\frac{2\,a\,d^4}{b^3} - \frac{4\,c\,d^3}{b^2}\right)}{b} - \frac{a^2\,d^4}{b^4} + \frac{6\,c^2\,d^2}{b^2}\right) - x^3\,\left(\frac{2\,a\,d^4}{3\,b^3} - \frac{4\,c\,d^3}{3\,b^2}\right) + \frac{d^4\,x^5}{5\,b^2} + \frac{x\,\left(a^4\,d^4 - 4\,a^3\,b\,c\,d^3 + 6\,a^2\,b^2\,c^2\,d^2 - 4\,a\,b^3\,c^3\,d + b^4\,c^4\right)}{2\,a\,\left(b^5\,x^2 + a\,b^4\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3\,\left(7\,a\,d+b\,c\right)}{\sqrt{a}\,\left(-7\,a^4\,d^4+20\,a^3\,b\,c\,d^3-18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}\right)\,{\left(a\,d - b\,c\right)}^3\,\left(7\,a\,d + b\,c\right)}{2\,a^{3/2}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^4/(a + b*x^2)^2,x)
```

```
[Out] x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^
2) - x^3*((2*a*d^4)/(3*b^3) - (4*c*d^3)/(3*b^2)) + (d^4*x^5)/(5*b^2) + (x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(2*
a*(a*b^4 + b^5*x^2)) + (atan((b^(1/2)*x*(a*d - b*c)^3*(7*a*d + b*c))/(a^(1/
2)*(b^4*c^4 - 7*a^4*d^4 - 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 20*a^3*b*c*d
^3)))*(a*d - b*c)^3*(7*a*d + b*c))/(2*a^(3/2)*b^(9/2))
```

sympy [B] time = 1.47, size = 403, normalized size = 2.84

$$x^3\left(-\frac{2ad^4}{3b^3}+\frac{4cd^3}{3b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right)+\frac{x\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{2a^2b^4+2ab^5x^2}+\frac{\sqrt{-\frac{1}{a^3b^9}}\,(ad-bc)^3\,(7ad+bc)\log\left(-\frac{a^2b^4\sqrt{-\frac{1}{a^3b^9}}\,(ad-bc)^3(7ad+bc)}{7a^4d^4-20a^3bcd^3+18a^2b^2c^2d^2-4ab^3c^3d-b^4c^4}+x\right)}{4}-\frac{\sqrt{-\frac{1}{a^3b^9}}\,(ad-bc)^3\,(7ad+bc)\log\left(\frac{a^2b^4\sqrt{-\frac{1}{a^3b^9}}\,(ad-bc)^3(7ad+bc)}{7a^4d^4-20a^3bcd^3+18a^2b^2c^2d^2-4ab^3c^3d-b^4c^4}+x\right)}{4}+\frac{d^4x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**2,x)
```

```
[Out] x**3*(-2*a*d**4/(3*b**3) + 4*c*d**3/(3*b**2)) + x*(3*a**2*d**4/b**4 - 8*a*c
*d**3/b**3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b*
*2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(2*a**2*b**4 + 2*a*b**5*x**2) +
 sqrt(-1/(a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)*log(-a**2*b**4*sqrt(-1/(
a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)/(7*a**4*d**4 - 20*a**3*b*c*d**3 +
18*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d - b**4*c**4) + x)/4 - sqrt(-1/(a**
3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)*log(a**2*b**4*sqrt(-1/(a**3*b**9))*(a
*d - b*c)**3*(7*a*d + b*c)/(7*a**4*d**4 - 20*a**3*b*c*d**3 + 18*a**2*b**2*c
**2*d**2 - 4*a*b**3*c**3*d - b**4*c**4) + x)/4 + d**4*x**5/(5*b**2)
```

3.29 $\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=106

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^2}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{b^3\left(a+bx^2\right)^2}\right)dx$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{\left(a+bx^2\right)^2}\,dx}{b^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\left(a+bx^2\right)}+\frac{\left((bc-ad)^2(bc+5ad)\right)\int \frac{1}{a+bx^2}\,dx}{2ab^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\left(a+bx^2\right)}+\frac{(bc-ad)^2(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 1.00

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2)^2, x]
```

fricas [B] time = 0.85, size = 442, normalized size = 4.17

$$\left[\frac{4\,a^2b^3d^3x^5+4\left(9\,a^2b^3cd^2-5\,a^3b^2d^3\right)x^3-3\left(ab^3c^3+3\,a^2b^2c^2d-9\,a^3bcd^2+5\,a^4d^3+\left(b^4c^3+3\,ab^3c^2d-9\,a^2b^2cd^2+5\,a^3bd^3\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+6\left(ab^4c^3-3\,a^2b^3c^2d+9\,a^3b^2cd^2-5\,a^4bd^3\right)x}{12\left(a^2b^5x^2+a^3b^4\right)},\ \frac{2\,a^2b^3d^3x^5+2\left(9\,a^2b^3cd^2-5\,a^3b^2d^3\right)x^3+3\left(ab^3c^3+3\,a^2b^2c^2d-9\,a^3bcd^2+5\,a^4d^3+\left(b^4c^3+3\,ab^3c^2d-9\,a^2b^2cd^2+5\,a^3bd^3\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\left(ab^4c^3-3\,a^2b^3c^2d+9\,a^3b^2cd^2-5\,a^4bd^3\right)x}{6\left(a^2b^5x^2+a^3b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*b^3*d^3*x^5 + 4*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 - 3*(a*b
^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c
^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 9*a^3*b^2*c*d^2
 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2*a^2*b^3*d^3*x^5 + 2*(9*a
^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 + 3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*
b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*
d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c^3 - 3*a^2*b^3*c^2*d
+ 9*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

giac [A] time = 0.57, size = 152, normalized size = 1.43

$$\frac{\left(b^{3} c^{3} + 3 \, a b^{2} c^{2} d - 9 \, a^{2} b c d^{2} + 5 \, a^{3} d^{3}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, \sqrt{a b} a b^{3}} + \frac{b^{3} c^{3} x - 3 \, a b^{2} c^{2} d x + 3 \, a^{2} b c d^{2} x - a^{3} d^{3} x}{2 \, {\left(b x^{2} + a\right)} a b^{3}} + \frac{b^{4} d^{3} x^{3} + 9 \, b^{4} c d^{2} x - 6 \, a b^{3} d^{3} x}{3 \, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*d^3*x^3 + 9*b^4*c*d^2*x - 6*a*
b^3*d^3*x)/b^6
```

maple [B] time = 0.01, size = 205, normalized size = 1.93

$$\frac{d^{3} x^{3}}{3 b^{2}} - \frac{a^{2} d^{3} x}{2 \left(b \,x^{2}+a \right) b^{3}} + \frac{5 a^{2} d^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\, b^{3}} + \frac{3 a c \,d^{2} x}{2 \left(b \,x^{2}+a \right) b^{2}} - \frac{9 a c \,d^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\, b^{2}} + \frac{c^{3} x}{2 \left(b \,x^{2}+a \right) a} + \frac{c^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\, a} - \frac{3 c^{2} d x}{2 \left(b \,x^{2}+a \right) b} + \frac{3 c^{2} d \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\, b} - \frac{2 a \,d^{3} x}{b^{3}} + \frac{3 c \,d^{2} x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/3/b^2*d^3*x^3-2*d^3/b^3*a*x+3/b^2*c*d^2*x-1/2/b^3*a^2*x/(b*x^2+a)*d^3+3/2
/b^2*a*x/(b*x^2+a)*c*d^2-3/2/b*x/(b*x^2+a)*c^2*d+1/2/a*x/(b*x^2+a)*c^3+5/2/
b^3*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^3-9/2/b^2*a/(a*b)^(1/2)*arc
tan(1/(a*b)^(1/2)*b*x)*c*d^2+3/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^
2*d+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^3
```

maxima [A] time = 2.99, size = 147, normalized size = 1.39

$$\frac{{\left(b^{3} c^{3} - 3 \, a b^{2} c^{2} d + 3 \, a^{2} b c d^{2} - a^{3} d^{3}\right)} x}{2 \, {\left(a b^{4} x^{2} + a^{2} b^{3}\right)}} + \frac{b d^{3} x^{3} + 3 \, {\left(3 \, b c d^{2} - 2 \, a d^{3}\right)} x}{3 \, b^{3}} + \frac{{\left(b^{3} c^{3} + 3 \, a b^{2} c^{2} d - 9 \, a^{2} b c d^{2} + 5 \, a^{3} d^{3}\right)} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, \sqrt{a b} a b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^2 + a^2*
b^3) + 1/3*(b*d^3*x^3 + 3*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/2*(b^3*c^3 + 3*a
*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*
b^3)
```

mupad [B] time = 0.10, size = 182, normalized size = 1.72

$$\frac{d^3\,x^3}{3\,b^2}-x\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)-\frac{x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,a\,\left(b^4\,x^2+a\,b^3\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(5\,a\,d+b\,c\right)}{\sqrt{a}\,\left(5\,a^3\,d^3-9\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(5\,a\,d+b\,c\right)}{2\,a^{3/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^2,x)
```

```
[Out] (d^3*x^3)/(3*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(1/
2)*x*(a*d - b*c)^2*(5*a*d + b*c))/(a^(1/2)*(5*a^3*d^3 + b^3*c^3 + 3*a*b^2*c
^2*d - 9*a^2*b*c*d^2)))*(a*d - b*c)^2*(5*a*d + b*c))/(2*a^(3/2)*b^(7/2))
```

sympy [B] time = 1.08, size = 314, normalized size = 2.96

$$x \left(- \frac{2 a d^{3}}{b^{3}} + \frac{3 c d^{2}}{b^{2}}\right) + \frac{x \left(- a^{3} d^{3} + 3 a^{2} b c d^{2} - 3 a b^{2} c^{2} d + b^{3} c^{3}\right)}{2 a^{2} b^{3} + 2 a b^{4} x^{2}} - \frac{\sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \left(5 a d + b c\right) \log{\left(- \frac{a^{2} b^{3} \sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \left(5 a d + b c\right)}{5 a^{3} d^{3} - 9 a^{2} b c d^{2} + 3 a b^{2} c^{2} d + b^{3} c^{3}} + x \right)}}{4} + \frac{\sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \left(5 a d + b c\right) \log{\left(\frac{a^{2} b^{3} \sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \left(5 a d + b c\right)}{5 a^{3} d^{3} - 9 a^{2} b c d^{2} + 3 a b^{2} c^{2} d + b^{3} c^{3}} + x \right)}}{4} + \frac{d^{3} x^{3}}{3 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(2*a**2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7
))*(a*d - b*c)**2*(5*a*d + b*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d -
b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b*
*3*c**3) + x)/4 + sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)*log(a**
2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a
**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3) + x)/4 + d**3*x**3/(3*b**2)
```

3.30 $\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=82

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}$$

Rubi [A] time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a*d)*(b*c +
 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx = \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{b^2\left(a+bx^2\right)^2}\right)dx$$
$$=\frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{\left(a+bx^2\right)^2}\,dx}{b^2}$$
$$=\frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)}+\frac{((bc-ad)(bc+3ad))\int \frac{1}{a+bx^2}\,dx}{2ab^2}$$
$$=\frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)}+\frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 1.07

$$\frac{\left(-3a^2d^2+2abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b^2*c^2 + 2*a*b*c
*d - 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2)^2, x]
```

fricas [A] time = 0.95, size = 297, normalized size = 3.62

$$\left[\frac{4\,a^2b^2d^2x^3+\left(ab^2c^2+2\,a^2bcd-3\,a^3d^2+\left(b^3c^2+2\,ab^2cd-3\,a^2bd^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(ab^3c^2-2\,a^2b^2cd+3\,a^3bd^2\right)x}{4\left(a^2b^4x^2+a^3b^3\right)},\ \frac{2\,a^2b^2d^2x^3+\left(ab^2c^2+2\,a^2bcd-3\,a^3d^2+\left(b^3c^2+2\,ab^2cd-3\,a^2bd^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(ab^3c^2-2\,a^2b^2cd+3\,a^3bd^2\right)x}{2\left(a^2b^4x^2+a^3b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 +
 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a
)/(b*x^2 + a)) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^
2 + a^3*b^3), 1/2*(2*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2
 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) + (a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^2 + a^3*b^3)]
```

giac [A] time = 0.58, size = 94, normalized size = 1.15

$$\frac{d^2x}{b^2}+\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^2}+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\left(bx^2+a\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a*b^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a*b
^2)
```

maple [A] time = 0.01, size = 129, normalized size = 1.57

$$\frac{a\,d^2 x}{2\left(b\,x^2+a\right)b^2}-\frac{3a\,d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2}+\frac{c^2 x}{2\left(b\,x^2+a\right)a}+\frac{c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{cdx}{\left(b\,x^2+a\right)b}+\frac{cd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/b^2*d^2*x+1/2/b^2*a*x/(b*x^2+a)*d^2-1/b*x/(b*x^2+a)*c*d+1/2/a*x/(b*x^2+a)
*c^2-3/2/b^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2+1/b/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x)*c*d+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2
```

maxima [A] time = 2.84, size = 95, normalized size = 1.16

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)x}{2\left(ab^3x^2+a^2b^2\right)}+\frac{d^2x}{b^2}+\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^2 + a^2*b^2) + d^2*x/b^2 + 1
/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2
)
```

mupad [B] time = 5.06, size = 124, normalized size = 1.51

$$\frac{d^2\,x}{b^2}+\frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,a\,\left(b^3\,x^2+a\,b^2\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+b\,c\right)}{\sqrt{a}\,\left(-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+b\,c\right)}{2\,a^{3/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*a*(a*b^2 + b^3*x^2)) +
 (atan((b^(1/2)*x*(a*d - b*c)*(3*a*d + b*c))/(a^(1/2)*(b^2*c^2 - 3*a^2*d^2
+ 2*a*b*c*d)))*(a*d - b*c)*(3*a*d + b*c))/(2*a^(3/2)*b^(5/2))
```

sympy [B] time = 0.72, size = 236, normalized size = 2.88

$$\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{2a^2b^2+2ab^3x^2}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)\,(3ad+bc)\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4}-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)\,(3ad+bc)\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4}+\frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + sqrt(
-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5
))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4 -
 sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(a**2*b**2*sqrt(-1/(a**3
*b**5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x
)/4 + d**2*x/b**2
```

3.31 $\int \frac{c+dx^2}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=63

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 205}

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{\left(a+bx^2\right)^2}\,dx &= \frac{(bc-ad)x}{2ab\left(a+bx^2\right)}+\frac{(bc+ad)\int \frac{1}{a+bx^2}\,dx}{2ab}\\ &= \frac{(bc-ad)x}{2ab\left(a+bx^2\right)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 1.00

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{x(ad-bc)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((-(b*c) + a*d)*x)/(a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2)^2, x]
```

fricas [A] time = 0.58, size = 181, normalized size = 2.87

$$\left[-\frac{\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\left(ab^2c-a^2bd\right)x}{4\left(a^2b^3x^2+a^3b^2\right)},\ \frac{\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(ab^2c-a^2bd\right)x}{2\left(a^2b^3x^2+a^3b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^
2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a) + (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

giac [A] time = 0.57, size = 57, normalized size = 0.90

$$\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}+\frac{bcx-adx}{2\left(bx^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(b*c*x - a*d*x)
/((b*x^2 + a)*a*b)
```

maple [A] time = 0.01, size = 68, normalized size = 1.08

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}+\frac{d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}-\frac{(ad-bc)\,x}{2\left(b\,x^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*(a*d-b*c)/a/b*x/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*
d+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 3.11, size = 57, normalized size = 0.90

$$\frac{(bc-ad)x}{2\left(ab^2x^2+a^2b\right)}+\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)*x/(a*b^2*x^2 + a^2*b) + 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a*b)
```

mupad [B] time = 5.04, size = 51, normalized size = 0.81

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d+b\,c)}{2\,a^{3/2}\,b^{3/2}}-\frac{x\,(a\,d-b\,c)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^2,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(a*d + b*c))/(2*a^(3/2)*b^(3/2)) - (x*(a*d - b*c
))/(2*a*b*(a + b*x^2))
```

sympy [B] time = 0.40, size = 112, normalized size = 1.78

$$\frac{x\,(-ad+bc)}{2a^2b+2ab^2x^2}-\frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad+bc)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad+bc)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

3.32 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{b}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 205}

$$\frac{\sqrt{b}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)}-\frac{\int \frac{-bc+2ad-bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}\\
&= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)}+\frac{d^2\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2}+\frac{(b(bc-3ad))\int \frac{1}{a+bx^2}\,dx}{2a(bc-ad)^2}\\
&= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)}+\frac{\sqrt{b}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 109, normalized size = 1.01

$$-\frac{\sqrt{b}(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(ad-bc)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}-\frac{bx}{2a\left(a+bx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/2*(b*x)/(a*(-(b*c) + a*d)*(a + b*x^2)) - (Sqrt[b]*(-(b*c) + 3*a*d)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*ArcTan[(Sqr
t[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [A] time = 1.12, size = 699, normalized size = 6.47

$$\left[-\frac{\left(abc-3a^2d+\left(b^2c-3abd\right)x^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\left(abdx^2+a^2d\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(b^2c-abd\right)x}{4\left(a^2b^2c^2-2a^3bcd+a^4d^2+\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)x^2\right)}, \frac{4\left(abdx^2+a^2d\right)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-\left(abc-3a^2d+\left(b^2c-3abd\right)x^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+2\left(b^2c-abd\right)x}{4\left(a^2b^2c^2-2a^3bcd+a^4d^2+\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)x^2\right)}, \frac{\left(abc-3a^2d+\left(b^2c-3abd\right)x^2\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+\left(abdx^2+a^2d\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+\left(b^2c-abd\right)x}{2\left(a^2b^2c^2-2a^3bcd+a^4d^2+\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)x^2\right)}, \frac{\left(abc-3a^2d+\left(b^2c-3abd\right)x^2\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\left(abdx^2+a^2d\right)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+\left(b^2c-abd\right)x}{2\left(a^2b^2c^2-2a^3bcd+a^4d^2+\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*
a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((d*
x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^
2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1
/4*(4*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (a*b*c - 3*a^2*d
+ (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) + 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b
^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c -
3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (a*b*d*x^2 + a^2*d)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (b^2*c - a*b*d)*x)/(a^2
*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*
x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (b^2*c - a*
b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^2)]
```

giac [A] time = 0.58, size = 121, normalized size = 1.12

$$\frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}}+\frac{\left(b^2c-3\,abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{ab}}+\frac{bx}{2\left(abc-a^2d\right)\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^
2)*sqrt(a*b)) + 1/2*b*x/((a*b*c - a^2*d)*(b*x^2 + a))
```

maple [A] time = 0.01, size = 144, normalized size = 1.33

$$\frac{b^{2}cx}{2\left(ad-bc\right)^{2}\left(b\,x^{2}+a\right)a}+\frac{b^{2}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{2}\sqrt{ab}\,a}-\frac{bdx}{2\left(ad-bc\right)^{2}\left(b\,x^{2}+a\right)}-\frac{3bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{2}\sqrt{ab}}+\frac{d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(ad-bc\right)^{2}\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] d^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)-1/2*b/(a*d-b*c)^2*x/(
b*x^2+a)*d+1/2*b^2/(a*d-b*c)^2/a*x/(b*x^2+a)*c-3/2*b/(a*d-b*c)^2/(a*b)^(1/2
)*arctan(1/(a*b)^(1/2)*b*x)*d+1/2*b^2/(a*d-b*c)^2/a/(a*b)^(1/2)*arctan(1/(a
*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.92, size = 132, normalized size = 1.22

$$\frac{d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{cd}}+\frac{bx}{2\left(a^{2}bc-a^{3}d+\left(ab^{2}c-a^{2}bd\right)x^{2}\right)}+\frac{\left(b^{2}c-3\,abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2}\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*(b^2*c - 3*a*b*d)*ar
ctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
```

mupad [B] time = 5.77, size = 3649, normalized size = 33.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (atan((((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*
a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^7
- 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*
d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*
b*c*d^2) - (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d
^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*
b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c
^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2
 - 2*a^4*b*c*d)))*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((-a^3*b)
^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^
2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4
*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) + (x*(-
a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*
c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d
)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))
)*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))/(((3*a*b^3*d^5)/2 - (b^4*c
*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) - ((-a^3
*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))
/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^7 - 2*a*b^7*c^5
*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*
b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) - (x
*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b
^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))
/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*
```

```
c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*
d))))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((-a^3*b)^(1/2)*(3*a*d -
b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*
b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c
*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d
^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) + (x*(-a^3*b)^(1/2)*(3*
a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3
*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*
b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/
2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))))/(4*(a^5*d^2 +
 a^3*b^2*c^2 - 2*a^4*b*c*d))))*(-a^3*b)^(1/2)*(3*a*d - b*c)*1i)/(2*(a^5*d^2
 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - (atan((((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 -
 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d
^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^
4*b*c*d^2)) - (x*(-c*d^3)^(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2
*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5
))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^
2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) - (x*(13*a^2
*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*
b*c*d)))*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d) - ((-c*d^3)^(1/2)*((((4*a^
6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^
3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c
^2*d - 3*a^4*b*c*d^2)) + (x*(-c*d^3)^(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d
^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*
b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2
 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) +
 (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)))*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d))/(((3*a*b^3*d^5
)/2 - (b^4*c*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d
^2) + ((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^
6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d
^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (x*(-c*d^3)^(1/2)*(1
6*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3
+ 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a
^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3
 + a^2*c*d^2 - 2*a*b*c^2*d)) - (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c
*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))))/(b^2*c^3 + a^2*c*d^2 - 2
*a*b*c^2*d) + ((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*
b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(
2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + (x*(-c*d^3)^
(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*
c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*
(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) + (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6
*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))))/(b^2*c^3 + a^2*c
*d^2 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2)*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d
) - (b*x)/(2*a*(a + b*x^2)*(a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
[Out] Timed out
```

3.33 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=167

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{dx(ad+bc)}{2ac\left(c+dx^2\right)(bc-$$

Rubi [A] time = 0.20, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {414, 527, 522, 205}

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{dx(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\int \frac{-bc+2ad-3bdx^2}{(a+bx^2)(c+dx^2)^2}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\int \frac{-2(b^2c^2-4abcd+a^2d^2)-2b}{(a+bx^2)(c+dx^2}}{4ac(bc-ad}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{(b^2(bc-5ad))\int \frac{1}{a+bx^2}}{2a(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{b^{3/2}(bc-5ad)\tan^{-1}(}{2a^{3/2}(bc-ad)^3}$$

Mathematica [A] time = 0.32, size = 136, normalized size = 0.81

$$\frac{1}{2}\left(\frac{b^{3/2}(5ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^3} + \frac{x(bc-ad)\left(\frac{b^2}{a^2+abx^2}+\frac{d^2}{c^2+cdx^2}\right) + \frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b^(3/2)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) +
a*d)^3) + ((b*c - a*d)*x*(b^2/(a^2 + a*b*x^2) + d^2/(c^2 + c*d*x^2)) + (d^(
3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c - a*d)^3)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)^2), x]
```

fricas [B] time = 2.41, size = 1681, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2
*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sq
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (5*a^2*b*c^2*d -
 a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d
^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c
)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*
a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^
3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d
 + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 +
```

```
2*(5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c
^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (a*b^2
*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2
*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^
3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*
a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2
*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d
^3)*x^3 + 2*(a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 +
(b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)
) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2
*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-
d/c) - c)/(d*x^2 + c)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*
x)/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*
c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c
^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/2*((b^3*c^2*d -
 a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*
x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5
*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ (b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b
^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2
 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*
a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)]
```

giac [A]   time = 0.58, size = 232, normalized size = 1.39

$$\frac{\left(b^{3} c - 5 \, a b^{2} d\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, {\left(a b^{3} c^{3} - 3 \, a^{2} b^{2} c^{2} d + 3 \, a^{3} b c d^{2} - a^{4} d^{3}\right)} \sqrt{a b}} + \frac{\left(5 \, b c d^{2} - a d^{3}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 \, {\left(b^{3} c^{4} - 3 \, a b^{2} c^{3} d + 3 \, a^{2} b c^{2} d^{2} - a^{3} c d^{3}\right)} \sqrt{c d}} + \frac{b^{2} c d x^{3} + a b d^{2} x^{3} + b^{2} c^{2} x + a^{2} d^{2} x}{2 \, {\left(a b^{2} c^{3} - 2 \, a^{2} b c^{2} d + a^{3} c d^{2}\right)} {\left(b d x^{4} + b c x^{2} + a d x^{2} + a c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c
*d)) + 1/2*(b^2*c*d*x^3 + a*b*d^2*x^3 + b^2*c^2*x + a^2*d^2*x)/((a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))
```

maple [A]   time = 0.02, size = 238, normalized size = 1.43

$$\frac{a d^{3} x}{2 \left(a d - b c\right)^{3} \left(d x^{2}+c\right) c}+\frac{a d^{3} \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 \left(a d - b c\right)^{3} \sqrt{c d}\, c}-\frac{b^{3} c x}{2 \left(a d - b c\right)^{3} \left(b x^{2}+a\right) a}-\frac{b^{3} c \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \left(a d - b c\right)^{3} \sqrt{a b}\, a}+\frac{b^{2} d x}{2 \left(a d - b c\right)^{3} \left(b x^{2}+a\right)}+\frac{5 b^{2} d \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \left(a d - b c\right)^{3} \sqrt{a b}}-\frac{b d^{2} x}{2 \left(a d - b c\right)^{3} \left(d x^{2}+c\right)}-\frac{5 b d^{2} \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 \left(a d - b c\right)^{3} \sqrt{c d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^3/(a*d-b*c)^3/c*x/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3*x/(d*x^2+c)*b+1/2*d
^3/(a*d-b*c)^3/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a-5/2*d^2/(a*d-b*c)^
3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b+1/2*b^2/(a*d-b*c)^3*x/(b*x^2+a)*d
-1/2*b^3/(a*d-b*c)^3/a*x/(b*x^2+a)*c+5/2*b^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan
(1/(a*b)^(1/2)*b*x)*d-1/2*b^3/(a*d-b*c)^3/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
)*b*x)*c
```

maxima [B]   time = 3.14, size = 294, normalized size = 1.76

$$\frac{\left(b^{3} c - 5 \, a b^{2} d\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, {\left(a b^{3} c^{3} - 3 \, a^{2} b^{2} c^{2} d + 3 \, a^{3} b c d^{2} - a^{4} d^{3}\right)} \sqrt{a b}} + \frac{\left(5 \, b c d^{2} - a d^{3}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 \, {\left(b^{3} c^{4} - 3 \, a b^{2} c^{3} d + 3 \, a^{2} b c^{2} d^{2} - a^{3} c d^{3}\right)} \sqrt{c d}} + \frac{{\left(b^{2} c d + a b d^{2}\right)} x^{3} + {\left(b^{2} c^{2} + a^{2} d^{2}\right)} x}{2 \, {\left(a^{2} b^{2} c^{4} - 2 \, a^{3} b c^{3} d + a^{4} c^{2} d^{2} + {\left(a b^{3} c^{3} d - 2 \, a^{2} b^{2} c^{2} d^{2} + a^{3} b c d^{3}\right)} x^{4} + {\left(a b^{3} c^{4} - a^{2} b^{2} c^{3} d - a^{3} b c^{2} d^{2} + a^{4} c d^{3}\right)} x^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c
*d)) + 1/2*((b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 -
 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d
^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)
```

mupad [B] time = 6.87, size = 6183, normalized size = 37.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^3
*(a*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2*(a*d + b*
c) + b*d*x^4) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 -
 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*
a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2
 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7
*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 -
 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b
*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) -
(x*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3
+ 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b
^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^
3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3
*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3
)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*
a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 1
0*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^
3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 +
 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c
^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 2
0*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c
^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x
*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 +
144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4
*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*
c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b
^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^
(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*
d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d -
 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^7)/4 + (5*b^7*c^3*d^4)/4 - (21*a*b^6*c^2*d
^5)/4 - (21*a^2*b^5*c*d^6)/4)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d
- 6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^
4*d^4) - (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*c*
d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d
- 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2 + 2*a^9*b^2*c
*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 220
*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*c^
2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 15*
a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) - (x*(5*a*d - b*
c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^7*
c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 - 8
0*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*
b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4
*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2))/(4*(a
```

```
^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*d - b*c)*(-a
^3*b^3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*c*d^6 +
 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a
^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 + 2*a^9*b^2*c*d^10
 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 220*a^5*
b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*c^2*d^9
)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 15*a^4*b
^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x*(5*a*d - b*c)*(-
a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^7*c^7*d
^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 - 80*a^8
*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c
^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a^5*
b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2))/(4*(a^6*d^
3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*d - b*c)*(-a^3*b^
3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(
5*a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(2*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2
*d - 3*a^5*b*c*d^2)) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^
3*d^4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d
^4 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*
c^9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*
a^4*b^7*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^
3*d^8 - 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d -
6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*
d^4) - (x*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c
^8*d^3 + 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 14
4*a^7*b^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 -
 a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4
 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-
c^3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d
)))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*1i)/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*
c^4*d^2 - 3*a*b^2*c^5*d)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*
d^4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4
 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^
9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^
4*b^7*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*
d^8 - 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*
a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^
4) + (x*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8
*d^3 + 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*
a^7*b^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 - a
^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4 -
4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-c^
3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))
)*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*1i)/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^
4*d^2 - 3*a*b^2*c^5*d)))/((((5*a^3*b^4*d^7)/4 + (5*b^7*c^3*d^4)/4 - (21*a*b^
6*c^2*d^5)/4 - (21*a^2*b^5*c*d^6)/4)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5
*c^7*d - 6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6
*b^2*c^4*d^4) - (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3
*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3
*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2 + 2*a^
9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^
5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8
*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^
5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) - (x*(a*d
 - 5*b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a
^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*
d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 - a^3*c^3*d^3 +
 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^
```

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5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)
)/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*(a*d - 5*b
*c)*(-c^3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2
*c^5*d)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*
c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*
d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 + 2*a^9*b^2
*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 2
20*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*
c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 1
5*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x*(a*d - 5*
b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^
7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 -
 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 - a^3*c^3*d^3 + 3*a^
2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d -
 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2))/(4*
(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*(a*d - 5*b*c)*(
-c^3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*
d))))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*1i)/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*
b*c^4*d^2 - 3*a*b^2*c^5*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.34 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=230

$$\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{}{2a\left(a+b\right.}$$

Rubi [A] time = 0.31, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {414, 527, 522, 205}

$$\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{dx(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*c + 3*a*d)*x)/(8*a*c
^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*(b*c - a*d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-bc+2ad-5bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-2\left(2b^2c^2-8abcd+3a^2d^2 \cdots\right.}{\left(a+bx^2\right)\left(c+ \cdots\right.}}{8ac(bc- \cdots}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+3a \cdots}{8ac^2(bc-ad)^3\left(c+ \cdots\right.}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+3a \cdots}{8ac^2(bc-ad)^3\left(c+ \cdots\right.}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+3a \cdots}{8ac^2(bc-ad)^3\left(c+ \cdots\right.}$$

Mathematica [A] time = 0.42, size = 197, normalized size = 0.86

$$\frac{1}{8}\left(\frac{4b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^4} + \frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^4} - \frac{4b^3x}{a\left(a+bx^2\right)(ad-bc)^3} + \frac{d^2x(11bc-3ad)}{c^2\left(c+dx^2\right)(bc-ad)^3} + \frac{2d^2x}{c\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^3*x)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2*x)/(c*(b*c - a*d)^2*(
c + d*x^2)^2) + (d^2*(11*b*c - 3*a*d)*x)/(c^2*(b*c - a*d)^3*(c + d*x^2)) +
(4*b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^
4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(c^(5/2)*(b*c - a*d)^4))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)^3), x]
```

fricas [B] time = 8.32, size = 3239, normalized size = 14.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4
 + 3*a^4*d^5)*x^3 - 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*b^3
*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 +
(b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 -
```

```
2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2
+ 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 +
 (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (
35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 -
 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a
^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^
4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*
c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b
^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^
5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c
^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^
2*b^2*c*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2
*c^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + (35*a^2*b^2*c^4*d - 14*a^3*b*c^
3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x
^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)
*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - 2*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4
*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b
^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(
-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^
2*c*d^4 + 3*a^3*b*d^5)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c
^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 8*(a*b^3*c^5 - 7*a^2*b^2*c^4*d +
(b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a
^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*s
qrt(b/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4
*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*
b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^
3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c
*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^
3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*
c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^
2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(b
/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4*c^2*
d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*b^3*c
^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^3*c^4
*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(d/c)*ar
ctan(x*sqrt(d/c)) + (4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^
3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^
6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3
+ 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7
*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^
3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2
 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2)]
```

giac [A] time = 0.58, size = 332, normalized size = 1.44

$$\frac{b^{3}x}{2\left(ab^{3}c^{3}-3a^{2}b^{2}c^{2}d+3a^{3}bcd^{2}-a^{4}d^{3}\right)\left(bx^{2}+a\right)}+\frac{\left(b^{4}c-7ab^{3}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^{4}c^{4}-4a^{2}b^{3}c^{3}d+6a^{3}b^{2}c^{2}d^{2}-4a^{4}bcd^{3}+a^{5}d^{4}\right)\sqrt{ab}}+\frac{\left(35b^{2}c^{2}d^{2}-14abcd^{3}+3a^{2}d^{4}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{4}c^{6}-4ab^{3}c^{5}d+6a^{2}b^{2}c^{4}d^{2}-4a^{3}bc^{3}d^{3}+a^{4}c^{2}d^{4}\right)\sqrt{cd}}+\frac{11bcd^{3}x^{3}-3ad^{4}x^{3}+13bc^{2}d^{2}x-5acd^{3}x}{8\left(b^{3}c^{5}-3ab^{2}c^{4}d+3a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)\left(dx^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b*x^2 +
 a)) + 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^
3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35
*b^2*c^2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 -
4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d
)) + 1/8*(11*b*c*d^3*x^3 - 3*a*d^4*x^3 + 13*b*c^2*d^2*x - 5*a*c*d^3*x)/((b^
3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

maple [A] time = 0.02, size = 403, normalized size = 1.75

$$\frac{3a^{2}d^{5}x^{3}}{8(ad-bc)^{4}\left(dx^{2}+c\right)^{2}c^{2}}-\frac{7abd^{4}x^{3}}{4(ad-bc)^{4}\left(dx^{2}+c\right)^{2}c}+\frac{11b^{2}d^{3}x^{3}}{8(ad-bc)^{4}\left(dx^{2}+c\right)^{2}}+\frac{5a^{2}d^{4}x}{8(ad-bc)^{4}\left(dx^{2}+c\right)^{2}c}-\frac{9abd^{3}x}{4(ad-bc)^{4}\left(dx^{2}+c\right)^{2}}+\frac{13b^{2}cd^{2}x}{8(ad-bc)^{4}\left(dx^{2}+c\right)^{2}}+\frac{3a^{2}d^{4}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8(ad-bc)^{4}\sqrt{cd}\,c^{2}}-\frac{7abd^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4(ad-bc)^{4}\sqrt{cd}\,c}+\frac{b^{4}cx}{2(ad-bc)^{4}\left(bx^{2}+a\right)a}+\frac{b^{4}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2(ad-bc)^{4}\sqrt{ab}\,a}-\frac{b^{3}dx}{2(ad-bc)^{4}\left(bx^{2}+a\right)}-\frac{7b^{3}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2(ad-bc)^{4}\sqrt{ab}}+\frac{35b^{2}d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8(ad-bc)^{4}\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^5/(a*d-b*c)^4/(d*x^2+c)^2/c^2*x^3*a^2-7/4*d^4/(a*d-b*c)^4/(d*x^2+c)^2
/c*x^3*a*b+11/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2+5/8*d^4/(a*d-b*c)^4/(d*
x^2+c)^2/c*x*a^2-9/4*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x*a*b+13/8*d^2/(a*d-b*c)^4
/(d*x^2+c)^2*c*x*b^2+3/8*d^4/(a*d-b*c)^4/c^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/
2)*d*x)*a^2-7/4*d^3/(a*d-b*c)^4/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b
+35/8*d^2/(a*d-b*c)^4/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-1/2*b^3/(a*
d-b*c)^4*x/(b*x^2+a)*d+1/2*b^4/(a*d-b*c)^4/a*x/(b*x^2+a)*c-7/2*b^3/(a*d-b*c
)^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+1/2*b^4/(a*d-b*c)^4/a/(a*b)^(1/
2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [B] time = 3.41, size = 529, normalized size = 2.30

$$\frac{\left(b^{4}c-7ab^{3}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^{4}c^{4}-4a^{2}b^{3}c^{3}d+6a^{3}b^{2}c^{2}d^{2}-4a^{4}bcd^{3}+a^{5}d^{4}\right)\sqrt{ab}}+\frac{\left(35b^{2}c^{2}d^{2}-14abcd^{3}+3a^{2}d^{4}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{4}c^{6}-4ab^{3}c^{5}d+6a^{2}b^{2}c^{4}d^{2}-4a^{3}bc^{3}d^{3}+a^{4}c^{2}d^{4}\right)\sqrt{cd}}+\frac{\left(4b^{3}c^{2}d^{2}+11ab^{2}cd^{3}-3a^{2}bd^{4}\right)x^{5}+\left(8b^{3}c^{3}d+13ab^{2}c^{2}d^{2}+6a^{2}bcd^{3}-3a^{3}d^{4}\right)x^{3}+\left(4b^{3}c^{4}+13a^{2}bc^{2}d^{2}-5a^{3}cd^{3}\right)x}{8\left(a^{2}b^{3}c^{7}-3a^{3}b^{2}c^{6}d+3a^{4}bc^{5}d^{2}-a^{5}c^{4}d^{3}+\left(ab^{4}c^{5}d^{2}-3a^{2}b^{3}c^{4}d^{3}+3a^{3}b^{2}c^{3}d^{4}-a^{4}bc^{2}d^{5}\right)x^{6}+\left(2ab^{4}c^{6}d-5a^{2}b^{3}c^{5}d^{2}+3a^{3}b^{2}c^{4}d^{3}+a^{4}bc^{3}d^{4}-a^{5}c^{2}d^{5}\right)x^{4}+\left(ab^{4}c^{7}-a^{2}b^{3}c^{6}d-3a^{3}b^{2}c^{5}d^{2}+5a^{4}bc^{4}d^{3}-2a^{5}c^{3}d^{4}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^3*c^3*d
 + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35*b^2*c^
2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 - 4*a*b^3
*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d)) + 1/
8*((4*b^3*c^2*d^2 + 11*a*b^2*c*d^3 - 3*a^2*b*d^4)*x^5 + (8*b^3*c^3*d + 13*a
*b^2*c^2*d^2 + 6*a^2*b*c*d^3 - 3*a^3*d^4)*x^3 + (4*b^3*c^4 + 13*a^2*b*c^2*d
^2 - 5*a^3*c*d^3)*x)/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5*d^2 - a^5
*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 - a^4*b*c
^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*d^3 + a^4*
b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^5*d
^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2)
```

mupad [B] time = 7.79, size = 8649, normalized size = 37.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] (atan(((((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^
4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7)
```

)/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*
a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) - (((2*a*b^13*c
^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*
c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7
*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 +
(35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^
4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*
b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d
^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14
*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^
11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^
7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 +
256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a
^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9
*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2
*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^
4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d))
)*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(16*(b^4*c^9 +
a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)) + (((x
*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2
009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*
b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8
*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) + (((2*a*b^13*c^13*d^2 - 2
8*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2
+ 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*
a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^
3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a
^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^
3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9
*b^2*c^6*d^7) + (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*(
256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 71
68*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^
8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^
2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*
d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*
b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)))
*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^4*c^9 + a^4
*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)))*(-c^5*d^3
)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(16*(b^4*c^9 + a^4*c^5*d^
4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)))/(((63*a^5*b^5*d^
9)/64 + (35*b^10*c^5*d^4)/16 - (651*a*b^9*c^4*d^5)/64 - (267*a^4*b^6*c*d^8)
/32 - (1275*a^2*b^8*c^3*d^6)/32 + (451*a^3*b^7*c^2*d^7)/16)/(a^2*b^9*c^13 -
a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 -
84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b
^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 2
24*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^
3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c
^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b
^2*c^6*d^4)) - (((2*a*b^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*
c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^
8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^
10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13
)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 3
6*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^
4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*
(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^1
0*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c
^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9
- 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5

```
*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 +
 a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*
a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^
2*c^2 - 14*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b
^2*c^7*d^2 - 4*a*b^3*c^8*d)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14
*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2
 - 4*a*b^3*c^8*d)) + (((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d
^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^
4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b
*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) +
 (((2*a*b^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 -
(987*a^4*b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 11
97*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b
^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*
c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11
*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84
*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) + (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 3
5*b^2*c^2 - 14*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5
120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*
a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b
^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b
*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 -
 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^
3 + 15*a^6*b^2*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b
*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4
*a*b^3*c^8*d)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16
*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8
*d))))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(8*(b^4*c
^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)) -
((x^5*(4*b^3*c^2*d^2 - 3*a^2*b*d^4 + 11*a*b^2*c*d^3))/(8*a*c^2*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(4*b^3*c^3 - 5*a^3*d^3 + 13*a
^2*b*c*d^2))/(8*a*c*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^3*(
8*b^3*c^3 - 3*a^3*d^3 + 13*a*b^2*c^2*d + 6*a^2*b*c*d^2))/(8*a*c^2*(a*d - b*
c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(
a*d^2 + 2*b*c*d) + b*d^2*x^6) + (atan(((((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3
 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^
6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b
^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a
^6*b^2*c^6*d^4)) - ((7*a*d - b*c)*(-a^3*b^5)^(1/2)*((2*a*b^13*c^13*d^2 - 28
*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2
+ 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*a
^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^3
*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^
3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3
 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*
b^2*c^6*d^7) - (x*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*(256*a^2*b^11*c^13*d^2 - 1
792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 358
4*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*
b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(128*(a^7*d^
4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)*(a^2
*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^
8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4))))/(4*(a^7*d^4 + a^3*b^4*c
^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)))*(7*a*d - b*c)*(
-a^3*b^5)^(1/2)*1i)/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2
*c^2*d^2 - 4*a^6*b*c*d^3)) + (((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b
^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6
+ 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d -
 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6
*d^4)) + ((7*a*d - b*c)*(-a^3*b^5)^(1/2)*((2*a*b^13*c^13*d^2 - 28*a^2*b^12*
```

```
c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2 + 978*a^5*
b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6
*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)
/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12
*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6
*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^
7) + (x*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^
10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*
c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^
9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(128*(a^7*d^4 + a^3*b^
4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)*(a^2*b^6*c^10
+ a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20
*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4))))/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4
*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)))*(7*a*d - b*c)*(-a^3*b^5)^
(1/2)*1i)/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 -
 4*a^6*b*c*d^3)))/(((63*a^5*b^5*d^9)/64 + (35*b^10*c^5*d^4)/16 - (651*a*b^9
*c^4*d^5)/64 - (267*a^4*b^6*c*d^8)/32 - (1275*a^2*b^8*c^3*d^6)/32 + (451*a^
3*b^7*c^2*d^7)/16)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10
*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^
4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (((x*(
9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 200
9*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^
6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d
^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) - ((7*a*d - b*c)*(-a^3*b^5)^
(1/2)*((2*a*b^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/
2 - (987*a^4*b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7
+ 1197*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^
10*b^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*
b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*
c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5
+ 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (x*(7*a*d - b*c)*(-a^3*b^5)^(1
/2)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4
 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 71
68*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^
11*b^2*c^4*d^11))/(128*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2
*c^2*d^2 - 4*a^6*b*c*d^3)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6
*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d
^4))))/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*
a^6*b*c*d^3)))*(7*a*d - b*c)*(-a^3*b^5)^(1/2))/(4*(a^7*d^4 + a^3*b^4*c^4 -
4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)) + (((x*(9*a^6*b^3*d^9
 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4
*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*
c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b
^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) + ((7*a*d - b*c)*(-a^3*b^5)^(1/2)*((2*a*b
^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*
b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^
7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^
11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^
11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84
*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*
c^7*d^6 - 36*a^9*b^2*c^6*d^7) + (x*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*(256*a^2*
b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b
^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^
7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^
11))/(128*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*
a^6*b*c*d^3)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^
5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4))))/(4*(a^
7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3))
)*(7*a*d - b*c)*(-a^3*b^5)^(1/2))/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3
```

```
*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3))))*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*1
i)/(2*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*
b*c*d^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.35 $\int \frac{\left(c+dx^2\right)^5}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=196

$$\frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5\left(a+bx^2\right)}+\frac{d^3x\left(6a^2d^2-15abcd+10b^2c^2\right)}{b^5}+\frac{\left(63a^2d^2+14abcd+3b^2c^2\right)(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}+$$

Rubi [A] time = 0.23, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {390, 1157, 385, 205}

$$\frac{d^3x\left(6a^2d^2-15abcd+10b^2c^2\right)}{b^5}+\frac{\left(63a^2d^2+14abcd+3b^2c^2\right)(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}+\frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5\left(a+bx^2\right)}+\frac{d^4x^3(5bc-3ad)}{3b^4}+\frac{x(bc-ad)^5}{4ab^5\left(a+bx^2\right)^2}+\frac{d^5x^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c - 3*a*d)*x^
3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^2)^2)
+ ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b^5*(a + b*x^2)) + ((b*c - a*d)
^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(5/2)*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^5}{\left(a+bx^2\right)^3}\,dx = \int \left(\frac{d^3\left(10b^2c^2-15abcd+6a^2d^2\right)}{b^5} + \frac{d^4(5bc-3ad)x^2}{b^4} + \frac{d^5x^4}{b^3} + \frac{(bc-ad)^3\left(b^2c^2+3abcd+6a\ldots\right.}{\ldots}\right.$$

$$= \frac{d^3\left(10b^2c^2-15abcd+6a^2d^2\right)x}{b^5} + \frac{d^4(5bc-3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{\int \frac{(bc-ad)^3\left(b^2c^2+3abcd+6a^2d^2\right)+5bd(bc-\ldots}{\left(a+bx^2\right)\ldots}}{b^5}$$

$$= \frac{d^3\left(10b^2c^2-15abcd+6a^2d^2\right)x}{b^5} + \frac{d^4(5bc-3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5\left(a+bx^2\right)^2} - \frac{\int \frac{-(bc-ad)^3\left(3b^2\ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{d^3\left(10b^2c^2-15abcd+6a^2d^2\right)x}{b^5} + \frac{d^4(5bc-3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5\left(a+bx^2\right)^2} + \frac{(bc-ad)^4(3b\ldots}{8a^2b^5\left(a\ldots\right.}$$

$$= \frac{d^3\left(10b^2c^2-15abcd+6a^2d^2\right)x}{b^5} + \frac{d^4(5bc-3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5\left(a+bx^2\right)^2} + \frac{(bc-ad)^4(3b\ldots}{8a^2b^5\left(a\ldots\right.}$$

Mathematica [A] time = 0.13, size = 196, normalized size = 1.00

$$\frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5\left(a+bx^2\right)} + \frac{d^3x\left(6a^2d^2-15abcd+10b^2c^2\right)}{b^5} + \frac{\left(63a^2d^2+14abcd+3b^2c^2\right)(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}} + \frac{x(bc-ad)^5}{4ab^5\left(a+bx^2\right)^2} + \frac{d^4x^3(5bc-3ad)}{3b^4} + \frac{d^5x^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c - 3*a*d)*x^
3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^2)^2)
+ ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b^5*(a + b*x^2)) + ((b*c - a*d)
^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(5/2)*b^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^5}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^5/(a + b*x^2)^3, x]
```

fricas [B] time = 0.92, size = 1044, normalized size = 5.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*a^3*b^5*d^5*x^9 + 16*(25*a^3*b^5*c*d^4 - 9*a^4*b^4*d^5)*x^7 + 16
*(150*a^3*b^5*c^2*d^3 - 175*a^4*b^4*c*d^4 + 63*a^5*b^3*d^5)*x^5 + 10*(9*a*b
^7*c^5 + 15*a^2*b^6*c^4*d - 150*a^3*b^5*c^3*d^2 + 750*a^4*b^4*c^2*d^3 - 875
*a^5*b^3*c*d^4 + 315*a^6*b^2*d^5)*x^3 + 15*(3*a^2*b^5*c^5 + 5*a^3*b^4*c^4*d
 + 30*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 175*a^6*b*c*d^4 - 63*a^7*d^5
```

```
+ (3*b^7*c^5 + 5*a*b^6*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b^4*c^2*d^3 + 1
75*a^4*b^3*c*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*a^2*b^5*c^4*d +
 30*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*c*d^4 - 63*a^6*b*d^
5)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(5*a^
2*b^6*c^5 - 5*a^3*b^5*c^4*d - 30*a^4*b^4*c^3*d^2 + 150*a^5*b^3*c^2*d^3 - 17
5*a^6*b^2*c*d^4 + 63*a^7*b*d^5)*x)/(a^3*b^8*x^4 + 2*a^4*b^7*x^2 + a^5*b^6),
 1/120*(24*a^3*b^5*d^5*x^9 + 8*(25*a^3*b^5*c*d^4 - 9*a^4*b^4*d^5)*x^7 + 8*(
150*a^3*b^5*c^2*d^3 - 175*a^4*b^4*c*d^4 + 63*a^5*b^3*d^5)*x^5 + 5*(9*a*b^7*
c^5 + 15*a^2*b^6*c^4*d - 150*a^3*b^5*c^3*d^2 + 750*a^4*b^4*c^2*d^3 - 875*a^
5*b^3*c*d^4 + 315*a^6*b^2*d^5)*x^3 + 15*(3*a^2*b^5*c^5 + 5*a^3*b^4*c^4*d +
30*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 175*a^6*b*c*d^4 - 63*a^7*d^5 + (
3*b^7*c^5 + 5*a*b^6*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b^4*c^2*d^3 + 175*
a^4*b^3*c*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*a^2*b^5*c^4*d + 30
*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*c*d^4 - 63*a^6*b*d^5)*
x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 15*(5*a^2*b^6*c^5 - 5*a^3*b^5*c^4*d
- 30*a^4*b^4*c^3*d^2 + 150*a^5*b^3*c^2*d^3 - 175*a^6*b^2*c*d^4 + 63*a^7*b*d
^5)*x)/(a^3*b^8*x^4 + 2*a^4*b^7*x^2 + a^5*b^6)]
```

giac [A] time = 0.58, size = 340, normalized size = 1.73

$$\frac{\left(3\,b^{5}c^{5} + 5\,ab^{4}c^{4}d + 30\,a^{2}b^{3}c^{3}d^{2} - 150\,a^{3}b^{2}c^{2}d^{3} + 175\,a^{4}bcd^{4} - 63\,a^{5}d^{5}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{2}b^{5}} + \frac{3\,b^{6}c^{5}x^{3} + 5\,ab^{5}c^{4}dx^{3} - 50\,a^{2}b^{4}c^{3}d^{2}x^{3} + 90\,a^{3}b^{3}c^{2}d^{3}x^{3} - 65\,a^{4}b^{2}cd^{4}x^{3} + 17\,a^{5}bd^{5}x^{3} + 5\,ab^{5}c^{5}x - 5\,a^{2}b^{4}c^{4}dx - 30\,a^{3}b^{3}c^{3}d^{2}x + 70\,a^{4}b^{2}c^{2}d^{3}x - 55\,a^{5}bcd^{4}x + 15\,a^{6}d^{5}x}{8\,\left(bx^{2} + a\right)^{2}a^{2}b^{5}} + \frac{3\,b^{12}d^{5}x^{5} + 25\,b^{12}cd^{4}x^{3} - 15\,ab^{11}d^{5}x^{3} + 150\,b^{12}c^{2}d^{3}x - 225\,ab^{11}cd^{4}x + 90\,a^{2}b^{10}d^{5}x}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^5*c^5 + 5*a*b^4*c^4*d + 30*a^2*b^3*c^3*d^2 - 150*a^3*b^2*c^2*d^3 +
 175*a^4*b*c*d^4 - 63*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^5) +
1/8*(3*b^6*c^5*x^3 + 5*a*b^5*c^4*d*x^3 - 50*a^2*b^4*c^3*d^2*x^3 + 90*a^3*b^
3*c^2*d^3*x^3 - 65*a^4*b^2*c*d^4*x^3 + 17*a^5*b*d^5*x^3 + 5*a*b^5*c^5*x - 5
*a^2*b^4*c^4*d*x - 30*a^3*b^3*c^3*d^2*x + 70*a^4*b^2*c^2*d^3*x - 55*a^5*b*c
*d^4*x + 15*a^6*d^5*x)/((b*x^2 + a)^2*a^2*b^5) + 1/15*(3*b^12*d^5*x^5 + 25*
b^12*c*d^4*x^3 - 15*a*b^11*d^5*x^3 + 150*b^12*c^2*d^3*x - 225*a*b^11*c*d^4*
x + 90*a^2*b^10*d^5*x)/b^15
```

maple [B] time = 0.02, size = 484, normalized size = 2.47

$$\frac{17a^{3}d^{5}x^{3}}{8\left(bx^{2}+a\right)^{2}b^{4}} - \frac{65a^{2}cd^{4}x^{3}}{8\left(bx^{2}+a\right)^{2}b^{3}} + \frac{45ac^{2}d^{3}x^{3}}{4\left(bx^{2}+a\right)^{2}b^{2}} + \frac{5c^{4}dx^{3}}{8\left(bx^{2}+a\right)^{2}a} + \frac{3bc^{5}x^{3}}{8\left(bx^{2}+a\right)^{2}a^{2}} - \frac{25c^{3}d^{2}x^{3}}{4\left(bx^{2}+a\right)^{2}b} + \frac{d^{5}x^{5}}{5b^{3}} + \frac{15a^{4}d^{5}x}{8\left(bx^{2}+a\right)^{2}b^{5}} - \frac{55a^{3}cd^{4}x}{8\left(bx^{2}+a\right)^{2}b^{4}} + \frac{35a^{2}c^{2}d^{3}x}{4\left(bx^{2}+a\right)^{2}b^{3}} - \frac{15ac^{3}d^{2}x}{4\left(bx^{2}+a\right)^{2}b^{2}} - \frac{ad^{5}x^{3}}{b^{4}} + \frac{5c^{5}x}{8\left(bx^{2}+a\right)^{2}a} - \frac{5c^{4}dx}{8\left(bx^{2}+a\right)^{2}b} + \frac{5cd^{4}x^{3}}{3b^{3}} - \frac{63a^{3}d^{5}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^{5}} + \frac{175a^{2}cd^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^{4}} - \frac{75ac^{2}d^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^{3}} + \frac{5c^{4}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab} + \frac{3c^{5}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^{2}} + \frac{15c^{3}d^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^{2}} + \frac{6a^{2}d^{5}x}{b^{5}} - \frac{15acd^{4}x}{b^{4}} + \frac{10c^{2}d^{3}x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^5/(b*x^2+a)^3,x)
```

```
[Out] 1/5*d^5*x^5/b^3-d^5/b^4*x^3*a+5/3*d^4/b^3*x^3*c+6*d^5/b^5*a^2*x-15*d^4/b^4*
a*c*x+10*d^3/b^3*c^2*x+17/8/b^4/(b*x^2+a)^2*a^3*x^3*d^5-65/8/b^3/(b*x^2+a)^
2*a^2*x^3*c*d^4+45/4/b^2/(b*x^2+a)^2*a*x^3*c^2*d^3-25/4/b/(b*x^2+a)^2*x^3*c
^3*d^2+5/8/(b*x^2+a)^2/a*x^3*c^4*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c^5+15/8/b^5/(
b*x^2+a)^2*x*a^4*d^5-55/8/b^4/(b*x^2+a)^2*x*a^3*c*d^4+35/4/b^3/(b*x^2+a)^2*
x*a^2*c^2*d^3-15/4/b^2/(b*x^2+a)^2*x*a*c^3*d^2-5/8/b/(b*x^2+a)^2*x*c^4*d+5/
8/(b*x^2+a)^2*x/a*c^5-63/8/b^5*a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^
5+175/8/b^4*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d^4-75/4/b^3*a/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2*d^3+15/4/b^2/(a*b)^(1/2)*arctan(1/(a*
b)^(1/2)*b*x)*c^3*d^2+5/8/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^4*d+3
/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^5
```

maxima [A] time = 3.04, size = 334, normalized size = 1.70

$$\frac{\left(3\,b^{6}c^{5} + 5\,ab^{5}c^{4}d - 50\,a^{2}b^{4}c^{3}d^{2} + 90\,a^{3}b^{3}c^{2}d^{3} - 65\,a^{4}b^{2}cd^{4} + 17\,a^{5}bd^{5}\right)x^{3} + 5\,\left(ab^{5}c^{5} - a^{2}b^{4}c^{4}d - 6\,a^{3}b^{3}c^{3}d^{2} + 14\,a^{4}b^{2}c^{2}d^{3} - 11\,a^{5}bcd^{4} + 3\,a^{6}d^{5}\right)x}{8\,\left(a^{2}b^{7}x^{4} + 2\,a^{3}b^{6}x^{2} + a^{4}b^{5}\right)} + \frac{3\,b^{2}d^{5}x^{5} + 5\,\left(5\,b^{2}cd^{4} - 3\,abd^{5}\right)x^{3} + 15\,\left(10\,b^{2}c^{2}d^{3} - 15\,abcd^{4} + 6\,a^{2}d^{5}\right)x}{15\,b^{5}} + \frac{\left(3\,b^{5}c^{5} + 5\,ab^{4}c^{4}d + 30\,a^{2}b^{3}c^{3}d^{2} - 150\,a^{3}b^{2}c^{2}d^{3} + 175\,a^{4}bcd^{4} - 63\,a^{5}d^{5}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{2}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^6*c^5 + 5*a*b^5*c^4*d - 50*a^2*b^4*c^3*d^2 + 90*a^3*b^3*c^2*d^3 -
 65*a^4*b^2*c*d^4 + 17*a^5*b*d^5)*x^3 + 5*(a*b^5*c^5 - a^2*b^4*c^4*d - 6*a^
3*b^3*c^3*d^2 + 14*a^4*b^2*c^2*d^3 - 11*a^5*b*c*d^4 + 3*a^6*d^5)*x)/(a^2*b^
7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5) + 1/15*(3*b^2*d^5*x^5 + 5*(5*b^2*c*d^4 - 3
*a*b*d^5)*x^3 + 15*(10*b^2*c^2*d^3 - 15*a*b*c*d^4 + 6*a^2*d^5)*x)/b^5 + 1/8
*(3*b^5*c^5 + 5*a*b^4*c^4*d + 30*a^2*b^3*c^3*d^2 - 150*a^3*b^2*c^2*d^3 + 17
5*a^4*b*c*d^4 - 63*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^5)
```

mupad [B] time = 5.02, size = 409, normalized size = 2.09

$$\frac{\frac{5\,x\,\left(3\,a^5\,d^5-11\,a^4\,b\,c\,d^4+14\,a^3\,b^2\,c^2\,d^3-6\,a^2\,b^3\,c^3\,d^2-a\,b^4\,c^4\,d+b^5\,c^5\right)}{8\,a}+\frac{x^3\,\left(17\,a^5\,b\,d^5-65\,a^4\,b^2\,c\,d^4+90\,a^3\,b^3\,c^2\,d^3-50\,a^2\,b^4\,c^3\,d^2+5\,a\,b^5\,c^4\,d+3\,b^6\,c^5\right)}{8\,a^2}}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}-x^3\,\left(\frac{a\,d^5}{b^4}-\frac{5\,c\,d^4}{3\,b^3}\right)+x\,\left(\frac{3\,a\,\left(\frac{3\,a\,d^5}{b^4}-\frac{5\,c\,d^4}{b^3}\right)}{b}-\frac{3\,a^2\,d^5}{b^5}+\frac{10\,c^2\,d^3}{b^3}\right)+\frac{d^5\,x^5}{5\,b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3\,\left(63\,a^2\,d^2+14\,a\,b\,c\,d+3\,b^2\,c^2\right)}{\sqrt{a}\,\left(-63\,a^5\,d^5+175\,a^4\,b\,c\,d^4-150\,a^3\,b^2\,c^2\,d^3+30\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d+3\,b^5\,c^5\right)}\right)\,{\left(a\,d-b\,c\right)}^3\,\left(63\,a^2\,d^2+14\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,a^{5/2}\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^5/(a + b*x^2)^3,x)
```

```
[Out] ((5*x*(3*a^5*d^5 + b^5*c^5 - 6*a^2*b^3*c^3*d^2 + 14*a^3*b^2*c^2*d^3 - a*b^4
*c^4*d - 11*a^4*b*c*d^4))/(8*a) + (x^3*(3*b^6*c^5 + 17*a^5*b*d^5 - 65*a^4*b
^2*c*d^4 - 50*a^2*b^4*c^3*d^2 + 90*a^3*b^3*c^2*d^3 + 5*a*b^5*c^4*d))/(8*a^2
))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) - x^3*((a*d^5)/b^4 - (5*c*d^4)/(3*b^3)
) + x*((3*a*((3*a*d^5)/b^4 - (5*c*d^4)/b^3))/b - (3*a^2*d^5)/b^5 + (10*c^2*
d^3)/b^3) + (d^5*x^5)/(5*b^3) + (atan((b^(1/2)*x*(a*d - b*c)^3*(63*a^2*d^2
+ 3*b^2*c^2 + 14*a*b*c*d))/(a^(1/2)*(3*b^5*c^5 - 63*a^5*d^5 + 30*a^2*b^3*c^
3*d^2 - 150*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d + 175*a^4*b*c*d^4)))*(a*d - b*c
)^3*(63*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(8*a^(5/2)*b^(11/2))
```

sympy [B] time = 4.41, size = 615, normalized size = 3.14

$$x^3\left(-\frac{ad^5}{b^4}+\frac{5cd^4}{3b^3}\right)+x\left(\frac{6a^2d^5}{b^5}-\frac{15acd^4}{b^4}+\frac{10c^2d^3}{b^3}\right)+\frac{\sqrt{-\frac{1}{a^5b^{11}}}\,(ad-bc)^3\left(63a^2d^2+14abcd+3b^2c^2\right)\log\left(-\frac{a^3b^5\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\left(63a^2d^2+14abcd+3b^2c^2\right)}{63a^5d^5-175a^4bcd^4+150a^3b^2c^2d^3-30a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5}+x\right)}{16}-\frac{\sqrt{-\frac{1}{a^5b^{11}}}\,(ad-bc)^3\left(63a^2d^2+14abcd+3b^2c^2\right)\log\left(\frac{a^3b^5\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\left(63a^2d^2+14abcd+3b^2c^2\right)}{63a^5d^5-175a^4bcd^4+150a^3b^2c^2d^3-30a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5}+x\right)}{16}+\frac{x^3\left(17a^5bd^5-65a^4b^2cd^4+90a^3b^3c^2d^3-50a^2b^4c^3d^2+5ab^5c^4d+3b^6c^5\right)+x\left(15a^6d^5-55a^5bcd^4+70a^4b^2c^2d^3-30a^3b^3c^3d^2-5a^2b^4c^4d+5ab^5c^5\right)}{8a^4b^5+16a^3b^6x^2+8a^2b^7x^4}+\frac{d^5x^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**5/(b*x**2+a)**3,x)
```

```
[Out] x**3*(-a*d**5/b**4 + 5*c*d**4/(3*b**3)) + x*(6*a**2*d**5/b**5 - 15*a*c*d**4
/b**4 + 10*c**2*d**3/b**3) + sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2*
d**2 + 14*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**5*sqrt(-1/(a**5*b**11))*(a*d
- b*c)**3*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c**2)/(63*a**5*d**5 - 175*a**
4*b*c*d**4 + 150*a**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 - 5*a*b**4*c*
*4*d - 3*b**5*c**5) + x)/16 - sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2
*d**2 + 14*a*b*c*d + 3*b**2*c**2)*log(a**3*b**5*sqrt(-1/(a**5*b**11))*(a*d
- b*c)**3*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c**2)/(63*a**5*d**5 - 175*a**
4*b*c*d**4 + 150*a**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 - 5*a*b**4*c*
*4*d - 3*b**5*c**5) + x)/16 + (x**3*(17*a**5*b*d**5 - 65*a**4*b**2*c*d**4 +
 90*a**3*b**3*c**2*d**3 - 50*a**2*b**4*c**3*d**2 + 5*a*b**5*c**4*d + 3*b**6
*c**5) + x*(15*a**6*d**5 - 55*a**5*b*c*d**4 + 70*a**4*b**2*c**2*d**3 - 30*a
**3*b**3*c**3*d**2 - 5*a**2*b**4*c**4*d + 5*a*b**5*c**5))/(8*a**4*b**5 + 16
*a**3*b**6*x**2 + 8*a**2*b**7*x**4) + d**5*x**5/(5*b**3)
```

3.36 $\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=160

$$\frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4\left(a+bx^2\right)}+\frac{(bc-ad)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}+\frac{d^3x(4bc-3ad)}{b^4}+\frac{x(bc-ad)^4}{4ab^4\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.20, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {390, 1157, 385, 205}

$$\frac{(bc-ad)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}+\frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4\left(a+bx^2\right)}+\frac{d^3x(4bc-3ad)}{b^4}+\frac{x(bc-ad)^4}{4ab^4\left(a+bx^2\right)^2}+\frac{d^4x^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*x)/(4*a*b^
4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(8*a^2*b^4*(a + b*x^2
)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^3}\,dx = \int \left(\frac{d^3(4bc-3ad)}{b^4} + \frac{d^4x^2}{b^3} + \frac{b^4c^4-4a^3bcd^3+3a^4d^4+4bd(bc-ad)^2(bc+2ad)x^2+6b^2d^2(bc}{b^4\left(a+bx^2\right)^3} \right.$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{\int \frac{b^4c^4-4a^3bcd^3+3a^4d^4+4bd(bc-ad)^2(bc+2ad)x^2+6b^2d^2(bc-ad)^2x^4}{\left(a+bx^2\right)^3}dx}{b^4}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2} - \frac{\int \frac{-(bc-ad)^2\left(3b^2c^2+10abcd+11a^2d^2\right)-24abd^2(bc-ad)^2x^2}{\left(a+bx^2\right)^2}dx}{4ab^4}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2} + \frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4\left(a+bx^2\right)} + \frac{\left((bc-ad)^2\left(3b^2c^2+1\right.\right.}{}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2} + \frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4\left(a+bx^2\right)} + \frac{(bc-ad)^2\left(3b^2c^2+10\right.}{8}$$

Mathematica [A] time = 0.10, size = 160, normalized size = 1.00

$$\frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4\left(a+bx^2\right)} + \frac{(bc-ad)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}} + \frac{d^3x(4bc-3ad)}{b^4} + \frac{x(bc-ad)^4}{4ab^4\left(a+bx^2\right)^2} + \frac{d^4x^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*x)/(4*a*b^
4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(8*a^2*b^4*(a + b*x^2
)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^4/(a + b*x^2)^3, x]
```

fricas [B] time = 0.87, size = 817, normalized size = 5.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^3*b^4*d^4*x^7 + 16*(12*a^3*b^4*c*d^3 - 7*a^4*b^3*d^4)*x^5 + 2*(
9*a*b^6*c^4 + 12*a^2*b^5*c^3*d - 90*a^3*b^4*c^2*d^2 + 300*a^4*b^3*c*d^3 - 1
75*a^5*b^2*d^4)*x^3 - 3*(3*a^2*b^4*c^4 + 4*a^3*b^3*c^3*d + 18*a^4*b^2*c^2*d
^2 - 60*a^5*b*c*d^3 + 35*a^6*d^4 + (3*b^6*c^4 + 4*a*b^5*c^3*d + 18*a^2*b^4*
c^2*d^2 - 60*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(3*a*b^5*c^4 + 4*a^2*b
```

```
^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 60*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(
-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*a^2*b^5*c^4 - 4*
a^3*b^4*c^3*d - 18*a^4*b^3*c^2*d^2 + 60*a^5*b^2*c*d^3 - 35*a^6*b*d^4)*x)/(a
^3*b^7*x^4 + 2*a^4*b^6*x^2 + a^5*b^5), 1/24*(8*a^3*b^4*d^4*x^7 + 8*(12*a^3*
b^4*c*d^3 - 7*a^4*b^3*d^4)*x^5 + (9*a*b^6*c^4 + 12*a^2*b^5*c^3*d - 90*a^3*b
^4*c^2*d^2 + 300*a^4*b^3*c*d^3 - 175*a^5*b^2*d^4)*x^3 + 3*(3*a^2*b^4*c^4 +
4*a^3*b^3*c^3*d + 18*a^4*b^2*c^2*d^2 - 60*a^5*b*c*d^3 + 35*a^6*d^4 + (3*b^6
*c^4 + 4*a*b^5*c^3*d + 18*a^2*b^4*c^2*d^2 - 60*a^3*b^3*c*d^3 + 35*a^4*b^2*d
^4)*x^4 + 2*(3*a*b^5*c^4 + 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 60*a^4*b^
2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*a^2*b^5
*c^4 - 4*a^3*b^4*c^3*d - 18*a^4*b^3*c^2*d^2 + 60*a^5*b^2*c*d^3 - 35*a^6*b*d
^4)*x)/(a^3*b^7*x^4 + 2*a^4*b^6*x^2 + a^5*b^5)]
```

giac [A] time = 0.58, size = 254, normalized size = 1.59

$$\frac{\left(3\,b^4c^4+4\,ab^3c^3d+18\,a^2b^2c^2d^2-60\,a^3bcd^3+35\,a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^4}+\frac{3\,b^5c^4x^3+4\,ab^4c^3dx^3-30\,a^2b^3c^2d^2x^3+36\,a^3b^2cd^3x^3-13\,a^4bd^4x^3+5\,ab^4c^4x-4\,a^2b^3c^3dx-18\,a^3b^2c^2d^2x+28\,a^4bcd^3x-11\,a^5d^4x}{8\left(bx^2+a\right)^2a^2b^4}+\frac{b^6d^4x^3+12\,b^6cd^3x-9\,ab^5d^4x}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^4*c^4 + 4*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 60*a^3*b*c*d^3 + 35*a
^4*d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^4) + 1/8*(3*b^5*c^4*x^3 + 4*
a*b^4*c^3*d*x^3 - 30*a^2*b^3*c^2*d^2*x^3 + 36*a^3*b^2*c*d^3*x^3 - 13*a^4*b*
d^4*x^3 + 5*a*b^4*c^4*x - 4*a^2*b^3*c^3*d*x - 18*a^3*b^2*c^2*d^2*x + 28*a^4
*b*c*d^3*x - 11*a^5*d^4*x)/((b*x^2 + a)^2*a^2*b^4) + 1/3*(b^6*d^4*x^3 + 12*
b^6*c*d^3*x - 9*a*b^5*d^4*x)/b^9
```

maple [B] time = 0.01, size = 367, normalized size = 2.29

$$-\frac{13a^2d^4x^3}{8\left(bx^2+a\right)^2b^3}+\frac{9acd^3x^3}{2\left(bx^2+a\right)^2b^2}+\frac{c^3dx^3}{2\left(bx^2+a\right)^2a}+\frac{3bc^4x^3}{8\left(bx^2+a\right)^2a^2}-\frac{15c^2d^2x^3}{4\left(bx^2+a\right)^2b}-\frac{11a^3d^4x}{8\left(bx^2+a\right)^2b^4}+\frac{7a^2cd^3x}{2\left(bx^2+a\right)^2b^3}-\frac{9ac^2d^2x}{4\left(bx^2+a\right)^2b^2}+\frac{5c^4x}{8\left(bx^2+a\right)^2a}-\frac{c^3dx}{2\left(bx^2+a\right)^2b}+\frac{d^4x^3}{3b^3}+\frac{35a^2d^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^4}-\frac{15acd^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^3}+\frac{c^3d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}+\frac{3c^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2}+\frac{9c^2d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^2}-\frac{3ad^4x}{b^4}+\frac{4cd^3x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a)^3,x)
```

```
[Out] 1/3*d^4*x^3/b^3-3*d^4/b^4*a*x+4*d^3/b^3*x*c-13/8/b^3/(b*x^2+a)^2*a^2*x^3*d^
4+9/2/b^2/(b*x^2+a)^2*a*x^3*c*d^3-15/4/b/(b*x^2+a)^2*x^3*c^2*d^2+1/2/(b*x^2
+a)^2/a*x^3*c^3*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c^4-11/8/b^4/(b*x^2+a)^2*x*a^3*
d^4+7/2/b^3/(b*x^2+a)^2*x*a^2*c*d^3-9/4/b^2/(b*x^2+a)^2*x*a*c^2*d^2-1/2/b/(
b*x^2+a)^2*x*c^3*d+5/8/(b*x^2+a)^2*x/a*c^4+35/8/b^4*a^2/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*d^4-15/2/b^3*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d
^3+9/4/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2*d^2+1/2/b/a/(a*b)^(1/2
)*arctan(1/(a*b)^(1/2)*b*x)*c^3*d+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x)*c^4
```

maxima [A] time = 3.07, size = 253, normalized size = 1.58

$$\frac{\left(3\,b^5c^4+4\,ab^4c^3d-30\,a^2b^3c^2d^2+36\,a^3b^2cd^3-13\,a^4bd^4\right)x^3+\left(5\,ab^4c^4-4\,a^2b^3c^3d-18\,a^3b^2c^2d^2+28\,a^4bcd^3-11\,a^5d^4\right)x}{8\left(a^2b^6x^4+2\,a^3b^5x^2+a^4b^4\right)}+\frac{bd^4x^3+3\left(4\,bcd^3-3\,ad^4\right)x}{3\,b^4}+\frac{\left(3\,b^4c^4+4\,ab^3c^3d+18\,a^2b^2c^2d^2-60\,a^3bcd^3+35\,a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^5*c^4 + 4*a*b^4*c^3*d - 30*a^2*b^3*c^2*d^2 + 36*a^3*b^2*c*d^3 - 1
3*a^4*b*d^4)*x^3 + (5*a*b^4*c^4 - 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 28
*a^4*b*c*d^3 - 11*a^5*d^4)*x)/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4) + 1/3
*(b*d^4*x^3 + 3*(4*b*c*d^3 - 3*a*d^4)*x)/b^4 + 1/8*(3*b^4*c^4 + 4*a*b^3*c^3
*d + 18*a^2*b^2*c^2*d^2 - 60*a^3*b*c*d^3 + 35*a^4*d^4)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a^2*b^4)
```

mupad [B] time = 0.14, size = 318, normalized size = 1.99

$$\frac{d^4\,x^3}{3\,b^3}-x\,\left(\frac{3\,a\,d^4}{b^4}-\frac{4\,c\,d^3}{b^3}\right)-\frac{\frac{x\,\left(11\,a^4\,d^4-28\,a^3\,b\,c\,d^3+18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d-5\,b^4\,c^4\right)}{8\,a}-\frac{x^3\,\left(-13\,a^4\,b\,d^4+36\,a^3\,b^2\,c\,d^3-30\,a^2\,b^3\,c^2\,d^2+4\,a\,b^4\,c^3\,d+3\,b^5\,c^4\right)}{8\,a^2}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(35\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{\sqrt{a}\,\left(35\,a^4\,d^4-60\,a^3\,b\,c\,d^3+18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d+3\,b^4\,c^4\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(35\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,a^{5/2}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^4/(a + b*x^2)^3,x)
```

```
[Out] (d^4*x^3)/(3*b^3) - x*((3*a*d^4)/b^4 - (4*c*d^3)/b^3) - ((x*(11*a^4*d^4 - 5
*b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 28*a^3*b*c*d^3))/(8*a) - (x
^3*(3*b^5*c^4 - 13*a^4*b*d^4 + 36*a^3*b^2*c*d^3 - 30*a^2*b^3*c^2*d^2 + 4*a*
b^4*c^3*d))/(8*a^2))/(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + (atan((b^(1/2)*x*(
a*d - b*c)^2*(35*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/(a^(1/2)*(35*a^4*d^4 +
3*b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 60*a^3*b*c*d^3)))*(a*d - b
*c)^2*(35*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/(8*a^(5/2)*b^(9/2))
```

sympy [B] time = 2.81, size = 515, normalized size = 3.22

$$x\left(-\frac{3ad^4}{b^4}+\frac{4cd^3}{b^3}\right)-\frac{\sqrt{-\frac{1}{a^5b^9}}\,(ad-bc)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\log\left(-\frac{a^3b^4\sqrt{-\frac{1}{a^5b^9}}\,(ad-bc)^2\left(35a^2d^2+10abcd+3b^2c^2\right)}{35a^4d^4-60a^3bcd^3+18a^2b^2c^2d^2+4ab^3c^3d+3b^4c^4}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^9}}\,(ad-bc)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\log\left(\frac{a^3b^4\sqrt{-\frac{1}{a^5b^9}}\,(ad-bc)^2\left(35a^2d^2+10abcd+3b^2c^2\right)}{35a^4d^4-60a^3bcd^3+18a^2b^2c^2d^2+4ab^3c^3d+3b^4c^4}+x\right)}{16}+\frac{x^3\left(-13a^4bd^4+36a^3b^2cd^3-30a^2b^3c^2d^2+4ab^4c^3d+3b^5c^4\right)+x\left(-11a^5d^4+28a^4bcd^3-18a^3b^2c^2d^2-4a^2b^3c^3d+5ab^4c^4\right)}{8a^4b^4+16a^3b^5x^2+8a^2b^6x^4}+\frac{d^4x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**3,x)
```

```
[Out] x*(-3*a*d**4/b**4 + 4*c*d**3/b**3) - sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(3
5*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**4*sqrt(-1/(a**5*b**9))
*(a*d - b*c)**2*(35*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)/(35*a**4*d**4 - 6
0*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d**2 + 4*a*b**3*c**3*d + 3*b**4*c**4) +
 x)/16 + sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2 + 10*a*b*c*d + 3
*b**2*c**2)*log(a**3*b**4*sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2
 + 10*a*b*c*d + 3*b**2*c**2)/(35*a**4*d**4 - 60*a**3*b*c*d**3 + 18*a**2*b**
2*c**2*d**2 + 4*a*b**3*c**3*d + 3*b**4*c**4) + x)/16 + (x**3*(-13*a**4*b*d*
*4 + 36*a**3*b**2*c*d**3 - 30*a**2*b**3*c**2*d**2 + 4*a*b**4*c**3*d + 3*b**
5*c**4) + x*(-11*a**5*d**4 + 28*a**4*b*c*d**3 - 18*a**3*b**2*c**2*d**2 - 4*
a**2*b**3*c**3*d + 5*a*b**4*c**4))/(8*a**4*b**4 + 16*a**3*b**5*x**2 + 8*a**
2*b**6*x**4) + d**4*x**3/(3*b**3)
```

3.37 $\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=130

$$\frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3(a+bx^2)}+\frac{3(bc-ad)\left(4a^2d^2+(ad+bc)^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}+\frac{x(bc-ad)^3}{4ab^3(a+bx^2)^2}+\frac{d^3x}{b^3}$$

Rubi [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {390, 1157, 385, 205}

$$\frac{3(bc-ad)\left(4a^2d^2+(ad+bc)^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}+\frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3(a+bx^2)}+\frac{x(bc-ad)^3}{4ab^3(a+bx^2)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c - a*d)^2*
(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b*c - a*d)*(4*a^2*d^2 + (b*c
 + a*d)^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^3}\,dx=\int \left(\frac{d^3}{b^3}+\frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{b^3\left(a+bx^2\right)^3}\right)dx$$

$$=\frac{d^3x}{b^3}+\frac{\int \frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{\left(a+bx^2\right)^3}dx}{b^3}$$

$$=\frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^2\right)^2}-\frac{\int \frac{-3(bc-ad)(bc+ad)^2-12abd^2(bc-ad)x^2}{\left(a+bx^2\right)^2}dx}{4ab^3}$$

$$=\frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^2\right)^2}+\frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3\left(a+bx^2\right)}+\frac{\left(3(bc-ad)\left(4a^2d^2+(bc+ad)^2\right)\right)\int \frac{1}{a+bx^2}}{8a^2b^3}$$

$$=\frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^2\right)^2}+\frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3\left(a+bx^2\right)}+\frac{3(bc-ad)\left(4a^2d^2+(bc+ad)^2\right)\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{8a^{5/2}b^{7/2}}$$

Mathematica [A] time = 0.08, size = 139, normalized size = 1.07

$$\frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3\left(a+bx^2\right)}+\frac{3\left(-5a^3d^3+3a^2bcd^2+ab^2c^2d+b^3c^3\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}+\frac{x(bc-ad)^3}{4ab^3\left(a+bx^2\right)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c - a*d)^2*
(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b^3*c^3 + a*b^2*c^2*d + 3*a^
2*b*c*d^2 - 5*a^3*d^3)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2)^3, x]
```

fricas [B] time = 0.64, size = 606, normalized size = 4.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^3*b^3*d^3*x^5 + 2*(3*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 15*a^3*b^3*c
*d^2 + 25*a^4*b^2*d^3)*x^3 + 3*(a^2*b^3*c^3 + a^3*b^2*c^2*d + 3*a^4*b*c*d^2
 - 5*a^5*d^3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^
4 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt
(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^4*c^3 - 3
*a^3*b^3*c^2*d - 9*a^4*b^2*c*d^2 + 15*a^5*b*d^3)*x)/(a^3*b^6*x^4 + 2*a^4*b^
```

```
5*x^2 + a^5*b^4), 1/8*(8*a^3*b^3*d^3*x^5 + (3*a*b^5*c^3 + 3*a^2*b^4*c^2*d -
 15*a^3*b^3*c*d^2 + 25*a^4*b^2*d^3)*x^3 + 3*(a^2*b^3*c^3 + a^3*b^2*c^2*d +
3*a^4*b*c*d^2 - 5*a^5*d^3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - 5*a^
3*b^2*d^3)*x^4 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - 5*a^4*b*d
^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (5*a^2*b^4*c^3 - 3*a^3*b^3*c^2*d
 - 9*a^4*b^2*c*d^2 + 15*a^5*b*d^3)*x)/(a^3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^
4)]
```

giac [A] time = 0.57, size = 178, normalized size = 1.37

$$\frac{d^3x}{b^3}+\frac{3\left(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^3}+\frac{3\,b^4c^3x^3+3\,ab^3c^2dx^3-15\,a^2b^2cd^2x^3+9\,a^3bd^3x^3+5\,ab^3c^3x-3\,a^2b^2c^2dx-9\,a^3bcd^2x+7\,a^4d^3x}{8\left(bx^2+a\right)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] d^3*x/b^3 + 3/8*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c^3*x^3 + 3*a*b^3*c^2*d*x^3
 - 15*a^2*b^2*c*d^2*x^3 + 9*a^3*b*d^3*x^3 + 5*a*b^3*c^3*x - 3*a^2*b^2*c^2*d
*x - 9*a^3*b*c*d^2*x + 7*a^4*d^3*x)/((b*x^2 + a)^2*a^2*b^3)
```

maple [B] time = 0.01, size = 266, normalized size = 2.05

$$\frac{9a\,d^3x^3}{8\left(b\,x^2+a\right)^2b^2}+\frac{3c^2d\,x^3}{8\left(b\,x^2+a\right)^2a}+\frac{3b\,c^3x^3}{8\left(b\,x^2+a\right)^2a^2}-\frac{15c\,d^2x^3}{8\left(b\,x^2+a\right)^2b}+\frac{7a^2d^3x}{8\left(b\,x^2+a\right)^2b^3}-\frac{9ac\,d^2x}{8\left(b\,x^2+a\right)^2b^2}+\frac{5c^3x}{8\left(b\,x^2+a\right)^2a}-\frac{3c^2dx}{8\left(b\,x^2+a\right)^2b}-\frac{15a\,d^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^3}+\frac{3c^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab}+\frac{3c^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2}+\frac{9c\,d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^2}+\frac{d^3x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^3,x)
```

```
[Out] 1/b^3*d^3*x+9/8/b^2/(b*x^2+a)^2*a*x^3*d^3-15/8/b/(b*x^2+a)^2*x^3*c*d^2+3/8/
(b*x^2+a)^2/a*x^3*c^2*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c^3+7/8/b^3/(b*x^2+a)^2*x
*a^2*d^3-9/8/b^2/(b*x^2+a)^2*x*a*c*d^2-3/8/b/(b*x^2+a)^2*x*c^2*d+5/8/(b*x^2
+a)^2*x/a*c^3-15/8/b^3*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^3+9/8/b^2/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d^2+3/8/b/a/(a*b)^(1/2)*arctan(1/(a
*b)^(1/2)*b*x)*c^2*d+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^3
```

maxima [A] time = 2.94, size = 185, normalized size = 1.42

$$\frac{d^3x}{b^3}+\frac{3\left(b^4c^3+ab^3c^2d-5\,a^2b^2cd^2+3\,a^3bd^3\right)x^3+\left(5\,ab^3c^3-3\,a^2b^2c^2d-9\,a^3bcd^2+7\,a^4d^3\right)x}{8\left(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3\right)}+\frac{3\left(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] d^3*x/b^3 + 1/8*(3*(b^4*c^3 + a*b^3*c^2*d - 5*a^2*b^2*c*d^2 + 3*a^3*b*d^3)*
x^3 + (5*a*b^3*c^3 - 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 7*a^4*d^3)*x)/(a^2*b
^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) + 3/8*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*
d^2 - 5*a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3)
```

mupad [B] time = 5.05, size = 240, normalized size = 1.85

$$\frac{\frac{x\left(7\,a^3\,d^3-9\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+5\,b^3\,c^3\right)}{8\,a}+\frac{3\,x^3\left(3\,a^3\,b\,d^3-5\,a^2\,b^2\,c\,d^2+a\,b^3\,c^2\,d+b^4\,c^3\right)}{8\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{d^3\,x}{b^3}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x\,\left(a\,d-b\,c\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2\right)}{\sqrt{a}\,\left(-5\,a^3\,d^3+3\,a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3\right)}\right)\,\left(a\,d-b\,c\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2\right)}{8\,a^{5/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^3,x)
```

```
[Out] ((x*(7*a^3*d^3 + 5*b^3*c^3 - 3*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(8*a) + (3*x^3
*(b^4*c^3 + 3*a^3*b*d^3 - 5*a^2*b^2*c*d^2 + a*b^3*c^2*d))/(8*a^2))/(a^2*b^3
 + b^5*x^4 + 2*a*b^4*x^2) + (d^3*x)/b^3 + (3*atan((b^(1/2)*x*(a*d - b*c)*(5
```

```
*a^2*d^2 + b^2*c^2 + 2*a*b*c*d))/(a^(1/2)*(b^3*c^3 - 5*a^3*d^3 + a*b^2*c^2*
d + 3*a^2*b*c*d^2)))*(a*d - b*c)*(5*a^2*d^2 + b^2*c^2 + 2*a*b*c*d))/(8*a^(5
/2)*b^(7/2))
```

sympy [B] time = 1.89, size = 422, normalized size = 3.25

$$\frac{3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)\log\left(-\frac{3a^3b^3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)}{15a^3d^3-9a^2bcd^2-3ab^2c^2d-3b^3c^3}+x\right)}{16}-\frac{3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)\log\left(\frac{3a^3b^3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)}{15a^3d^3-9a^2bcd^2-3ab^2c^2d-3b^3c^3}+x\right)}{16}+\frac{x^3\left(9a^3bd^3-15a^2b^2cd^2+3ab^3c^2d+3b^4c^3\right)+x\left(7a^4d^3-9a^3bcd^2-3a^2b^2c^2d+5ab^3c^3\right)}{8a^4b^3+16a^3b^4x^2+8a^2b^5x^4}+\frac{d^3x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**3,x)
```

```
[Out] 3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b**2*c**2)*lo
g(-3*a**3*b**3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d +
b**2*c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 3*b**3*c**3)
 + x)/16 - 3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b*
*2*c**2)*log(3*a**3*b**3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*
a*b*c*d + b**2*c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 3*
b**3*c**3) + x)/16 + (x**3*(9*a**3*b*d**3 - 15*a**2*b**2*c*d**2 + 3*a*b**3*
c**2*d + 3*b**4*c**3) + x*(7*a**4*d**3 - 9*a**3*b*c*d**2 - 3*a**2*b**2*c**2
*d + 5*a*b**3*c**3))/(8*a**4*b**3 + 16*a**3*b**4*x**2 + 8*a**2*b**5*x**4) +
 d**3*x/b**3
```

3.38 $\int \frac{(c+dx^2)^2}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=116

$$\frac{3x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{8\left(a+bx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 385, 205}

$$\frac{3x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{8\left(a+bx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

```
[Out] (3*(c^2/a^2 - d^2/b^2)*x)/(8*(a + b*x^2)) + ((b*c - a*d)*x*(c + d*x^2))/(4*
a*b*(a + b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^3}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)}{4ab\left(a+bx^2\right)^2} + \frac{\int \frac{c(3bc+ad)+d(bc+3ad)x^2}{\left(a+bx^2\right)^2}\,dx}{4ab}$$
$$= \frac{3\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{8\left(a+bx^2\right)} + \frac{(bc-ad)x\left(c+dx^2\right)}{4ab\left(a+bx^2\right)^2} + \frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\int \frac{1}{a+bx^2}\,dx}{8a^2b^2}$$
$$= \frac{3\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{8\left(a+bx^2\right)} + \frac{(bc-ad)x\left(c+dx^2\right)}{4ab\left(a+bx^2\right)^2} + \frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

Mathematica [A] time = 0.10, size = 124, normalized size = 1.07

$$\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}} + \frac{x\left(-3a^3d^2-a^2bd\left(2c+5dx^2\right)+ab^2c\left(5c+2dx^2\right)+3b^3c^2x^2\right)}{8a^2b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

```
[Out] (x*(-3*a^3*d^2 + 3*b^3*c^2*x^2 + a*b^2*c*(5*c + 2*d*x^2) - a^2*b*d*(2*c + 5
*d*x^2)))/(8*a^2*b^2*(a + b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2)^3, x]
```

fricas [B] time = 0.81, size = 449, normalized size = 3.87

$$\left[\frac{2\left(3ab^4c^2+2a^2b^3cd-5a^3b^2d^2\right)x^3-\left(3a^2b^2c^2+2a^3bcd+3a^4d^2+\left(3b^4c^2+2ab^3cd+3a^2b^2d^2\right)x^4+2\left(3ab^3c^2+2a^2b^2cd+3a^3bd^2\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(5a^2b^3c^2-2a^3b^2cd-3a^4bd^2\right)x}{16\left(a^3b^5x^4+2a^4b^4x^2+a^5b^3\right)}, \frac{\left(3ab^4c^2+2a^2b^3cd-5a^3b^2d^2\right)x^3+\left(3a^2b^2c^2+2a^3bcd+3a^4d^2+\left(3b^4c^2+2ab^3cd+3a^2b^2d^2\right)x^4+2\left(3ab^3c^2+2a^2b^2cd+3a^3bd^2\right)x^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(5a^2b^3c^2-2a^3b^2cd-3a^4bd^2\right)x}{8\left(a^3b^5x^4+2a^4b^4x^2+a^5b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a*b^4*c^2 + 2*a^2*b^3*c*d - 5*a^3*b^2*d^2)*x^3 - (3*a^2*b^2*c^2
 + 2*a^3*b*c*d + 3*a^4*d^2 + (3*b^4*c^2 + 2*a*b^3*c*d + 3*a^2*b^2*d^2)*x^4
+ 2*(3*a*b^3*c^2 + 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2
- 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^3*c^2 - 2*a^3*b^2*c*d - 3*a
^4*b*d^2)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5*b^3), 1/8*((3*a*b^4*c^2 + 2
*a^2*b^3*c*d - 5*a^3*b^2*d^2)*x^3 + (3*a^2*b^2*c^2 + 2*a^3*b*c*d + 3*a^4*d^
2 + (3*b^4*c^2 + 2*a*b^3*c*d + 3*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 + 2*a^2*
b^2*c*d + 3*a^3*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (5*a^2*b^3*c^
2 - 2*a^3*b^2*c*d - 3*a^4*b*d^2)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5*b^3)
]
```

giac [A] time = 0.57, size = 126, normalized size = 1.09

$$\frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{2}b^{2}}+\frac{3\,b^{3}c^{2}x^{3}+2\,ab^{2}cdx^{3}-5\,a^{2}bd^{2}x^{3}+5\,ab^{2}c^{2}x-2\,a^{2}bcdx-3\,a^{3}d^{2}x}{8\,\left(bx^{2}+a\right)^{2}a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
2*b^2) + 1/8*(3*b^3*c^2*x^3 + 2*a*b^2*c*d*x^3 - 5*a^2*b*d^2*x^3 + 5*a*b^2*c
^2*x - 2*a^2*b*c*d*x - 3*a^3*d^2*x)/((b*x^2 + a)^2*a^2*b^2)
```

maple [A] time = 0.01, size = 147, normalized size = 1.27

$$\frac{cd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{4\sqrt{ab}\;ab}+\frac{3c^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^{2}}+\frac{3d^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^{2}}+\frac{-\frac{\left(5a^{2}d^{2}-2abcd-3b^{2}c^{2}\right)x^{3}}{8a^{2}b}-\frac{\left(3a^{2}d^{2}+2abcd-5b^{2}c^{2}\right)x}{8a\,b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^3,x)
```

```
[Out] (-1/8*(5*a^2*d^2-2*a*b*c*d-3*b^2*c^2)/a^2/b*x^3-1/8*(3*a^2*d^2+2*a*b*c*d-5*
b^2*c^2)/a/b^2*x)/(b*x^2+a)^2+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
*d^2+1/4/a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d+3/8/a^2/(a*b)^(1/2)*
arctan(1/(a*b)^(1/2)*b*x)*c^2
```

maxima [A] time = 3.01, size = 138, normalized size = 1.19

$$\frac{\left(3\,b^{3}c^{2}+2\,ab^{2}cd-5\,a^{2}bd^{2}\right)x^{3}+\left(5\,ab^{2}c^{2}-2\,a^{2}bcd-3\,a^{3}d^{2}\right)x}{8\,\left(a^{2}b^{4}x^{4}+2\,a^{3}b^{3}x^{2}+a^{4}b^{2}\right)}+\frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^3*c^2 + 2*a*b^2*c*d - 5*a^2*b*d^2)*x^3 + (5*a*b^2*c^2 - 2*a^2*b*c
*d - 3*a^3*d^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(3*b^2*c^2
 + 2*a*b*c*d + 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

mupad [B] time = 5.02, size = 130, normalized size = 1.12

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(3\,a^{2}\,d^{2}+2\,a\,b\,c\,d+3\,b^{2}\,c^{2}\right)}{8\,a^{5/2}\,b^{5/2}}-\frac{\frac{x\,\left(3\,a^{2}\,d^{2}+2\,a\,b\,c\,d-5\,b^{2}\,c^{2}\right)}{8\,a\,b^{2}}-\frac{x^{3}\,\left(-5\,a^{2}\,d^{2}+2\,a\,b\,c\,d+3\,b^{2}\,c^{2}\right)}{8\,a^{2}\,b}}{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^3,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(8*a^(5/2)*
b^(5/2)) - ((x*(3*a^2*d^2 - 5*b^2*c^2 + 2*a*b*c*d))/(8*a*b^2) - (x^3*(3*b^2
*c^2 - 5*a^2*d^2 + 2*a*b*c*d))/(8*a^2*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

sympy [B] time = 1.05, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{a^{5}b^{5}}}\left(3a^{2}d^{2}+2abcd+3b^{2}c^{2}\right)\log\left(-a^{3}b^{2}\sqrt{-\frac{1}{a^{5}b^{5}}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^{5}b^{5}}}\left(3a^{2}d^{2}+2abcd+3b^{2}c^{2}\right)\log\left(a^{3}b^{2}\sqrt{-\frac{1}{a^{5}b^{5}}}+x\right)}{16}+\frac{x^{3}\left(-5a^{2}bd^{2}+2ab^{2}cd+3b^{3}c^{2}\right)+x\left(-3a^{3}d^{2}-2a^{2}bcd+5ab^{2}c^{2}\right)}{8a^{4}b^{2}+16a^{3}b^{3}x^{2}+8a^{2}b^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**
2*sqrt(-1/(a**5*b**5)) + x)/16 + sqrt(-1/(a**5*b**5))*(3*a**2*d**2 + 2*a*b*
c*d + 3*b**2*c**2)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/16 + (x**3*(-5*a
**2*b*d**2 + 2*a*b**2*c*d + 3*b**3*c**2) + x*(-3*a**3*d**2 - 2*a**2*b*c*d +
 5*a*b**2*c**2))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b**4*x**4)
```

3.39 $\int \frac{c+dx^2}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=92

$$\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(ad+3bc)}{8a^2b\,(a+bx^2)}+\frac{x(bc-ad)}{4ab\,(a+bx^2)^2}$$

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {385, 199, 205}

$$\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(ad+3bc)}{8a^2b\,(a+bx^2)}+\frac{x(bc-ad)}{4ab\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^3,x]
```

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^2)^2) + ((3*b*c + a*d)*x)/(8*a^2*b*(a + b*x
^2)) + ((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{(a+bx^2)^3}\,dx &= \frac{(bc-ad)x}{4ab\,(a+bx^2)^2}+\frac{(3bc+ad)\int \frac{1}{(a+bx^2)^2}\,dx}{4ab}\\ &= \frac{(bc-ad)x}{4ab\,(a+bx^2)^2}+\frac{(3bc+ad)x}{8a^2b\,(a+bx^2)}+\frac{(3bc+ad)\int \frac{1}{a+bx^2}\,dx}{8a^2b}\\ &= \frac{(bc-ad)x}{4ab\,(a+bx^2)^2}+\frac{(3bc+ad)x}{8a^2b\,(a+bx^2)}+\frac{(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 84, normalized size = 0.91

$$\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x\left(a^2(-d)+ab\left(5c+dx^2\right)+3b^2cx^2\right)}{8a^2b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^3,x]
```

```
[Out] (x*(-(a^2*d) + 3*b^2*c*x^2 + a*b*(5*c + d*x^2)))/(8*a^2*b*(a + b*x^2)^2) +
((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2)^3, x]
```

fricas [A] time = 0.79, size = 301, normalized size = 3.27

$$\left[\frac{2\left(3ab^3c+a^2b^2d\right)x^3-\left(\left(3b^3c+ab^2d\right)x^4+3a^2bc+a^3d+2\left(3ab^2c+a^2bd\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(5a^2b^2c-a^3bd\right)x}{16\left(a^3b^4x^4+2a^4b^3x^2+a^5b^2\right)},\frac{\left(3ab^3c+a^2b^2d\right)x^3+\left(\left(3b^3c+ab^2d\right)x^4+3a^2bc+a^3d+2\left(3ab^2c+a^2bd\right)x^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(5a^2b^2c-a^3bd\right)x}{8\left(a^3b^4x^4+2a^4b^3x^2+a^5b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a*b^3*c + a^2*b^2*d)*x^3 - ((3*b^3*c + a*b^2*d)*x^4 + 3*a^2*b*c
 + a^3*d + 2*(3*a*b^2*c + a^2*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^2*c - a^3*b*d)*x)/(a^3*b^4*x^4 + 2*a^4*b
^3*x^2 + a^5*b^2), 1/8*((3*a*b^3*c + a^2*b^2*d)*x^3 + ((3*b^3*c + a*b^2*d)*
x^4 + 3*a^2*b*c + a^3*d + 2*(3*a*b^2*c + a^2*b*d)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) + (5*a^2*b^2*c - a^3*b*d)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5
*b^2)]
```

giac [A] time = 0.58, size = 78, normalized size = 0.85

$$\frac{(3\,bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}+\frac{3\,b^2cx^3+abdx^3+5\,abcx-a^2dx}{8\left(bx^2+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(3*b^2*c*x^
3 + a*b*d*x^3 + 5*a*b*c*x - a^2*d*x)/((b*x^2 + a)^2*a^2*b)
```

maple [A] time = 0.01, size = 89, normalized size = 0.97

$$\frac{d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;ab}+\frac{3c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^2}+\frac{\frac{(ad+3bc)x^3}{8a^2}-\frac{(ad-5bc)x}{8ab}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(a*d+3*b*c)/a^2*x^3-1/8*(a*d-5*b*c)/a/b*x)/(b*x^2+a)^2+1/8/a/b/(a*b)^(
1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)*c
```

maxima [A] time = 3.00, size = 92, normalized size = 1.00

$$\frac{\left(3\,b^2c+abd\right)x^3+\left(5\,abc-a^2d\right)x}{8\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)}+\frac{(3\,bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^2*c + a*b*d)*x^3 + (5*a*b*c - a^2*d)*x)/(a^2*b^3*x^4 + 2*a^3*b^2*
x^2 + a^4*b) + 1/8*(3*b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

mupad [B] time = 5.02, size = 81, normalized size = 0.88

$$\frac{\frac{x^3\,(a\,d+3\,b\,c)}{8\,a^2}-\frac{x\,(a\,d-5\,b\,c)}{8\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a\,d+3\,b\,c)}{8\,a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^3,x)
```

```
[Out] ((x^3*(a*d + 3*b*c))/(8*a^2) - (x*(a*d - 5*b*c))/(8*a*b))/(a^2 + b^2*x^4 +
2*a*b*x^2) + (atan((b^(1/2)*x)/a^(1/2))*(a*d + 3*b*c))/(8*a^(5/2)*b^(3/2))
```

sympy [A] time = 0.58, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad+3bc)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad+3bc)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{x^3\left(abd+3b^2c\right)+x\left(-a^2d+5abc\right)}{8a^4b+16a^3b^2x^2+8a^2b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (x**3*(a*b*d + 3*b**2*c) + x*(-a**2*d + 5*a*b*c))/(8*a**4*b + 16*a**3
*b**2*x**2 + 8*a**2*b**3*x**4)
```

3.40 $\int \frac{1}{(a+bx^2)^3 (c+dx^2)} \, dx$

Optimal. Leaf size=161

$$\frac{bx(3bc-7ad)}{8a^2(a+bx^2)(bc-ad)^2}+\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^3}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}+\frac{bx}{4a(a+bx^2)^2(bc-ad)}$$

Rubi [A] time = 0.20, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {414, 527, 522, 205}

$$\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^3}+\frac{bx(3bc-7ad)}{8a^2(a+bx^2)(bc-ad)^2}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}+\frac{bx}{4a(a+bx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(3*b*c - 7*a*d)*x)/(8*a^2*(b*c -
 a*d)^2*(a + b*x^2)) + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2} - \frac{\int \frac{-3bc+4ad-3bdx^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)} + \frac{\int \frac{3b^2c^2-7abcd+8a^2d^2+bd(3bc-7ad)}{\left(a+bx^2\right)\left(c+dx^2\right)}}{8a^2(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)} - \frac{d^3\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^3} + \frac{\left(b\left(3b^2c^2-1\right.\right.}{}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)} + \frac{\sqrt{b}\left(3b^2c^2-10abcd+15a^2d\right.}{8a^{5/2}(bc-ad)}$$

Mathematica [A] time = 0.28, size = 158, normalized size = 0.98

$$\frac{1}{8}\left(\frac{bx(3bc-7ad)}{a^2\left(a+bx^2\right)(bc-ad)^2} - \frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^3} - \frac{8d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3} - \frac{2bx}{a\left(a+bx^2\right)^2(ad-bc)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] ((-2*b*x)/(a*(-(b*c) + a*d)*(a + b*x^2)^2) + (b*(3*b*c - 7*a*d)*x)/(a^2*(b*
c - a*d)^2*(a + b*x^2)) - (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*d)^3) - (8*d^(5/2)*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^3))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)^3*(c + d*x^2)), x]
```

fricas [B] time = 2.48, size = 1587, normalized size = 9.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 - (3*a^2*b^2*c^2 -
10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15*a^2*b^2*d^2)*x^4
 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sqrt(-b/a)*log((b*x
^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 8*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*
x^2 + a^4*d^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) +
 2*(5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2
*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b
^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*
c*d^2 - a^6*b*d^3)*x^2), 1/16*(2*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)
*x^3 - 16*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(d/c)*arctan(x*
```

```
sqrt(d/c)) - (3*a^2*b^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a
*b^3*c*d + 15*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b
*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(
5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2
*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c
*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^
2 - a^6*b*d^3)*x^2), 1/8*((3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 +
(3*a^2*b^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15
*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sq
rt(b/a)*arctan(x*sqrt(b/a)) - 4*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^
2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (5*a*b^3*c^
2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6
*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5
*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*
d^3)*x^2), 1/8*((3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 + (3*a^2*b^2
*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15*a^2*b^2*d
^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sqrt(b/a)*ar
ctan(x*sqrt(b/a)) - 8*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(d/
c)*arctan(x*sqrt(d/c)) + (5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a
^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a
^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*
b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x^2)]
```

giac [A] time = 0.58, size = 218, normalized size = 1.35

$$-\frac{d^3 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{cd}} + \frac{\left(3\,b^3c^2 - 10\,ab^2cd + 15\,a^2bd^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3\right)\sqrt{ab}} + \frac{3\,b^3cx^3 - 7\,ab^2dx^3 + 5\,ab^2cx - 9\,a^2bdx}{8\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2\right)\left(bx^2 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(c*d)) + 1/8*(3*b^3*c^2 - 10*a*b^2*c*d + 15*a^2*b*d^2)*arctan(b*x/
sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(
a*b)) + 1/8*(3*b^3*c*x^3 - 7*a*b^2*d*x^3 + 5*a*b^2*c*x - 9*a^2*b*d*x)/((a^2
*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(b*x^2 + a)^2)
```

maple [B] time = 0.01, size = 309, normalized size = 1.92

$$\frac{5b^3cd\,x^3}{4\,(ad-bc)^3\left(b\,x^2+a\right)^2 a} - \frac{3b^4c^2x^3}{8\,(ad-bc)^3\left(b\,x^2+a\right)^2 a^2} - \frac{7b^2d^2x^3}{8\,(ad-bc)^3\left(b\,x^2+a\right)^2} - \frac{9ab\,d^2x}{8\,(ad-bc)^3\left(b\,x^2+a\right)^2} - \frac{5b^3c^2x}{8\,(ad-bc)^3\left(b\,x^2+a\right)^2 a} + \frac{7b^2cdx}{4\,(ad-bc)^3\left(b\,x^2+a\right)^2} + \frac{5b^2cd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{4\,(ad-bc)^3\sqrt{ab}\;a} - \frac{3b^3c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(ad-bc)^3\sqrt{ab}\;a^2} - \frac{15b\,d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(ad-bc)^3\sqrt{ab}} + \frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^3\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] d^3/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)-7/8*b^2/(a*d-b*c)^3/(
b*x^2+a)^2*x^3*d^2+5/4*b^3/(a*d-b*c)^3/(b*x^2+a)^2/a*x^3*c*d-3/8*b^4/(a*d-b
*c)^3/(b*x^2+a)^2/a^2*x^3*c^2-9/8*b/(a*d-b*c)^3/(b*x^2+a)^2*x*a*d^2+7/4*b^2
/(a*d-b*c)^3/(b*x^2+a)^2*x*c*d-5/8*b^3/(a*d-b*c)^3/(b*x^2+a)^2*x/a*c^2-15/8
*b/(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2+5/4*b^2/(a*d-b*c)^
3/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d-3/8*b^3/(a*d-b*c)^3/a^2/(a*b)
^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2
```

maxima [A] time = 3.16, size = 278, normalized size = 1.73

$$-\frac{d^3 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{cd}} + \frac{\left(3\,b^3c^2 - 10\,ab^2cd + 15\,a^2bd^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3\right)\sqrt{ab}} + \frac{\left(3\,b^3c - 7\,ab^2d\right)x^3 + \left(5\,ab^2c - 9\,a^2bd\right)x}{8\left(a^4b^2c^2 - 2\,a^5bcd + a^6d^2 + \left(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2\right)x^4 + 2\left(a^3b^3c^2 - 2\,a^4b^2cd + a^5bd^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(c*d)) + 1/8*(3*b^3*c^2 - 10*a*b^2*c*d + 15*a^2*b*d^2)*arctan(b*x/
sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(
a*b)) + 1/8*((3*b^3*c - 7*a*b^2*d)*x^3 + (5*a*b^2*c - 9*a^2*b*d)*x)/(a^4*b^
2*c^2 - 2*a^5*b*c*d + a^6*d^2 + (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)
*x^4 + 2*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x^2)
```

mupad [B] time = 6.89, size = 6033, normalized size = 37.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^3*(c + d*x^2)),x)
```

```
[Out] ((x^3*(3*b^3*c - 7*a*b^2*d))/(8*a^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(
5*b^2*c - 9*a*b*d))/(8*a*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a^2 + b^2*x^4 +
 2*a*b*x^2) + (atan(((((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d
^4 - 300*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 -
 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - ((-c*d^5)^(1/2)*((
256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*
c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^
6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^
6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*
c^2*d^4 - 6*a^9*b*c*d^5)) - (x*(-c*d^5)^(1/2)*(256*a^11*b^2*d^9 - 1280*a^10
*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5*
d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))/
(64*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^8*d^4 + a^4*
b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3))))/(2*(b^3*c
^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(-c*d^5)^(1/2)*1i)/(2*(
b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) + (((x*(289*a^4*b^3
*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 + 190*a^2*b^5*c
^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 -
 4*a^7*b*c*d^3)) + ((-c*d^5)^(1/2)*((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9
 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*
a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^
4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d
^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) + (x*(-c*d^5
)^(1/2)*(256*a^11*b^2*d^9 - 1280*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 128
0*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*
b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))/(64*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2
*d^2 - 3*a*b^2*c^3*d)*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*
c^2*d^2 - 4*a^7*b*c*d^3))))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a
*b^2*c^3*d)))*(-c*d^5)^(1/2)*1i)/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2
 - 3*a*b^2*c^3*d)))/((105*a^3*b^3*d^8 - 9*b^6*c^3*d^5 + 51*a*b^5*c^2*d^6 - 1
15*a^2*b^4*c*d^7)/(32*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^
4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) - (((
x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 +
 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6
*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - ((-c*d^5)^(1/2)*((256*a^10*b^2*d^10 - 1760
*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c
^6*d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7
 + 5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15
*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)
) - (x*(-c*d^5)^(1/2)*(256*a^11*b^2*d^9 - 1280*a^10*b^3*c*d^8 + 256*a^4*b^9
*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d
^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))/(64*(b^3*c^4 - a^3*c*d^3
 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*
d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3))))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b
*c^2*d^2 - 3*a*b^2*c^3*d)))*(-c*d^5)^(1/2))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2
*b*c^2*d^2 - 3*a*b^2*c^3*d)) + (((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a
```

*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4
*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) + ((-c*d^5
)^(1/2)*((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 80
0*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*
b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 +
a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 1
5*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) + (x*(-c*d^5)^(1/2)*(256*a^11*b^2*d^9 -
 1280*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^
6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*
c^2*d^7))/(64*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^8*
d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3))))
/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(-c*d^5)^(1/2
))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d))))*(-c*d^5)^(
1/2)*1i)/(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d) + (atan(((
((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6
 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a
^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - (((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^
9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816
*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b
^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*
d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) - (x*(-a^5*
b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d)*(256*a^11*b^2*d^9 - 1280*a^1
0*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5
*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))
/(512*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)*(a^8*d^4 +
a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)))*(-a^5*
b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d))/(16*(a^8*d^3 - a^5*b^3*c^3
+ 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2
 - 10*a*b*c*d)*1i)/(16*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c
d^2)) + (((x(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3
*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c
^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) + (((256*a^10*b^2*d^10 - 1760*a^
9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*
d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 +
5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^
6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) +
 (x*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d)*(256*a^11*b^2*d^9
- 1280*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a
^6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4
*c^2*d^7))/(512*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)*(
a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3
)))*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d))/(16*(a^8*d^3 - a^
5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b)^(1/2)*(15*a^2*d^2 +
 3*b^2*c^2 - 10*a*b*c*d)*1i)/(16*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d -
 3*a^7*b*c*d^2)))/((105*a^3*b^3*d^8 - 9*b^6*c^3*d^5 + 51*a*b^5*c^2*d^6 - 11
5*a^2*b^4*c*d^7)/(32*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4
*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) - (((x
*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 +
190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*
b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - (((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9 +
 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*a^
5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^4*
c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2
 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) - (x*(-a^5*b)^
(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d)*(256*a^11*b^2*d^9 - 1280*a^10*b
^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5*d^
4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))/(5
12*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)*(a^8*d^4 + a^4
*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)))*(-a^5*b)^

```
(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d))/(16*(a^8*d^3 - a^5*b^3*c^3 + 3
*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 -
10*a*b*c*d))/(16*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2))
 + (((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c
*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d +
 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) + (((256*a^10*b^2*d^10 - 1760*a^9*b^3*
c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 -
6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a
^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*
c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) + (x*(-
a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d)*(256*a^11*b^2*d^9 - 1280
*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7
*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d
^7))/(512*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)*(a^8*d^
4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)))*(-
a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d))/(16*(a^8*d^3 - a^5*b^3*
c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2
*c^2 - 10*a*b*c*d))/(16*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*
c*d^2))))*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d)*1i)/(8*(a^8*
d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.41 $\int \frac{1}{(a+bx^2)^3 (c+dx^2)^2} \, dx$

Optimal. Leaf size=236

$$\frac{dx(bc-4ad)(ad+3bc)}{8a^2c\left(c+dx^2\right)(bc-ad)^3}+\frac{3bx(bc-3ad)}{8a^2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)^2}+\frac{b^{3/2}\left(35a^2d^2-14abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^4}-\frac{d^{5/2}(7\cdots}{\cdots}$$

Rubi [A] time = 0.31, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {414, 527, 522, 205}

$$\frac{b^{3/2}\left(35a^2d^2-14abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^4}+\frac{dx(bc-4ad)(ad+3bc)}{8a^2c\left(c+dx^2\right)(bc-ad)^3}+\frac{3bx(bc-3ad)}{8a^2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)^2}-\frac{d^{5/2}(7bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^4}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c - 4*a*d)*(3*b*c + a*d)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x^2)) + (b*
x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)) + (3*b*(b*c - 3*a*d)*x)/(8*a
^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(3*b^2*c^2 - 14*a*b*c*
d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^4) - (d
^(5/2)*(7*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^2}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} - \frac{\int \frac{-3bc+4ad-5bdx^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\int \frac{3b^2c^2-\cdots}{\cdots}}{\cdots}$$

$$= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(bc-\cdots}{8a^2(bc-ad)^2\left(a\cdots\right.}$$

$$= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(bc-\cdots}{8a^2(bc-ad)^2\left(a\cdots\right.}$$

$$= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(bc-\cdots}{8a^2(bc-ad)^2\left(a\cdots\right.}$$

Mathematica [A] time = 0.42, size = 197, normalized size = 0.83

$$\frac{1}{8}\left(\frac{b^2x(11ad-3bc)}{a^2\left(a+bx^2\right)(ad-bc)^3} + \frac{b^{3/2}\left(35a^2d^2-14abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^4} + \frac{2b^2x}{a\left(a+bx^2\right)^2(bc-ad)^2} + \frac{4d^{5/2}(ad-7bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^4} - \frac{4d^3x}{c\left(c+dx^2\right)(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

```
[Out] ((2*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^2)^2) + (b^2*(-3*b*c + 11*a*d)*x)/(a^2
*(-(b*c) + a*d)^3*(a + b*x^2)) - (4*d^3*x)/(c*(b*c - a*d)^3*(c + d*x^2)) +
(b^(3/2)*(3*b^2*c^2 - 14*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(a^(5/2)*(b*c - a*d)^4) + (4*d^(5/2)*(-7*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(c^(3/2)*(b*c - a*d)^4))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)^3*(c + d*x^2)^2), x]
```

fricas [B] time = 8.54, size = 3251, normalized size = 13.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d^3 + 4*a^3*b^2*d^4)
*x^5 + 2*(3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 +
 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 35*a^4*b*c^2*d^2 +
(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 + (3*b^5*c^4 - 8*a*
b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6*a*b^4*c^4 - 25*a
```

```
^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*(7*a^4*b*c^2*d^2 - a^5*c*d^3 +
 (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^
3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^4*c^4
 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a
^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^
2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*
c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c
^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^
3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7
*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/16*(2*(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a
^2*b^3*c*d^3 + 4*a^3*b^2*d^4)*x^5 + 2*(3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^
3*c^2*d^2 + 5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 - 8*(7*a^4*b*c^2*d^2 - a^5*c
*d^3 + (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^
2*c*d^3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)
*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 3
5*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 +
 (3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (
6*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*
sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(5*a*b^4*c^4
 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a
^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^
2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*
c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c
^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^
3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7
*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/8*((3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*
b^3*c*d^3 + 4*a^3*b^2*d^4)*x^5 + (3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2
*d^2 + 5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3
*d + 35*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)
*x^6 + (3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x
^4 + (6*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)
*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 2*(7*a^4*b*c^2*d^2 - a^5*c*d^3 + (7*a
^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^3 - 2
*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(
-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (5*a*b^4*c^4 - 18*a
^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a^4*b^4*
c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^2*d^4 +
 (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*c^2*d^4
 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c^4*d^2
+ 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^3*b^5*c
^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7*b*c^2*
d^4 + a^8*c*d^5)*x^2), 1/8*((3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d
^3 + 4*a^3*b^2*d^4)*x^5 + (3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 +
5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 35
*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 +
(3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6
*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*s
qrt(b/a)*arctan(x*sqrt(b/a)) - 4*(7*a^4*b*c^2*d^2 - a^5*c*d^3 + (7*a^2*b^3*
c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^3 - 2*a^4*b*
d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(d/c)*ar
ctan(x*sqrt(d/c)) + (5*a*b^4*c^4 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 -
4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^
4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2
+ 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6
 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*
d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4
*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7*b*c^2*d^4 + a^8*c*d^5)*x^2)]
```

giac [A] time = 0.61, size = 333, normalized size = 1.41

$$-\frac{d^3 x}{2\left(b^3c^4 - 3\,ab^2c^3d + 3\,a^2bc^2d^2 - a^3cd^3\right)\left(dx^2 + c\right)} + \frac{\left(3\,b^4c^2 - 14\,ab^3cd + 35\,a^2b^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(a^2b^4c^4 - 4\,a^3b^3c^3d + 6\,a^4b^2c^2d^2 - 4\,a^5bcd^3 + a^6d^4\right)\sqrt{ab}} - \frac{\left(7\,bcd^3 - ad^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^4c^5 - 4\,ab^3c^4d + 6\,a^2b^2c^3d^2 - 4\,a^3bc^2d^3 + a^4cd^4\right)\sqrt{cd}} + \frac{3\,b^4cx^3 - 11\,ab^3dx^3 + 5\,ab^3cx - 13\,a^2b^2dx}{8\left(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3\right)\left(bx^2 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*x/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2
+ c)) + 1/8*(3*b^4*c^2 - 14*a*b^3*c*d + 35*a^2*b^2*d^2)*arctan(b*x/sqrt(a*b
))/((a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^
6*d^4)*sqrt(a*b)) - 1/2*(7*b*c*d^3 - a*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^5
 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*
d)) + 1/8*(3*b^4*c*x^3 - 11*a*b^3*d*x^3 + 5*a*b^3*c*x - 13*a^2*b^2*d*x)/((a
^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*(b*x^2 + a)^2)
```

maple [A] time = 0.01, size = 403, normalized size = 1.71

$$-\frac{7b^4cd\,x^3}{4\left(ad-bc\right)^4\left(b\,x^2+a\right)^2a} + \frac{3b^5c^2x^3}{8\left(ad-bc\right)^4\left(b\,x^2+a\right)^2a^2} + \frac{11b^3d^2x^3}{8\left(ad-bc\right)^4\left(b\,x^2+a\right)^2} + \frac{13a\,b^2d^2x}{8\left(ad-bc\right)^4\left(b\,x^2+a\right)^2} + \frac{5b^4c^2x}{8\left(ad-bc\right)^4\left(b\,x^2+a\right)^2a} - \frac{9b^3cdx}{4\left(ad-bc\right)^4\left(b\,x^2+a\right)^2} + \frac{a\,d^4x}{2\left(ad-bc\right)^4\left(d\,x^2+c\right)c} + \frac{a\,d^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^4\sqrt{cd}\;c} - \frac{7b^3cd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{4\left(ad-bc\right)^4\sqrt{ab}\;a} + \frac{3b^4c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(ad-bc\right)^4\sqrt{ab}\;a^2} + \frac{35b^2d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(ad-bc\right)^4\sqrt{ab}} - \frac{b\,d^3x}{2\left(ad-bc\right)^4\left(d\,x^2+c\right)} - \frac{7b\,d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^4\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^4/(a*d-b*c)^4/c*x/(d*x^2+c)*a-1/2*d^3/(a*d-b*c)^4*x/(d*x^2+c)*b+1/2*d
^4/(a*d-b*c)^4/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a-7/2*d^3/(a*d-b*c)^
4/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b+11/8*b^3/(a*d-b*c)^4/(b*x^2+a)^2*
x^3*d^2-7/4*b^4/(a*d-b*c)^4/(b*x^2+a)^2/a*x^3*c*d+3/8*b^5/(a*d-b*c)^4/(b*x^
2+a)^2/a^2*x^3*c^2+13/8*b^2/(a*d-b*c)^4/(b*x^2+a)^2*x*a*d^2-9/4*b^3/(a*d-b*
c)^4/(b*x^2+a)^2*x*c*d+5/8*b^4/(a*d-b*c)^4/(b*x^2+a)^2*x/a*c^2+35/8*b^2/(a*
d-b*c)^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2-7/4*b^3/(a*d-b*c)^4/a/(a
*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d+3/8*b^4/(a*d-b*c)^4/a^2/(a*b)^(1/2)
*arctan(1/(a*b)^(1/2)*b*x)*c^2
```

maxima [B] time = 3.18, size = 530, normalized size = 2.25

$$\frac{\left(3\,b^4c^2 - 14\,ab^3cd + 35\,a^2b^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(a^2b^4c^4 - 4\,a^3b^3c^3d + 6\,a^4b^2c^2d^2 - 4\,a^5bcd^3 + a^6d^4\right)\sqrt{ab}} - \frac{\left(7\,bcd^3 - ad^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^4c^5 - 4\,ab^3c^4d + 6\,a^2b^2c^3d^2 - 4\,a^3bc^2d^3 + a^4cd^4\right)\sqrt{cd}} + \frac{\left(3\,b^4c^2d - 11\,ab^3cd^2 - 4\,a^2b^2d^3\right)x^5 + \left(3\,b^4c^3 - 6\,ab^3c^2d - 13\,a^2b^2cd^2 - 8\,a^3bd^3\right)x^3 + \left(5\,ab^3c^3 - 13\,a^2b^2c^2d - 4\,a^4d^3\right)x}{8\left(a^4b^3c^5 - 3\,a^5b^2c^4d + 3\,a^6bc^3d^2 - a^7c^2d^3 + \left(a^2b^5c^4d - 3\,a^3b^4c^3d^2 + 3\,a^4b^3c^2d^3 - a^5b^2cd^4\right)x^6 + \left(a^2b^5c^5 - a^3b^4c^4d - 3\,a^4b^3c^3d^2 + 5\,a^5b^2c^2d^3 - 2\,a^6bcd^4\right)x^4 + \left(2\,a^3b^4c^5 - 5\,a^4b^3c^4d + 3\,a^5b^2c^3d^2 + a^6bc^2d^3 - a^7cd^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b^4*c^2 - 14*a*b^3*c*d + 35*a^2*b^2*d^2)*arctan(b*x/sqrt(a*b))/((a^2
*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*s
qrt(a*b)) - 1/2*(7*b*c*d^3 - a*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b
^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) + 1/
8*((3*b^4*c^2*d - 11*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*x^5 + (3*b^4*c^3 - 6*a*b^
3*c^2*d - 13*a^2*b^2*c*d^2 - 8*a^3*b*d^3)*x^3 + (5*a*b^3*c^3 - 13*a^2*b^2*c
^2*d - 4*a^4*d^3)*x)/(a^4*b^3*c^5 - 3*a^5*b^2*c^4*d + 3*a^6*b*c^3*d^2 - a^7
*c^2*d^3 + (a^2*b^5*c^4*d - 3*a^3*b^4*c^3*d^2 + 3*a^4*b^3*c^2*d^3 - a^5*b^2
*c*d^4)*x^6 + (a^2*b^5*c^5 - a^3*b^4*c^4*d - 3*a^4*b^3*c^3*d^2 + 5*a^5*b^2*
c^2*d^3 - 2*a^6*b*c*d^4)*x^4 + (2*a^3*b^4*c^5 - 5*a^4*b^3*c^4*d + 3*a^5*b^2
*c^3*d^2 + a^6*b*c^2*d^3 - a^7*c*d^4)*x^2)
```

mupad [B] time = 7.85, size = 8635, normalized size = 36.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^3*(c + d*x^2)^2),x)
```

```
[Out] ((x^5*(4*a^2*b^2*d^3 - 3*b^4*c^2*d + 11*a*b^3*c*d^2))/(8*a^2*c*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(4*a^3*d^3 - 5*b^3*c^3 + 13*a
```

```
*b^2*c^2*d))/(8*a*c*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*x^3*(
8*a^3*d^3 - 3*b^3*c^3 + 6*a*b^2*c^2*d + 13*a^2*b*c*d^2))/(8*a^2*c*(a*d - b*
c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a^2*c + x^2*(a^2*d + 2*a*b*c) + x^4*(
b^2*c + 2*a*b*d) + b^2*d*x^6) - (atan(((((x*(16*a^6*b^3*d^9 + 9*b^9*c^6*d^3
 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6
*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b
^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a
^8*b^2*c^4*d^4)) - (((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3
*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b
^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*
d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c
^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8
 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*
b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (x*(-a^5*b^3)^(1
/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*a^4*b^11*c^11*d^2 - 1792*a^5
*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7
*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4
*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(512*(a^9*d^4 + a^5
*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)*(a^4*b^6*c^
8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 -
 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 +
3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a
^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2
- 14*a*b*c*d)*1i)/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*
c^2*d^2 - 4*a^8*b*c*d^3)) + (((x*(16*a^6*b^3*d^9 + 9*b^9*c^6*d^3 - 84*a*b^8
*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 +
2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d -
6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*
d^4)) + (((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*
d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6
+ 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*
a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a
^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b
^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5
 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) + (x*(-a^5*b^3)^(1/2)*(35*a^2
*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*a^4*b^11*c^11*d^2 - 1792*a^5*b^10*c^10*
d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 +
3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792
*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(512*(a^9*d^4 + a^5*b^4*c^4 -
4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)*(a^4*b^6*c^8 + a^10*c^
2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3
*c^5*d^3 + 15*a^8*b^2*c^4*d^4)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 -
 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*
d^2 - 4*a^8*b*c*d^3)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*
d)*1i)/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4
*a^8*b*c*d^3)))/(((35*a^5*b^4*d^10)/16 + (63*b^9*c^5*d^5)/64 - (267*a*b^8*c
^4*d^6)/32 - (651*a^4*b^5*c*d^9)/64 + (451*a^2*b^7*c^3*d^7)/16 - (1275*a^3*
b^6*c^2*d^8)/32)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b
*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 -
126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (((x*(16
*a^6*b^3*d^9 + 9*b^9*c^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a
^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*
c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2
 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)) - (((2*a^13*b^2*c*d^13 - (3*a^
2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*
a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^
7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4
*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b
^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 12
```

6*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2
*c^4*d^7) - (x*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*
a^4*b^11*c^11*d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^
7*b^8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5
*c^5*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^
2*d^11))/(512*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2
- 4*a^8*b*c*d^3)*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^
3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)))*(-a
^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4
*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))*(-a^5*b^3)^(1
/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a
^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)) + (((x*(16*a^6*b^3*d^9 +
9*b^9*c^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5
- 980*a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2
*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*
c^5*d^3 + 15*a^8*b^2*c^4*d^4)) + (((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^
2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d
^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 97
8*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 -
28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*
a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*
d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) + (x
*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*a^4*b^11*c^11*
d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5
+ 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 512
0*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(512*
(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^
3)*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6
*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)))*(-a^5*b^3)^(1/2)*
(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b
^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))*(-a^5*b^3)^(1/2)*(35*a^2*d^
2 + 3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d +
6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3))))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2
*c^2 - 14*a*b*c*d)*1i)/(8*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*
b^2*c^2*d^2 - 4*a^8*b*c*d^3)) - (atan(((((x*(16*a^6*b^3*d^9 + 9*b^9*c^6*d^3
- 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6
*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b
^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a
^8*b^2*c^4*d^4)) - ((a*d - 7*b*c)*(-c^3*d^5)^(1/2)*((2*a^13*b^2*c*d^13 - (3
*a^2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 3
36*a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8
*b^7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*
b^4*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^
5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 +
126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*
b^2*c^4*d^7) - (x*(a*d - 7*b*c)*(-c^3*d^5)^(1/2)*(256*a^4*b^11*c^11*d^2 - 1
792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*
a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*
b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(128*(b^4*c^
7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)*(a^4
*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^
6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4))))/(4*(b^4*c^7 + a^4*c^3*d
^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)))*(a*d - 7*b*c)*(
-c^3*d^5)^(1/2)*1i)/(4*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2
*c^5*d^2 - 4*a*b^3*c^6*d)) + (((x*(16*a^6*b^3*d^9 + 9*b^9*c^6*d^3 - 84*a*b^
8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 +
2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d -
6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4
*d^4)) + ((a*d - 7*b*c)*(-c^3*d^5)^(1/2)*((2*a^13*b^2*c*d^13 - (3*a^2*b^13*

```
c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^1
0*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d
^8 + 978*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^
11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10
*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b
^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^
7) + (x*(a*d - 7*b*c)*(-c^3*d^5)^(1/2)*(256*a^4*b^11*c^11*d^2 - 1792*a^5*b^
10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7*c^
7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4*d^
9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(128*(b^4*c^7 + a^4*c^
3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)*(a^4*b^6*c^8 +
 a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20
*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4))))/(4*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3
*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)))*(a*d - 7*b*c)*(-c^3*d^5)^
(1/2)*1i)/(4*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 -
 4*a*b^3*c^6*d)))/(((35*a^5*b^4*d^10)/16 + (63*b^9*c^5*d^5)/64 - (267*a*b^8
*c^4*d^6)/32 - (651*a^4*b^5*c*d^9)/64 + (451*a^2*b^7*c^3*d^7)/16 - (1275*a^
3*b^6*c^2*d^8)/32)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12
*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4
- 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (((x*(
16*a^6*b^3*d^9 + 9*b^9*c^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406
*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^
6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d
^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)) - ((a*d - 7*b*c)*(-c^3*d^5)^
(1/2)*((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3
)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1
197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*a^1
0*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*
b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*
c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 +
84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (x*(a*d - 7*b*c)*(-c^3*d^5)^(1
/2)*(256*a^4*b^11*c^11*d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4
- 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168
*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^
13*b^2*c^2*d^11))/(128*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2
*c^5*d^2 - 4*a*b^3*c^6*d)*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6
*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d
^4))))/(4*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*
a*b^3*c^6*d)))*(a*d - 7*b*c)*(-c^3*d^5)^(1/2))/(4*(b^4*c^7 + a^4*c^3*d^4 -
4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)) + (((x*(16*a^6*b^3*d^
9 + 9*b^9*c^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*
d^5 - 980*a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*
c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b
^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)) + ((a*d - 7*b*c)*(-c^3*d^5)^(1/2)*((2*a^1
3*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*
b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c
^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^1
0)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^
13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*
a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c
^5*d^6 - 36*a^11*b^2*c^4*d^7) + (x*(a*d - 7*b*c)*(-c^3*d^5)^(1/2)*(256*a^4*
b^11*c^11*d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^
8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5
*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^
11))/(128*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*
a*b^3*c^6*d)*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^
5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4))))/(4*(b^
4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d))
)*(a*d - 7*b*c)*(-c^3*d^5)^(1/2))/(4*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d
```

```
^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d))))*(a*d - 7*b*c)*(-c^3*d^5)^(1/2)*1
i)/(2*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^
3*c^6*d))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.42 $\int \frac{1}{(a+bx^2)^3 (c+dx^2)^3} \, dx$

Optimal. Leaf size=315

$$\frac{3dx(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{8a^2c^2\left(c+dx^2\right)(bc-ad)^4}+\frac{dx\left(-2a^2d^2-13abcd+3b^2c^2\right)}{8a^2c\left(c+dx^2\right)^2(bc-ad)^3}-\frac{3d^{5/2}\left(a^2d^2-6abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^5}+$$

Rubi [A] time = 0.45, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {414, 527, 522, 205}

$$\frac{3dx(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{8a^2c^2\left(c+dx^2\right)(bc-ad)^4}+\frac{dx\left(-2a^2d^2-13abcd+3b^2c^2\right)}{8a^2c\left(c+dx^2\right)^2(bc-ad)^3}+\frac{3b^{5/2}\left(21a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^5}-\frac{3d^{5/2}\left(a^2d^2-6abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^5}+\frac{bx(3bc-11ad)}{8a^2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)^2}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

```
[Out] (d*(3*b^2*c^2 - 13*a*b*c*d - 2*a^2*d^2)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x^
2)^2) + (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)^2) + (b*(3*b*c - 1
1*a*d)*x)/(8*a^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)^2) + (3*d*(b*c + a*d
)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x)/(8*a^2*c^2*(b*c - a*d)^4*(c + d*x^2))
+ (3*b^(5/2)*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*a^(5/2)*(b*c - a*d)^5) - (3*d^(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)
*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^5)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^3}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} - \frac{\int \frac{-3bc+4ad-7bdx^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} + \frac{b(3bc-11ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)^2} + \int \frac{3b^2c}{}$$

$$= \frac{d\left(3b^2c^2-13abcd-2a^2d^2\right)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)^2} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} + \frac{b}{8a^2(bc-ad}$$

$$= \frac{d\left(3b^2c^2-13abcd-2a^2d^2\right)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)^2} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} + \frac{b}{8a^2(bc-ad}$$

$$= \frac{d\left(3b^2c^2-13abcd-2a^2d^2\right)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)^2} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} + \frac{b}{8a^2(bc-ad}$$

$$= \frac{d\left(3b^2c^2-13abcd-2a^2d^2\right)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)^2} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} + \frac{b}{8a^2(bc-ad}$$

Mathematica [A] time = 0.93, size = 233, normalized size = 0.74

$$\frac{1}{8}\left(\frac{x(bc-ad)\left(\frac{3b^4c}{a^2\left(a+bx^2\right)} + \frac{b^3\left(-17ad+2bc-15bdx^2\right)}{a\left(a+bx^2\right)^2} - \frac{d^3\left(-2ad+17bc+15bdx^2\right)}{c\left(c+dx^2\right)^2} + \frac{3ad^4}{c^2\left(c+dx^2\right)}\right) - \frac{3d^{5/2}\left(a^2d^2-6abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}}}{(bc-ad)^5} - \frac{3b^{5/2}\left(21a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

```
[Out] ((-3*b^(5/2)*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(a^(5/2)*(-(b*c) + a*d)^5) + ((b*c - a*d)*x*((3*b^4*c)/(a^2*(a + b*x^2))
+ (3*a*d^4)/(c^2*(c + d*x^2)) + (b^3*(2*b*c - 17*a*d - 15*b*d*x^2))/(a*(a +
 b*x^2)^2) - (d^3*(17*b*c - 2*a*d + 15*b*d*x^2))/(c*(c + d*x^2)^2)) - (3*d^
(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(5/
2))/(b*c - a*d)^5)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)^3*(c + d*x^2)^3), x]
```

fricas [B] time = 32.35, size = 5070, normalized size = 16.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b^2*d^6)*x^
7 + 2*(6*b^6*c^5*d - 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d^3 + 9*a^3*b^3*c^2*d
^4 + 31*a^4*b^2*c*d^5 - 6*a^5*b*d^6)*x^5 + 2*(3*b^6*c^6 - 8*a*b^5*c^5*d - 2
9*a^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*d^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^3 - 3
*(a^2*b^4*c^6 - 6*a^3*b^3*c^5*d + 21*a^4*b^2*c^4*d^2 + (b^6*c^4*d^2 - 6*a*b
^5*c^3*d^3 + 21*a^2*b^4*c^2*d^4)*x^8 + 2*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 15*
a^2*b^4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*x^6 + (b^6*c^6 - 2*a*b^5*c^5*d - 2*a^
2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*d^4)*x^4 + 2*(a*b^5*c^6
 - 5*a^2*b^4*c^5*d + 15*a^3*b^3*c^4*d^2 + 21*a^4*b^2*c^3*d^3)*x^2)*sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*(21*a^4*b^2*c^4*d^2
- 6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^2*d^4 - 6*a^3*b^3*c*d^5 + a
^4*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3*b^3*c^2*d^4 - 5*a^4*b^2*c*
d^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 - 2*a^4*b^2
*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*a^3*b^3*c^4*d^2 + 15*a^4*b^
2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x
*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^5*c^6 - 22*a^2*b^4*c^5*d + 17*a^3*
b^3*c^4*d^2 - 17*a^4*b^2*c^3*d^3 + 22*a^5*b*c^2*d^4 - 5*a^6*c*d^5)*x)/(a^4*
b^5*c^9 - 5*a^5*b^4*c^8*d + 10*a^6*b^3*c^7*d^2 - 10*a^7*b^2*c^6*d^3 + 5*a^8
*b*c^5*d^4 - a^9*c^4*d^5 + (a^2*b^7*c^7*d^2 - 5*a^3*b^6*c^6*d^3 + 10*a^4*b^
5*c^5*d^4 - 10*a^5*b^4*c^4*d^5 + 5*a^6*b^3*c^3*d^6 - a^7*b^2*c^2*d^7)*x^8 +
 2*(a^2*b^7*c^8*d - 4*a^3*b^6*c^7*d^2 + 5*a^4*b^5*c^6*d^3 - 5*a^6*b^3*c^4*d
^5 + 4*a^7*b^2*c^3*d^6 - a^8*b*c^2*d^7)*x^6 + (a^2*b^7*c^9 - a^3*b^6*c^8*d
- 9*a^4*b^5*c^7*d^2 + 25*a^5*b^4*c^6*d^3 - 25*a^6*b^3*c^5*d^4 + 9*a^7*b^2*c
^4*d^5 + a^8*b*c^3*d^6 - a^9*c^2*d^7)*x^4 + 2*(a^3*b^6*c^9 - 4*a^4*b^5*c^8*
d + 5*a^5*b^4*c^7*d^2 - 5*a^7*b^2*c^5*d^4 + 4*a^8*b*c^4*d^5 - a^9*c^3*d^6)*
x^2), 1/16*(6*(b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b^2*d^
6)*x^7 + 2*(6*b^6*c^5*d - 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d^3 + 9*a^3*b^3*
c^2*d^4 + 31*a^4*b^2*c*d^5 - 6*a^5*b*d^6)*x^5 + 2*(3*b^6*c^6 - 8*a*b^5*c^5*
d - 29*a^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*d^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^
3 - 6*(21*a^4*b^2*c^4*d^2 - 6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^2
*d^4 - 6*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3*
b^3*c^2*d^4 - 5*a^4*b^2*c*d^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*a
^3*b^3*c^3*d^3 - 2*a^4*b^2*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*a
^3*b^3*c^4*d^2 + 15*a^4*b^2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sqr
t(d/c)*arctan(x*sqrt(d/c)) - 3*(a^2*b^4*c^6 - 6*a^3*b^3*c^5*d + 21*a^4*b^2*
c^4*d^2 + (b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 21*a^2*b^4*c^2*d^4)*x^8 + 2*(b^6
*c^5*d - 5*a*b^5*c^4*d^2 + 15*a^2*b^4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*x^6 + (
b^6*c^6 - 2*a*b^5*c^5*d - 2*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 + 21*a^4*b
^2*c^2*d^4)*x^4 + 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d + 15*a^3*b^3*c^4*d^2 + 21*
a^4*b^2*c^3*d^3)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
+ a)) + 2*(5*a*b^5*c^6 - 22*a^2*b^4*c^5*d + 17*a^3*b^3*c^4*d^2 - 17*a^4*b^2
*c^3*d^3 + 22*a^5*b*c^2*d^4 - 5*a^6*c*d^5)*x)/(a^4*b^5*c^9 - 5*a^5*b^4*c^8*
d + 10*a^6*b^3*c^7*d^2 - 10*a^7*b^2*c^6*d^3 + 5*a^8*b*c^5*d^4 - a^9*c^4*d^5
 + (a^2*b^7*c^7*d^2 - 5*a^3*b^6*c^6*d^3 + 10*a^4*b^5*c^5*d^4 - 10*a^5*b^4*c
^4*d^5 + 5*a^6*b^3*c^3*d^6 - a^7*b^2*c^2*d^7)*x^8 + 2*(a^2*b^7*c^8*d - 4*a^
3*b^6*c^7*d^2 + 5*a^4*b^5*c^6*d^3 - 5*a^6*b^3*c^4*d^5 + 4*a^7*b^2*c^3*d^6 -
 a^8*b*c^2*d^7)*x^6 + (a^2*b^7*c^9 - a^3*b^6*c^8*d - 9*a^4*b^5*c^7*d^2 + 25
*a^5*b^4*c^6*d^3 - 25*a^6*b^3*c^5*d^4 + 9*a^7*b^2*c^4*d^5 + a^8*b*c^3*d^6 -
 a^9*c^2*d^7)*x^4 + 2*(a^3*b^6*c^9 - 4*a^4*b^5*c^8*d + 5*a^5*b^4*c^7*d^2 -
5*a^7*b^2*c^5*d^4 + 4*a^8*b*c^4*d^5 - a^9*c^3*d^6)*x^2), 1/16*(6*(b^6*c^4*d
^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b^2*d^6)*x^7 + 2*(6*b^6*c^5*d
- 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d^3 + 9*a^3*b^3*c^2*d^4 + 31*a^4*b^2*c*d
^5 - 6*a^5*b*d^6)*x^5 + 2*(3*b^6*c^6 - 8*a*b^5*c^5*d - 29*a^2*b^4*c^4*d^2 +
 29*a^4*b^2*c^2*d^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^3 + 6*(a^2*b^4*c^6 - 6*a
^3*b^3*c^5*d + 21*a^4*b^2*c^4*d^2 + (b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 21*a^2
*b^4*c^2*d^4)*x^8 + 2*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 15*a^2*b^4*c^3*d^3 + 2
1*a^3*b^3*c^2*d^4)*x^6 + (b^6*c^6 - 2*a*b^5*c^5*d - 2*a^2*b^4*c^4*d^2 + 78*
a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*d^4)*x^4 + 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d
+ 15*a^3*b^3*c^4*d^2 + 21*a^4*b^2*c^3*d^3)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a
```

```
)) - 3*(21*a^4*b^2*c^4*d^2 - 6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^
2*d^4 - 6*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3
*b^3*c^2*d^4 - 5*a^4*b^2*c*d^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*
a^3*b^3*c^3*d^3 - 2*a^4*b^2*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*
a^3*b^3*c^4*d^2 + 15*a^4*b^2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sq
rt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^5*c^6 -
 22*a^2*b^4*c^5*d + 17*a^3*b^3*c^4*d^2 - 17*a^4*b^2*c^3*d^3 + 22*a^5*b*c^2*
d^4 - 5*a^6*c*d^5)*x)/(a^4*b^5*c^9 - 5*a^5*b^4*c^8*d + 10*a^6*b^3*c^7*d^2 -
 10*a^7*b^2*c^6*d^3 + 5*a^8*b*c^5*d^4 - a^9*c^4*d^5 + (a^2*b^7*c^7*d^2 - 5*
a^3*b^6*c^6*d^3 + 10*a^4*b^5*c^5*d^4 - 10*a^5*b^4*c^4*d^5 + 5*a^6*b^3*c^3*d
^6 - a^7*b^2*c^2*d^7)*x^8 + 2*(a^2*b^7*c^8*d - 4*a^3*b^6*c^7*d^2 + 5*a^4*b^
5*c^6*d^3 - 5*a^6*b^3*c^4*d^5 + 4*a^7*b^2*c^3*d^6 - a^8*b*c^2*d^7)*x^6 + (a
^2*b^7*c^9 - a^3*b^6*c^8*d - 9*a^4*b^5*c^7*d^2 + 25*a^5*b^4*c^6*d^3 - 25*a^
6*b^3*c^5*d^4 + 9*a^7*b^2*c^4*d^5 + a^8*b*c^3*d^6 - a^9*c^2*d^7)*x^4 + 2*(a
^3*b^6*c^9 - 4*a^4*b^5*c^8*d + 5*a^5*b^4*c^7*d^2 - 5*a^7*b^2*c^5*d^4 + 4*a^
8*b*c^4*d^5 - a^9*c^3*d^6)*x^2), 1/8*(3*(b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 6*
a^3*b^3*c*d^5 - a^4*b^2*d^6)*x^7 + (6*b^6*c^5*d - 31*a*b^5*c^4*d^2 - 9*a^2*
b^4*c^3*d^3 + 9*a^3*b^3*c^2*d^4 + 31*a^4*b^2*c*d^5 - 6*a^5*b*d^6)*x^5 + (3*
b^6*c^6 - 8*a*b^5*c^5*d - 29*a^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*d^4 + 8*a^5*b
*c*d^5 - 3*a^6*d^6)*x^3 + 3*(a^2*b^4*c^6 - 6*a^3*b^3*c^5*d + 21*a^4*b^2*c^4
*d^2 + (b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 21*a^2*b^4*c^2*d^4)*x^8 + 2*(b^6*c^
5*d - 5*a*b^5*c^4*d^2 + 15*a^2*b^4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*x^6 + (b^6
*c^6 - 2*a*b^5*c^5*d - 2*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 + 21*a^4*b^2*
c^2*d^4)*x^4 + 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d + 15*a^3*b^3*c^4*d^2 + 21*a^4
*b^2*c^3*d^3)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*(21*a^4*b^2*c^4*d^2 -
6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^2*d^4 - 6*a^3*b^3*c*d^5 + a^4
*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3*b^3*c^2*d^4 - 5*a^4*b^2*c*d^
5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 - 2*a^4*b^2*c
^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*a^3*b^3*c^4*d^2 + 15*a^4*b^2*
c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) +
 (5*a*b^5*c^6 - 22*a^2*b^4*c^5*d + 17*a^3*b^3*c^4*d^2 - 17*a^4*b^2*c^3*d^3
+ 22*a^5*b*c^2*d^4 - 5*a^6*c*d^5)*x)/(a^4*b^5*c^9 - 5*a^5*b^4*c^8*d + 10*a^
6*b^3*c^7*d^2 - 10*a^7*b^2*c^6*d^3 + 5*a^8*b*c^5*d^4 - a^9*c^4*d^5 + (a^2*b
^7*c^7*d^2 - 5*a^3*b^6*c^6*d^3 + 10*a^4*b^5*c^5*d^4 - 10*a^5*b^4*c^4*d^5 +
5*a^6*b^3*c^3*d^6 - a^7*b^2*c^2*d^7)*x^8 + 2*(a^2*b^7*c^8*d - 4*a^3*b^6*c^7
*d^2 + 5*a^4*b^5*c^6*d^3 - 5*a^6*b^3*c^4*d^5 + 4*a^7*b^2*c^3*d^6 - a^8*b*c^
2*d^7)*x^6 + (a^2*b^7*c^9 - a^3*b^6*c^8*d - 9*a^4*b^5*c^7*d^2 + 25*a^5*b^4*
c^6*d^3 - 25*a^6*b^3*c^5*d^4 + 9*a^7*b^2*c^4*d^5 + a^8*b*c^3*d^6 - a^9*c^2*
d^7)*x^4 + 2*(a^3*b^6*c^9 - 4*a^4*b^5*c^8*d + 5*a^5*b^4*c^7*d^2 - 5*a^7*b^2
*c^5*d^4 + 4*a^8*b*c^4*d^5 - a^9*c^3*d^6)*x^2)]
```

giac [A] time = 0.62, size = 574, normalized size = 1.82

$$\frac{3\left(b^{5}c^{2}-6\,ab^{4}cd+21\,a^{2}b^{3}d^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(a^{2}b^{5}c^{5}-5\,a^{3}b^{4}c^{4}d+10\,a^{4}b^{3}c^{3}d^{2}-10\,a^{5}b^{2}c^{2}d^{3}+5\,a^{6}bcd^{4}-a^{7}d^{5}\right)\sqrt{ab}}-\frac{3\left(21\,b^{2}c^{2}d^{3}-6\,abcd^{4}+a^{2}d^{5}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{5}c^{7}-5\,ab^{4}c^{6}d+10\,a^{2}b^{3}c^{5}d^{2}-10\,a^{3}b^{2}c^{4}d^{3}+5\,a^{4}bc^{3}d^{4}-a^{5}c^{2}d^{5}\right)\sqrt{cd}}+\frac{3\,b^{5}c^{3}d^{2}x^{7}-15\,ab^{4}c^{2}d^{3}x^{7}-15\,a^{2}b^{3}cd^{4}x^{7}+3\,a^{3}b^{2}d^{5}x^{7}+6\,b^{5}c^{4}dx^{5}-25\,ab^{4}c^{3}d^{2}x^{5}-34\,a^{2}b^{3}c^{2}d^{3}x^{5}-25\,a^{3}b^{2}cd^{4}x^{5}+6\,a^{4}bd^{5}x^{5}+3\,b^{5}c^{5}x^{3}-5\,ab^{4}c^{4}dx^{3}-34\,a^{2}b^{3}c^{3}d^{2}x^{3}-34\,a^{3}b^{2}c^{2}d^{3}x^{3}-5\,a^{4}bcd^{4}x^{3}+3\,a^{5}d^{5}x^{3}+5\,ab^{4}c^{5}x-17\,a^{2}b^{3}c^{4}dx-17\,a^{4}bc^{2}d^{3}x+5\,a^{5}cd^{4}x}{8\left(a^{2}b^{4}c^{6}-4\,a^{3}b^{3}c^{5}d+6\,a^{4}b^{2}c^{4}d^{2}-4\,a^{5}bc^{3}d^{3}+a^{6}c^{2}d^{4}\right)\left(bdx^{4}+bcx^{2}+adx^{2}+ac\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 3/8*(b^5*c^2 - 6*a*b^4*c*d + 21*a^2*b^3*d^2)*arctan(b*x/sqrt(a*b))/((a^2*b^
5*c^5 - 5*a^3*b^4*c^4*d + 10*a^4*b^3*c^3*d^2 - 10*a^5*b^2*c^2*d^3 + 5*a^6*b
*c*d^4 - a^7*d^5)*sqrt(a*b)) - 3/8*(21*b^2*c^2*d^3 - 6*a*b*c*d^4 + a^2*d^5)
*arctan(d*x/sqrt(c*d))/((b^5*c^7 - 5*a*b^4*c^6*d + 10*a^2*b^3*c^5*d^2 - 10*
a^3*b^2*c^4*d^3 + 5*a^4*b*c^3*d^4 - a^5*c^2*d^5)*sqrt(c*d)) + 1/8*(3*b^5*c^
3*d^2*x^7 - 15*a*b^4*c^2*d^3*x^7 - 15*a^2*b^3*c*d^4*x^7 + 3*a^3*b^2*d^5*x^7
 + 6*b^5*c^4*d*x^5 - 25*a*b^4*c^3*d^2*x^5 - 34*a^2*b^3*c^2*d^3*x^5 - 25*a^3
*b^2*c*d^4*x^5 + 6*a^4*b*d^5*x^5 + 3*b^5*c^5*x^3 - 5*a*b^4*c^4*d*x^3 - 34*a
^2*b^3*c^3*d^2*x^3 - 34*a^3*b^2*c^2*d^3*x^3 - 5*a^4*b*c*d^4*x^3 + 3*a^5*d^5
*x^3 + 5*a*b^4*c^5*x - 17*a^2*b^3*c^4*d*x - 17*a^4*b*c^2*d^3*x + 5*a^5*c*d^
4*x)/((a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3
+ a^6*c^2*d^4)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)^2)
```

maple [A] time = 0.02, size = 568, normalized size = 1.80

$$\frac{3a^2d^6x^3}{8(ad-bc)^5(dx^2+c)^2c^2}-\frac{9abd^5x^3}{4(ad-bc)^5(dx^2+c)^2c}+\frac{9b^5cdx^3}{4(ad-bc)^5(bx^2+a)^2a}-\frac{3b^6c^2x^3}{8(ad-bc)^5(bx^2+a)^2a^2}-\frac{15b^4d^2x^3}{8(ad-bc)^5(bx^2+a)^2}+\frac{15b^2d^4x^3}{8(ad-bc)^5(dx^2+c)^2}+\frac{5a^2d^5x}{8(ad-bc)^5(dx^2+c)^2c}-\frac{17ab^3d^2x}{8(ad-bc)^5(bx^2+a)^2}-\frac{11abd^4x}{4(ad-bc)^5(dx^2+c)^2}-\frac{5b^5c^2x}{8(ad-bc)^5(bx^2+a)^2a}+\frac{11b^4cdx}{4(ad-bc)^5(bx^2+a)^2}+\frac{17b^2cd^3x}{8(ad-bc)^5(dx^2+c)^2}+\frac{3a^2d^5\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8(ad-bc)^5\sqrt{cd}\,c^2}-\frac{9abd^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4(ad-bc)^5\sqrt{cd}\,c}+\frac{9b^4cd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{4(ad-bc)^5\sqrt{ab}\,a}-\frac{3b^5c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8(ad-bc)^5\sqrt{ab}\,a^2}-\frac{63b^3d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8(ad-bc)^5\sqrt{ab}}+\frac{63b^2d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8(ad-bc)^5\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^6/(a*d-b*c)^5/(d*x^2+c)^2/c^2*x^3*a^2-9/4*d^5/(a*d-b*c)^5/(d*x^2+c)^2
/c*x^3*a*b+15/8*d^4/(a*d-b*c)^5/(d*x^2+c)^2*x^3*b^2+5/8*d^5/(a*d-b*c)^5/(d*
x^2+c)^2/c*x*a^2-11/4*d^4/(a*d-b*c)^5/(d*x^2+c)^2*x*a*b+17/8*d^3/(a*d-b*c)^
5/(d*x^2+c)^2*c*x*b^2+3/8*d^5/(a*d-b*c)^5/c^2/(c*d)^(1/2)*arctan(1/(c*d)^(1
/2)*d*x)*a^2-9/4*d^4/(a*d-b*c)^5/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*
b+63/8*d^3/(a*d-b*c)^5/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-15/8*b^4/(
a*d-b*c)^5/(b*x^2+a)^2*x^3*d^2+9/4*b^5/(a*d-b*c)^5/(b*x^2+a)^2/a*x^3*c*d-3/
8*b^6/(a*d-b*c)^5/(b*x^2+a)^2/a^2*x^3*c^2-17/8*b^3/(a*d-b*c)^5/(b*x^2+a)^2*
x*a*d^2+11/4*b^4/(a*d-b*c)^5/(b*x^2+a)^2*x*c*d-5/8*b^5/(a*d-b*c)^5/(b*x^2+a
)^2*x/a*c^2-63/8*b^3/(a*d-b*c)^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2+
9/4*b^4/(a*d-b*c)^5/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d-3/8*b^5/(a*
d-b*c)^5/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2
```

maxima [B] time = 3.39, size = 820, normalized size = 2.60

$$\frac{3\left(b^5c^2-6ab^4cd+21a^2b^3d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(a^2b^5c^5-5a^3b^4c^4d+10a^4b^3c^3d^2-10a^5b^2c^2d^3+5a^6bcd^4-a^7d^5\right)\sqrt{ab}}-\frac{3\left(21b^2c^2d^3-6abcd^4+a^2d^5\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^5c^7-5ab^4c^6d+10a^2b^3c^5d^2-10a^3b^2c^4d^3+5a^4bc^3d^4-a^5c^2d^5\right)\sqrt{cd}}+\frac{3\left(b^5c^3d^2-5ab^4c^2d^3-5a^2b^3cd^4+a^3b^2d^5\right)x^7+\left(6b^5c^4d-25ab^4c^3d^2-34a^2b^3c^2d^3-25a^3b^2cd^4+6a^4bd^5\right)x^5+\left(3b^5c^5-5ab^4c^4d-34a^2b^3c^3d^2-34a^3b^2c^2d^3-5a^4bcd^4+3a^5d^5\right)x^3+\left(5ab^4c^5-17a^2b^3c^4d-17a^4bc^2d^3+5a^5cd^4\right)x}{8\left(a^4b^4c^8-4a^5b^3c^7d+6a^6b^2c^6d^2-4a^7bc^5d^3+a^8c^4d^4+\left(a^2b^6c^6d^2-4a^3b^5c^5d^3+6a^4b^4c^4d^4-4a^5b^3c^3d^5+a^6b^2c^2d^6\right)x^8+2\left(a^2b^6c^7d-3a^3b^5c^6d^2+2a^4b^4c^5d^3+2a^5b^3c^4d^4-3a^6b^2c^3d^5+a^7bc^2d^6\right)x^6+\left(a^2b^6c^8-9a^4b^4c^6d^2+16a^5b^3c^5d^3-9a^6b^2c^4d^4+a^8c^2d^6\right)x^4+2\left(a^3b^5c^8-3a^4b^4c^7d+2a^5b^3c^6d^2+2a^6b^2c^5d^3-3a^7bc^4d^4+a^8c^3d^5\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 3/8*(b^5*c^2 - 6*a*b^4*c*d + 21*a^2*b^3*d^2)*arctan(b*x/sqrt(a*b))/((a^2*b^
5*c^5 - 5*a^3*b^4*c^4*d + 10*a^4*b^3*c^3*d^2 - 10*a^5*b^2*c^2*d^3 + 5*a^6*b
*c*d^4 - a^7*d^5)*sqrt(a*b)) - 3/8*(21*b^2*c^2*d^3 - 6*a*b*c*d^4 + a^2*d^5)
*arctan(d*x/sqrt(c*d))/((b^5*c^7 - 5*a*b^4*c^6*d + 10*a^2*b^3*c^5*d^2 - 10*
a^3*b^2*c^4*d^3 + 5*a^4*b*c^3*d^4 - a^5*c^2*d^5)*sqrt(c*d)) + 1/8*(3*(b^5*c
^3*d^2 - 5*a*b^4*c^2*d^3 - 5*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^7 + (6*b^5*c^4*
d - 25*a*b^4*c^3*d^2 - 34*a^2*b^3*c^2*d^3 - 25*a^3*b^2*c*d^4 + 6*a^4*b*d^5)
*x^5 + (3*b^5*c^5 - 5*a*b^4*c^4*d - 34*a^2*b^3*c^3*d^2 - 34*a^3*b^2*c^2*d^3
 - 5*a^4*b*c*d^4 + 3*a^5*d^5)*x^3 + (5*a*b^4*c^5 - 17*a^2*b^3*c^4*d - 17*a^
4*b*c^2*d^3 + 5*a^5*c*d^4)*x)/(a^4*b^4*c^8 - 4*a^5*b^3*c^7*d + 6*a^6*b^2*c^
6*d^2 - 4*a^7*b*c^5*d^3 + a^8*c^4*d^4 + (a^2*b^6*c^6*d^2 - 4*a^3*b^5*c^5*d^
3 + 6*a^4*b^4*c^4*d^4 - 4*a^5*b^3*c^3*d^5 + a^6*b^2*c^2*d^6)*x^8 + 2*(a^2*b
^6*c^7*d - 3*a^3*b^5*c^6*d^2 + 2*a^4*b^4*c^5*d^3 + 2*a^5*b^3*c^4*d^4 - 3*a^
6*b^2*c^3*d^5 + a^7*b*c^2*d^6)*x^6 + (a^2*b^6*c^8 - 9*a^4*b^4*c^6*d^2 + 16*
a^5*b^3*c^5*d^3 - 9*a^6*b^2*c^4*d^4 + a^8*c^2*d^6)*x^4 + 2*(a^3*b^5*c^8 - 3
*a^4*b^4*c^7*d + 2*a^5*b^3*c^6*d^2 + 2*a^6*b^2*c^5*d^3 - 3*a^7*b*c^4*d^4 +
a^8*c^3*d^5)*x^2)
```

mupad [B] time = 8.55, size = 11150, normalized size = 35.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^3*(c + d*x^2)^3),x)
```

```
[Out] ((x*(5*a^4*d^4 + 5*b^4*c^4 - 17*a*b^3*c^3*d - 17*a^3*b*c*d^3))/(8*a*c*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (x^3*
(34*a^2*b^3*c^3*d^2 - 3*b^5*c^5 - 3*a^5*d^5 + 34*a^3*b^2*c^2*d^3 + 5*a*b^4*
c^4*d + 5*a^4*b*c*d^4))/(8*a^2*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 -
 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (x^5*(25*a*b^4*c^3*d^2 - 6*b^5*c^4*d - 6
*a^4*b*d^5 + 25*a^3*b^2*c*d^4 + 34*a^2*b^3*c^2*d^3))/(8*a^2*c^2*(a^4*d^4 +
b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (3*b*d*x^7*
(a^3*b*d^4 + b^4*c^3*d - 5*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3))/(8*a^2*c^2*(a^
```

```
4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))/(x^4
*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + x^2*(2*a*b*c^2 + 2*a^2*c*d) + x^6*(2*a*b
*d^2 + 2*b^2*c*d) + a^2*c^2 + b^2*d^2*x^8) - (atan(((((x*(9*a^8*b^3*d^11 +
9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 + 702*a^2*b^9*c^6*
d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^5*b^6*c^3*d^8 +
702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d -
 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c
^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (3*(((3*a^2*b^16*c^16
*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4)/2 - 765*a^5*b^
13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*c^11*d^7)/2 + (1
6425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a^10*b^8*c^8*d^10
)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12)/2 - 765*a^13*b
^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*d^15)/2 + (3*a^1
6*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12
*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^
8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^
9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)
 - (3*x*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)*(256*a^4*b^13*c
^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 - 19200*a^7*b^10*
c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7 - 10752*a^10*b^7
*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10 + 8960*a^13*b^4
*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13))/(512*(a^10*d^5
 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3
- 5*a^9*b*c*d^4)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b
*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 -
56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-a^5*b^5)^(1/2)*(21*a^2*d^2 +
b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^
7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)))*(-a^5*b^5)^(1/2)*(21*
a^2*d^2 + b^2*c^2 - 6*a*b*c*d)*3i)/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*
c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)) + (((x*(9
*a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 +
702*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^
5*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*
a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^
3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + (3*((
(3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4
)/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*
c^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a
^10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12
)/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*
d^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*
b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*
d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 -
 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^
14*b^2*c^6*d^10) + (3*x*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)
*(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 -
 19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7
- 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10
 + 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13)
)/(512*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*
a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c
^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^
8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-a^5*b^5)^(1/2
)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b
^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)))*(-a^5
*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)*3i)/(16*(a^10*d^5 - a^5*b^5*
c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c
*d^4)))/(((567*a^7*b^5*d^12)/256 + (567*b^12*c^7*d^5)/256 - (6399*a*b^11*c^
6*d^6)/256 - (6399*a^6*b^6*c*d^11)/256 + (27891*a^2*b^10*c^5*d^7)/256 - (49
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707*a^3*b^9*c^4*d^8)/256 - (49707*a^4*b^8*c^3*d^9)/256 + (27891*a^5*b^7*c^2
*d^10)/256)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b
*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*
d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 +
 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10) + (3*(
(x*(9*a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^
10 + 702*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 22
68*a^5*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8
 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c
^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) -
(3*(((3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^1
4*d^4)/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*
b^11*c^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16
425*a^10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6
*d^12)/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3
*c^3*d^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12
*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*
c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*
d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 +
66*a^14*b^2*c^6*d^10) - (3*x*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b
*c*d)*(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*
d^4 - 19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10
*d^7 - 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7
*d^10 + 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*
d^13))/(512*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2
+ 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*
b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 +
70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-a^5*b^5)
^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*
a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)))*
(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a^5*b^
5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b
*c*d^4)) - (3*((x*(9*a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 1
08*a^7*b^4*c*d^10 + 702*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b
^7*c^4*d^7 - 2268*a^5*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12
 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2
 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b
^2*c^6*d^6)) + (3*(((3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (3
33*a^4*b^14*c^14*d^4)/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/
2 - (10371*a^7*b^11*c^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^
9*c^9*d^9 + (16425*a^10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (47
43*a^12*b^6*c^6*d^12)/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2
 - (45*a^15*b^3*c^3*d^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^1
6*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2
 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924
*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13
*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10) + (3*x*(-a^5*b^5)^(1/2)*(21*a^2*d^2 +
b^2*c^2 - 6*a*b*c*d)*(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960
*a^6*b^11*c^13*d^4 - 19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 107
52*a^9*b^8*c^10*d^7 - 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 192
00*a^12*b^5*c^7*d^10 + 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 25
6*a^15*b^2*c^4*d^13))/(512*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a
^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)*(a^4*b^8*c^12 + a^12*c
^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7
*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^
6)))*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a
^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*
a^9*b*c*d^4)))*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^
10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^
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2*d^3 - 5*a^9*b*c*d^4))))*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*
d)*3i)/(8*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 +
10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)) - (atan(((((x*(9*a^8*b^3*d^11 + 9*b^11
*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 + 702*a^2*b^9*c^6*d^5 -
2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^5*b^6*c^3*d^8 + 702*a^
6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^1
1*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4
 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (3*(((3*a^2*b^16*c^16*d^2)/
2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4)/2 - 765*a^5*b^13*c^1
3*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*c^11*d^7)/2 + (16425*a
^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a^10*b^8*c^8*d^10)/2 -
(10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12)/2 - 765*a^13*b^5*c^5
*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*d^15)/2 + (3*a^16*b^2*
c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*
b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12
*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7
+ 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10) - (3*
x*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d)*(256*a^4*b^13*c^15*d^
2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 - 19200*a^7*b^10*c^12*d
^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7 - 10752*a^10*b^7*c^9*d
^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10 + 8960*a^13*b^4*c^6*d
^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13))/(512*(b^5*c^10 - a^5
*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*
b^4*c^9*d)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d
^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9
*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^
2 - 6*a*b*c*d))/(16*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*
c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)))*(-c^5*d^5)^(1/2)*(a^2*d^2 +
 21*b^2*c^2 - 6*a*b*c*d)*3i)/(16*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4
+ 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)) + (((x*(9*a^8*b
^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 + 702*a^
2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^5*b^6*
c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^
7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70
*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + (3*(((3*a^2
*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4)/2 -
765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*c^11*d
^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a^10*b^
8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12)/2 -
765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*d^15)/
2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c
^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 +
495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a
^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2
*c^6*d^10) + (3*x*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d)*(256*
a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 - 19200
*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7 - 1075
2*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10 + 896
0*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13))/(512
*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^
2*c^7*d^3 - 5*a*b^4*c^9*d)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d
- 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*
c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-c^5*d^5)^(1/2)*(a^2
*d^2 + 21*b^2*c^2 - 6*a*b*c*d))/(16*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d
^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)))*(-c^5*d^5)^
(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d)*3i)/(16*(b^5*c^10 - a^5*c^5*d^5 +
5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d))
)/((((567*a^7*b^5*d^12)/256 + (567*b^12*c^7*d^5)/256 - (6399*a*b^11*c^6*d^6)
/256 - (6399*a^6*b^6*c*d^11)/256 + (27891*a^2*b^10*c^5*d^7)/256 - (49707*a^
```

3*b^9*c^4*d^8)/256 - (49707*a^4*b^8*c^3*d^9)/256 + (27891*a^5*b^7*c^2*d^10)
/256)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d
^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 -
792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a
^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10) + (3*((x*(9*
a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 + 7
02*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^5
*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a
^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3
 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (3*(((
3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4)
/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*c
^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a^
10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12)
/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*d
^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b
^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d
^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 -
792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^1
4*b^2*c^6*d^10) - (3*x*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d)*
(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 -
19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7 -
 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10
+ 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13))
/(512*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a
^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^
11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8
*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-c^5*d^5)^(1/2)
*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d))/(16*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*
c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)))*(-c^5*
d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d))/(16*(b^5*c^10 - a^5*c^5*d^5
+ 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d
)) - (3*((x*(9*a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7
*b^4*c*d^10 + 702*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4
*d^7 - 2268*a^5*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^1
2*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*
a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6
d^6)) + (3(((3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4
*b^14*c^14*d^4)/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (1
0371*a^7*b^11*c^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*
d^9 + (16425*a^10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^1
2*b^6*c^6*d^12)/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45
*a^15*b^3*c^3*d^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*
d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220
*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*
b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c
^7*d^9 + 66*a^14*b^2*c^6*d^10) + (3*x*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^
2 - 6*a*b*c*d)*(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b
^11*c^13*d^4 - 19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9
*b^8*c^10*d^7 - 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^1
2*b^5*c^7*d^10 + 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15
*b^2*c^4*d^13))/(512*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3
*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)*(a^4*b^8*c^12 + a^12*c^4*d^8
 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c
^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(
-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d))/(16*(b^5*c^10 - a^5*c^5
*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*
c^9*d)))*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d))/(16*(b^5*c^10
 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3

```
- 5*a*b^4*c^9*d))))*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d)*3i)
/(8*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3
*b^2*c^7*d^3 - 5*a*b^4*c^9*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.43 $\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^4} \, dx$

Optimal. Leaf size=34

$$-\frac{x\left(1-x^2\right)^2}{3\left(x^2+1\right)^3}-\frac{2x}{3\left(x^2+1\right)}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {413, 21, 383}

$$-\frac{x\left(1-x^2\right)^2}{3\left(x^2+1\right)^3}-\frac{2x}{3\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^3/(1 + x^2)^4,x]

[Out] -(x*(1 - x^2)^2)/(3*(1 + x^2)^3) - (2*x)/(3*(1 + x^2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^4} \, dx &= -\frac{x\left(1-x^2\right)^2}{3\left(1+x^2\right)^3}+\frac{1}{6}\int \frac{\left(-1+x^2\right)\left(4+4x^2\right)}{\left(1+x^2\right)^3} \, dx\\
&= -\frac{x\left(1-x^2\right)^2}{3\left(1+x^2\right)^3}+\frac{2}{3}\int \frac{-1+x^2}{\left(1+x^2\right)^2} \, dx\\
&= -\frac{x\left(1-x^2\right)^2}{3\left(1+x^2\right)^3}-\frac{2x}{3\left(1+x^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$-\frac{x\left(3x^4+2x^2+3\right)}{3\left(x^2+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^3/(1 + x^2)^4,x]
[Out] -1/3*(x*(3 + 2*x^2 + 3*x^4))/(1 + x^2)^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x^2)^3/(1 + x^2)^4,x]
[Out] IntegrateAlgebraic[(-1 + x^2)^3/(1 + x^2)^4, x]
```

fricas [A] time = 1.04, size = 33, normalized size = 0.97

$$-\frac{3\,x^5+2\,x^3+3\,x}{3\left(x^6+3\,x^4+3\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="fricas")
[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

giac [A] time = 0.58, size = 20, normalized size = 0.59

$$-\frac{3\left(x+\frac{1}{x}\right)^2-4}{3\left(x+\frac{1}{x}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="giac")
[Out] -1/3*(3*(x + 1/x)^2 - 4)/(x + 1/x)^3
```

maple [A] time = 0.01, size = 23, normalized size = 0.68

$$\frac{-x^5-\frac{2}{3}x^3-x}{\left(x^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^3/(x^2+1)^4,x)
[Out] (-x^5-2/3*x^3-x)/(x^2+1)^3
```

maxima [A] time = 1.32, size = 33, normalized size = 0.97

$$-\frac{3\,x^5+2\,x^3+3\,x}{3\left(x^6+3\,x^4+3\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="maxima")
[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

mupad [B] time = 5.00, size = 31, normalized size = 0.91

$$\frac{4x}{3\left(x^2+1\right)^2}-\frac{x}{x^2+1}-\frac{4x}{3\left(x^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^3/(x^2 + 1)^4,x)
[Out] (4*x)/(3*(x^2 + 1)^2) - x/(x^2 + 1) - (4*x)/(3*(x^2 + 1)^3)
```

sympy [A] time = 0.13, size = 31, normalized size = 0.91

$$\frac{-3x^5-2x^3-3x}{3x^6+9x^4+9x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**3/(x**2+1)**4,x)
[Out] (-3*x**5 - 2*x**3 - 3*x)/(3*x**6 + 9*x**4 + 9*x**2 + 3)
```

3.44 $\int \frac{\left(-1+x^2\right)^4}{\left(1+x^2\right)^5} \, dx$

Optimal. Leaf size=47

$$\frac{x\left(1-x^2\right)^3}{4\left(x^2+1\right)^4}+\frac{3x\left(1-x^2\right)}{8\left(x^2+1\right)^2}+\frac{3}{8}\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {413, 21, 203}

$$\frac{x\left(1-x^2\right)^3}{4\left(x^2+1\right)^4}+\frac{3x\left(1-x^2\right)}{8\left(x^2+1\right)^2}+\frac{3}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^4/(1 + x^2)^5,x]
```

```
[Out] (x*(1 - x^2)^3)/(4*(1 + x^2)^4) + (3*x*(1 - x^2))/(8*(1 + x^2)^2) + (3*ArcT
an[x])/8
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-1+x^2\right)^4}{\left(1+x^2\right)^5}\,dx &= \frac{x\left(1-x^2\right)^3}{4\left(1+x^2\right)^4}+\frac{1}{8}\int \frac{\left(-1+x^2\right)^2\left(6+6x^2\right)}{\left(1+x^2\right)^4}\,dx\\ &= \frac{x\left(1-x^2\right)^3}{4\left(1+x^2\right)^4}+\frac{3}{4}\int \frac{\left(-1+x^2\right)^2}{\left(1+x^2\right)^3}\,dx\\ &= \frac{x\left(1-x^2\right)^3}{4\left(1+x^2\right)^4}+\frac{3x\left(1-x^2\right)}{8\left(1+x^2\right)^2}+\frac{3}{16}\int \frac{2+2x^2}{\left(1+x^2\right)^2}\,dx\\ &= \frac{x\left(1-x^2\right)^3}{4\left(1+x^2\right)^4}+\frac{3x\left(1-x^2\right)}{8\left(1+x^2\right)^2}+\frac{3}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x\left(1-x^2\right)^3}{4\left(1+x^2\right)^4}+\frac{3x\left(1-x^2\right)}{8\left(1+x^2\right)^2}+\frac{3}{8}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.87

$$\frac{-5x^7+3x^5-3x^3+3\left(x^2+1\right)^4\tan^{-1}(x)+5x}{8\left(x^2+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^4/(1 + x^2)^5,x]
[Out] (5*x - 3*x^3 + 3*x^5 - 5*x^7 + 3*(1 + x^2)^4*ArcTan[x])/(8*(1 + x^2)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^2\right)^4}{\left(1+x^2\right)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x^2)^4/(1 + x^2)^5,x]
[Out] IntegrateAlgebraic[(-1 + x^2)^4/(1 + x^2)^5, x]
```

fricas [A] time = 1.02, size = 67, normalized size = 1.43

$$-\frac{5\,x^7-3\,x^5+3\,x^3-3\left(x^8+4\,x^6+6\,x^4+4\,x^2+1\right)\arctan(x)-5\,x}{8\left(x^8+4\,x^6+6\,x^4+4\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="fricas")
[Out] -1/8*(5*x^7 - 3*x^5 + 3*x^3 - 3*(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)*arctan(x)
 - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)
```

giac [A] time = 0.58, size = 54, normalized size = 1.15

$$\frac{3}{32}\,\pi\mathrm{sgn}(x)-\frac{5\left(x-\frac{1}{x}\right)^3+12\,x-\frac{12}{x}}{8\left(\left(x-\frac{1}{x}\right)^2+4\right)^2}+\frac{3}{16}\,\arctan\left(\frac{x^2-1}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="giac")
```

```
[Out] 3/32*pi*sgn(x) - 1/8*(5*(x - 1/x)^3 + 12*x - 12/x)/((x - 1/x)^2 + 4)^2 + 3/
16*arctan(1/2*(x^2 - 1)/x)
```

maple [A] time = 0.01, size = 33, normalized size = 0.70

$$\frac{3\arctan(x)}{8}+\frac{-\frac{5}{8}x^7+\frac{3}{8}x^5-\frac{3}{8}x^3+\frac{5}{8}x}{\left(x^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^4/(x^2+1)^5,x)
```

```
[Out] (-5/8*x^7+3/8*x^5-3/8*x^3+5/8*x)/(x^2+1)^4+3/8*arctan(x)
```

maxima [A] time = 2.99, size = 48, normalized size = 1.02

$$-\frac{5\,x^7-3\,x^5+3\,x^3-5\,x}{8\left(x^8+4\,x^6+6\,x^4+4\,x^2+1\right)}+\frac{3}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(5*x^7 - 3*x^5 + 3*x^3 - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1) + 3/8*
arctan(x)
```

mupad [B] time = 0.04, size = 47, normalized size = 1.00

$$\frac{3\operatorname{atan}(x)}{8}+\frac{-\frac{5\,x^7}{8}+\frac{3\,x^5}{8}-\frac{3\,x^3}{8}+\frac{5\,x}{8}}{x^8+4\,x^6+6\,x^4+4\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^4/(x^2 + 1)^5,x)
```

```
[Out] (3*atan(x))/8 + ((5*x)/8 - (3*x^3)/8 + (3*x^5)/8 - (5*x^7)/8)/(4*x^2 + 6*x^
4 + 4*x^6 + x^8 + 1)
```

sympy [A] time = 0.17, size = 46, normalized size = 0.98

$$\frac{-5x^7+3x^5-3x^3+5x}{8x^8+32x^6+48x^4+32x^2+8}+\frac{3\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**4/(x**2+1)**5,x)
```

```
[Out] (-5*x**7 + 3*x**5 - 3*x**3 + 5*x)/(8*x**8 + 32*x**6 + 48*x**4 + 32*x**2 + 8
) + 3*atan(x)/8
```

3.45 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^3 dx$

Optimal. Leaf size=231

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{192b^3}+\frac{x\sqrt{a+bx^2}\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{128b^3}+\frac{a\left(-5a^3d^3+\right.}{}$$

Rubi [A] time = 0.18, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {416, 528, 388, 195, 217, 206}

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{192b^3}+\frac{x\sqrt{a+bx^2}\left(24a^2bcd^2-5a^3d^3-48ab^2c^2d+64b^3c^3\right)}{128b^3}+\frac{a\left(24a^2bcd^2-5a^3d^3-48ab^2c^2d+64b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(12bc-5ad)}{48b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

```
[Out] ((64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a + b*x^
2])/(128*b^3) + (d*(72*b^2*c^2 - 52*a*b*c*d + 15*a^2*d^2)*x*(a + b*x^2)^(3/
2))/(192*b^3) + (d*(12*b*c - 5*a*d)*x*(a + b*x^2)^(3/2)*(c + d*x^2))/(48*b^
2) + (d*x*(a + b*x^2)^(3/2)*(c + d*x^2)^2)/(8*b) + (a*(64*b^3*c^3 - 48*a*b^
2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(128*b^(7/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx = \frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b} + \frac{\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(c(8bc-ad)+d(12bc-5ad)\right.}{8b}$$

$$= \frac{d(12bc-5ad)x\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{48b^2} + \frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b} + \frac{\int \sqrt{a+bx^2}}{}$$

$$= \frac{d\left(72b^2c^2-52abcd+15a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{192b^3} + \frac{d(12bc-5ad)x\left(a+bx^2\right)^{3/2}\left(c+\right.}{48b^2}$$

$$= \frac{\left(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{128b^3} + \frac{d\left(72b^2c^2-52abcd+15\right.}{192b^3}$$

$$= \frac{\left(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{128b^3} + \frac{d\left(72b^2c^2-52abcd+15\right.}{192b^3}$$

$$= \frac{\left(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{128b^3} + \frac{d\left(72b^2c^2-52abcd+15\right.}{192b^3}$$

Mathematica [A] time = 5.11, size = 181, normalized size = 0.78

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(15a^3d^3-2a^2bd^2\left(36c+5dx^2\right)+8ab^2d\left(18c^2+6cdx^2+d^2x^4\right)+48b^3\left(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6\right)\right)-3a\left(5a^3d^3-24a^2bcd^2+48ab^2c^2d-64b^3c^3\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{384b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^3*d^3 - 2*a^2*b*d^2*(36*c + 5*d*x^2) + 8*a
*b^2*d*(18*c^2 + 6*c*d*x^2 + d^2*x^4) + 48*b^3*(4*c^3 + 6*c^2*d*x^2 + 4*c*d
^2*x^4 + d^3*x^6)) - 3*a*(-64*b^3*c^3 + 48*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 5
*a^3*d^3)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(384*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.30, size = 202, normalized size = 0.87

$$\frac{\sqrt{a+bx^2}\left(15a^3d^3x-72a^2bcd^2x-10a^2bd^3x^3+144ab^2c^2dx+48ab^2cd^2x^3+8ab^2d^3x^5+192b^3c^3x+288b^3c^2dx^3+192b^3cd^2x^5+48b^3d^3x^7\right)}{384b^3}+\frac{\left(5a^4d^3-24a^3bcd^2+48a^2b^2c^2d-64ab^3c^3\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{128b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(192*b^3*c^3*x + 144*a*b^2*c^2*d*x - 72*a^2*b*c*d^2*x + 15
*a^3*d^3*x + 288*b^3*c^2*d*x^3 + 48*a*b^2*c*d^2*x^3 - 10*a^2*b*d^3*x^3 + 19
2*b^3*c*d^2*x^5 + 8*a*b^2*d^3*x^5 + 48*b^3*d^3*x^7))/(384*b^3) + ((-64*a*b^
3*c^3 + 48*a^2*b^2*c^2*d - 24*a^3*b*c*d^2 + 5*a^4*d^3)*Log[-(Sqrt[b]*x) + S
qrt[a + b*x^2]])/(128*b^(7/2))
```

fricas [A] time = 0.85, size = 398, normalized size = 1.72

$$\left[-\frac{3\left(64ab^3c^3-48a^2b^2c^2d+24a^3bcd^2-5a^4d^3\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48b^4d^3x^7+8\left(24b^4cd^2+ab^3d^3\right)x^5+2\left(144b^4c^2d+24ab^3cd^2-5a^2b^2d^3\right)x^3+3\left(64b^4c^3+48ab^3c^2d-24a^2b^2cd^2+5a^3bd^3\right)x\right)\sqrt{bx^2+a}}{768b^4}, -\frac{3\left(64ab^3c^3-48a^2b^2c^2d+24a^3bcd^2-5a^4d^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(48b^4d^3x^7+8\left(24b^4cd^2+ab^3d^3\right)x^5+2\left(144b^4c^2d+24ab^3cd^2-5a^2b^2d^3\right)x^3+3\left(64b^4c^3+48ab^3c^2d-24a^2b^2cd^2+5a^3bd^3\right)x\right)\sqrt{bx^2+a}}{384b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*s
qrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*b^4*d^3*x^7
+ 8*(24*b^4*c*d^2 + a*b^3*d^3)*x^5 + 2*(144*b^4*c^2*d + 24*a*b^3*c*d^2 - 5*
a^2*b^2*d^3)*x^3 + 3*(64*b^4*c^3 + 48*a*b^3*c^2*d - 24*a^2*b^2*c*d^2 + 5*a^
3*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/384*(3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*
d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (48*b^4*d^3*x^7 + 8*(24*b^4*c*d^2 + a*b^3*d^3)*x^5 + 2*(144*b^4*c^2*d +
24*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^3 + 3*(64*b^4*c^3 + 48*a*b^3*c^2*d - 24*a
^2*b^2*c*d^2 + 5*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

giac [A] time = 0.64, size = 201, normalized size = 0.87

$$\frac{1}{384}\left(2\left(4\left(6\,d^3x^2+\frac{24\,b^6cd^2+ab^5d^3}{b^6}\right)x^2+\frac{144\,b^6c^2d+24\,ab^5cd^2-5\,a^2b^4d^3}{b^6}\right)x^2+\frac{3\left(64\,b^6c^3+48\,ab^5c^2d-24\,a^2b^4cd^2+5\,a^3b^3d^3\right)}{b^6}\right)\sqrt{bx^2+a}\,x-\frac{\left(64\,ab^3c^3-48\,a^2b^2c^2d+24\,a^3bcd^2-5\,a^4d^3\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*d^3*x^2 + (24*b^6*c*d^2 + a*b^5*d^3)/b^6)*x^2 + (144*b^6*c^2
*d + 24*a*b^5*c*d^2 - 5*a^2*b^4*d^3)/b^6)*x^2 + 3*(64*b^6*c^3 + 48*a*b^5*c^
2*d - 24*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)/b^6)*sqrt(b*x^2 + a)*x - 1/128*(64*
a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.02, size = 310, normalized size = 1.34

$$\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}d^3x^5}{8b}-\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}a\,d^3x^3}{48b^2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}c\,d^2x^3}{2b}-\frac{5a^4d^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{128b^{\frac{7}{2}}}+\frac{3a^3c\,d^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{5}{2}}}-\frac{3a^2c^2d\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}+\frac{a\,c^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}-\frac{5\sqrt{b\,x^2+a}\,a^3d^3x}{128b^3}+\frac{3\sqrt{b\,x^2+a}\,a^2c\,d^2x}{16b^2}-\frac{3\sqrt{b\,x^2+a}\,a\,c^2dx}{8b}+\frac{\sqrt{b\,x^2+a}\,c^3x}{2}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}a^2d^3x}{64b^3}-\frac{3\left(b\,x^2+a\right)^{\frac{3}{2}}ac\,d^2x}{8b^2}+\frac{3\left(b\,x^2+a\right)^{\frac{3}{2}}c^2dx}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^3,x)
```

```
[Out] 1/8*d^3*x^5*(b*x^2+a)^(3/2)/b-5/48*d^3*a/b^2*x^3*(b*x^2+a)^(3/2)+5/64*d^3*a
^2/b^3*x*(b*x^2+a)^(3/2)-5/128*d^3*a^3/b^3*x*(b*x^2+a)^(1/2)-5/128*d^3*a^4/
b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*c*d^2*x^3*(b*x^2+a)^(3/2)/b-3/8*c
*d^2*a/b^2*x*(b*x^2+a)^(3/2)+3/16*c*d^2*a^2/b^2*x*(b*x^2+a)^(1/2)+3/16*c*d^
2*a^3/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+3/4*c^2*d*x*(b*x^2+a)^(3/2)/b-3
/8*c^2*d*a/b*x*(b*x^2+a)^(1/2)-3/8*c^2*d*a^2/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)
^(1/2))+1/2*c^3*x*(b*x^2+a)^(1/2)+1/2*c^3*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^
(1/2))
```

maxima [A] time = 1.39, size = 281, normalized size = 1.22

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}d^3x^5}{8\,b}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}cd^2x^3}{2\,b}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}ad^3x^3}{48\,b^2}+\frac{1}{2}\sqrt{bx^2+a}\,c^3x+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}c^2dx}{4\,b}-\frac{3\sqrt{bx^2+a}\,ac^2dx}{8\,b}-\frac{3\left(bx^2+a\right)^{\frac{3}{2}}acd^2x}{8\,b^2}+\frac{3\sqrt{bx^2+a}\,a^2cd^2x}{16\,b^2}+\frac{5\left(bx^2+a\right)^{\frac{3}{2}}a^2d^3x}{64\,b^3}-\frac{5\sqrt{bx^2+a}\,a^3d^3x}{128\,b^3}+\frac{ac^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}-\frac{3\,a^2c^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{3\,a^3cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}-\frac{5\,a^4d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(3/2)*d^3*x^5/b + 1/2*(b*x^2 + a)^(3/2)*c*d^2*x^3/b - 5/48*
(b*x^2 + a)^(3/2)*a*d^3*x^3/b^2 + 1/2*sqrt(b*x^2 + a)*c^3*x + 3/4*(b*x^2 +
a)^(3/2)*c^2*d*x/b - 3/8*sqrt(b*x^2 + a)*a*c^2*d*x/b - 3/8*(b*x^2 + a)^(3/2
)*a*c*d^2*x/b^2 + 3/16*sqrt(b*x^2 + a)*a^2*c*d^2*x/b^2 + 5/64*(b*x^2 + a)^(
3/2)*a^2*d^3*x/b^3 - 5/128*sqrt(b*x^2 + a)*a^3*d^3*x/b^3 + 1/2*a*c^3*arcsin
h(b*x/sqrt(a*b))/sqrt(b) - 3/8*a^2*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3
/16*a^3*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/128*a^4*d^3*arcsinh(b*x/sq
rt(a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2)^3, x)
```

sympy [B] time = 20.38, size = 484, normalized size = 2.10

$$\frac{5a^{\frac{7}{2}}d^3x}{128b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{\frac{5}{2}}cd^2x}{16b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{5a^{\frac{5}{2}}d^3x^3}{384b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{\frac{3}{2}}c^2dx}{8b\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{3}{2}}cd^2x^3}{16b\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{3}{2}}d^3x^5}{192b\sqrt{1+\frac{bx^2}{a}}} + \frac{\sqrt{a}c^3x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{9\sqrt{a}c^2dx^3}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{5\sqrt{a}cd^2x^5}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{7\sqrt{a}d^3x^7}{48\sqrt{1+\frac{bx^2}{a}}} - \frac{5a^4d^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}} + \frac{3a^3cd^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}} - \frac{3a^2c^2d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} + \frac{ac^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{3bc^2dx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{bcd^2x^7}{2\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{bd^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3,x)
```

```
[Out] 5*a**(7/2)*d**3*x/(128*b**3*sqrt(1 + b*x**2/a)) - 3*a**(5/2)*c*d**2*x/(16*b
**2*sqrt(1 + b*x**2/a)) + 5*a**(5/2)*d**3*x**3/(384*b**2*sqrt(1 + b*x**2/a)
) + 3*a**(3/2)*c**2*d*x/(8*b*sqrt(1 + b*x**2/a)) - a**(3/2)*c*d**2*x**3/(16
*b*sqrt(1 + b*x**2/a)) - a**(3/2)*d**3*x**5/(192*b*sqrt(1 + b*x**2/a)) + sq
rt(a)*c**3*x*sqrt(1 + b*x**2/a)/2 + 9*sqrt(a)*c**2*d*x**3/(8*sqrt(1 + b*x**
2/a)) + 5*sqrt(a)*c*d**2*x**5/(8*sqrt(1 + b*x**2/a)) + 7*sqrt(a)*d**3*x**7/
(48*sqrt(1 + b*x**2/a)) - 5*a**4*d**3*asinh(sqrt(b)*x/sqrt(a))/(128*b**(7/2
)) + 3*a**3*c*d**2*asinh(sqrt(b)*x/sqrt(a))/(16*b**(5/2)) - 3*a**2*c**2*d*a
sinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + a*c**3*asinh(sqrt(b)*x/sqrt(a))/(2*s
qrt(b)) + 3*b*c**2*d*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + b*c*d**2*x**7/(2
*sqrt(a)*sqrt(1 + b*x**2/a)) + b*d**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.46 $\int \sqrt{a+bx^2} \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=149

$$\frac{x\sqrt{a+bx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16b^2}+\frac{a\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{24b^2}+\frac{dx\left(\right.}{}$$

Rubi [A] time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {416, 388, 195, 217, 206}

$$\frac{x\sqrt{a+bx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16b^2}+\frac{a\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{24b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

```
[Out] ((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(16*b^2) + (d*(8*b*c
- 3*a*d)*x*(a + b*x^2)^(3/2))/(24*b^2) + (d*x*(a + b*x^2)^(3/2)*(c + d*x^2)
)/(6*b) + (a*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(16*b^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^2}\left(c+dx^2\right)^2 dx &= \frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}+\frac{\int \sqrt{a+bx^2}\left(c(6bc-ad)+d(8bc-3ad)x^2\right) dx}{6b}\\ &= \frac{d(8bc-3ad)x\left(a+bx^2\right)^{3/2}}{24b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}+\frac{\left(8b^2c^2-4abcd+a^2d^2\right)}{8b^2}\\ &= \frac{\left(8b^2c^2-4abcd+a^2d^2\right)x\sqrt{a+bx^2}}{16b^2}+\frac{d(8bc-3ad)x\left(a+bx^2\right)^{3/2}}{24b^2}+\frac{dx\left(a+bx^2\right)}{6b}\\ &= \frac{\left(8b^2c^2-4abcd+a^2d^2\right)x\sqrt{a+bx^2}}{16b^2}+\frac{d(8bc-3ad)x\left(a+bx^2\right)^{3/2}}{24b^2}+\frac{dx\left(a+bx^2\right)}{6b}\\ &= \frac{\left(8b^2c^2-4abcd+a^2d^2\right)x\sqrt{a+bx^2}}{16b^2}+\frac{d(8bc-3ad)x\left(a+bx^2\right)^{3/2}}{24b^2}+\frac{dx\left(a+bx^2\right)}{6b}\end{aligned}$$

Mathematica [C] time = 2.67, size = 160, normalized size = 1.07

$$\frac{x\sqrt{a+bx^2}\left(2bx^2\left(c+dx^2\right)^2 {}_3F_2\left(\frac{1}{2},\frac{3}{2},2;1,\frac{9}{2};-\frac{bx^2}{a}\right)+4bx^2\left(2c^2+3cdx^2+d^2x^4\right) {}_2F_1\left(\frac{1}{2},\frac{3}{2};\frac{9}{2};-\frac{bx^2}{a}\right)+7a\left(15c^2+10cdx^2+3d^2x^4\right) {}_2F_1\left(-\frac{1}{2},\frac{1}{2};\frac{7}{2};-\frac{bx^2}{a}\right)\right)}{105a\sqrt{\frac{bx^2}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(7*a*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*Hypergeometric2F1
[-1/2, 1/2, 7/2, -((b*x^2)/a)] + 4*b*x^2*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*Hype
rgeometric2F1[1/2, 3/2, 9/2, -((b*x^2)/a)] + 2*b*x^2*(c + d*x^2)^2*Hypergeo
metricPFQ[{1/2, 3/2, 2}, {1, 9/2}, -((b*x^2)/a)]))/(105*a*Sqrt[1 + (b*x^2)/
a])
```

IntegrateAlgebraic [A] time = 0.19, size = 132, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(-3a^2d^2x+12abcdx+2abd^2x^3+24b^2c^2x+24b^2cdx^3+8b^2d^2x^5\right)}{48b^2}+\frac{\left(a^3\left(-d^2\right)+4a^2bcd-8ab^2c^2\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(24*b^2*c^2*x + 12*a*b*c*d*x - 3*a^2*d^2*x + 24*b^2*c*d*x^
3 + 2*a*b*d^2*x^3 + 8*b^2*d^2*x^5))/(48*b^2) + ((-8*a*b^2*c^2 + 4*a^2*b*c*d
 - a^3*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(5/2))
```

fricas [A] time = 0.98, size = 264, normalized size = 1.77

$$\left[\frac{3\left(8ab^2c^2-4a^2bcd+a^3d^2\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(8b^3d^2x^5+2\left(12b^3cd+ab^2d^2\right)x^3+3\left(8b^3c^2+4ab^2cd-a^2bd^2\right)x\right)\sqrt{bx^2+a}}{96b^3}, -\frac{3\left(8ab^2c^2-4a^2bcd+a^3d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(8b^3d^2x^5+2\left(12b^3cd+ab^2d^2\right)x^3+3\left(8b^3c^2+4ab^2cd-a^2bd^2\right)x\right)\sqrt{bx^2+a}}{48b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) + 2*(8*b^3*d^2*x^5 + 2*(12*b^3*c*d + a*b^2*d^2)
*x^3 + 3*(8*b^3*c^2 + 4*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/
48*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) - (8*b^3*d^2*x^5 + 2*(12*b^3*c*d + a*b^2*d^2)*x^3 + 3*(8*b^3*c
^2 + 4*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.61, size = 129, normalized size = 0.87

$$\frac{1}{48}\left(2\left(4\,d^2x^2+\frac{12\,b^4cd+ab^3d^2}{b^4}\right)x^2+\frac{3\left(8\,b^4c^2+4\,ab^3cd-a^2b^2d^2\right)}{b^4}\right)\sqrt{bx^2+a}\,x-\frac{\left(8\,ab^2c^2-4\,a^2bcd+a^3d^2\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^2*x^2 + (12*b^4*c*d + a*b^3*d^2)/b^4)*x^2 + 3*(8*b^4*c^2 + 4*a
*b^3*c*d - a^2*b^2*d^2)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(8*a*b^2*c^2 - 4*a^2*
b*c*d + a^3*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 190, normalized size = 1.28

$$\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}d^2x^3}{6b}+\frac{a^3d^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{5}{2}}}-\frac{a^2cd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{4b^{\frac{3}{2}}}+\frac{a\,c^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\;a^2d^2x}{16b^2}-\frac{\sqrt{b\,x^2+a}\;acdx}{4b}+\frac{\sqrt{b\,x^2+a}\;c^2x}{2}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}a\,d^2x}{8b^2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}cdx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^2,x)
```

```
[Out] 1/6*d^2*x^3*(b*x^2+a)^(3/2)/b-1/8*d^2*a/b^2*x*(b*x^2+a)^(3/2)+1/16*d^2*a^2/
b^2*x*(b*x^2+a)^(1/2)+1/16*d^2*a^3/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/
2*c*d*x*(b*x^2+a)^(3/2)/b-1/4*c*d*a/b*x*(b*x^2+a)^(1/2)-1/4*c*d*a^2/b^(3/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*c^2*x*(b*x^2+a)^(1/2)+1/2*c^2*a/b^(1/2)*
ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.35, size = 168, normalized size = 1.13

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}d^2x^3}{6\,b}+\frac{1}{2}\,\sqrt{bx^2+a}\,c^2x+\frac{\left(bx^2+a\right)^{\frac{3}{2}}cdx}{2\,b}-\frac{\sqrt{bx^2+a}\,acdx}{4\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}ad^2x}{8\,b^2}+\frac{\sqrt{bx^2+a}\,a^2d^2x}{16\,b^2}+\frac{ac^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}-\frac{a^2cd\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{4\,b^{\frac{3}{2}}}+\frac{a^3d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(3/2)*d^2*x^3/b + 1/2*sqrt(b*x^2 + a)*c^2*x + 1/2*(b*x^2 +
a)^(3/2)*c*d*x/b - 1/4*sqrt(b*x^2 + a)*a*c*d*x/b - 1/8*(b*x^2 + a)^(3/2)*a*
d^2*x/b^2 + 1/16*sqrt(b*x^2 + a)*a^2*d^2*x/b^2 + 1/2*a*c^2*arcsinh(b*x/sqrt
(a*b))/sqrt(b) - 1/4*a^2*c*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/16*a^3*d^2*
arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2)^2, x)
```

sympy [B] time = 11.39, size = 291, normalized size = 1.95

$$-\frac{a^{\frac{5}{2}}d^2x}{16b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}cdx}{4b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{3}{2}}d^2x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,c^2x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3\sqrt{a}\,cdx^3}{4\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}\,d^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{a^3d^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}-\frac{a^2cd\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+\frac{ac^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{bcdx^5}{2\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{bd^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2,x)
```

```
[Out] -a**(5/2)*d**2*x/(16*b**2*sqrt(1 + b*x**2/a)) + a**(3/2)*c*d*x/(4*b*sqrt(1
+ b*x**2/a)) - a**(3/2)*d**2*x**3/(48*b*sqrt(1 + b*x**2/a)) + sqrt(a)*c**2*
```

```
x*sqrt(1 + b*x**2/a)/2 + 3*sqrt(a)*c*d*x**3/(4*sqrt(1 + b*x**2/a)) + 5*sqrt
(a)*d**2*x**5/(24*sqrt(1 + b*x**2/a)) + a**3*d**2*asinh(sqrt(b)*x/sqrt(a))/
(16*b**(5/2)) - a**2*c*d*asinh(sqrt(b)*x/sqrt(a))/(4*b**(3/2)) + a*c**2*asi
nh(sqrt(b)*x/sqrt(a))/(2*sqrt(b)) + b*c*d*x**5/(2*sqrt(a)*sqrt(1 + b*x**2/a
)) + b*d**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.47 $\int \sqrt{a+bx^2} \left(c+dx^2\right) dx$

Optimal. Leaf size=87

$$\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}\,(4bc-ad)}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}$$

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {388, 195, 217, 206}

$$\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}\,(4bc-ad)}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

```
[Out] ((4*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b) + (d*x*(a + b*x^2)^(3/2))/(4*b) + (
a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)dx = \frac{dx\left(a+bx^2\right)^{3/2}}{4b} - \frac{(-4bc+ad)\int \sqrt{a+bx^2}\,dx}{4b}$$

$$= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b} + \frac{dx\left(a+bx^2\right)^{3/2}}{4b} + \frac{(a(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b}$$

$$= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b} + \frac{dx\left(a+bx^2\right)^{3/2}}{4b} + \frac{(a(4bc-ad))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+b\cdots}}\right.}{8b}$$

$$= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b} + \frac{dx\left(a+bx^2\right)^{3/2}}{4b} + \frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

Mathematica [A] time = 0.16, size = 85, normalized size = 0.98

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(ad+4bc+2bdx^2\right) - \frac{\sqrt{a}\,(ad-4bc)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(4*b*c + a*d + 2*b*d*x^2) - (Sqrt[a]*(-4*b*c +
a*d)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(8*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 77, normalized size = 0.89

$$\frac{\left(a^2d-4abc\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{3/2}} + \frac{\sqrt{a+bx^2}\left(adx+4bcx+2bdx^3\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(4*b*c*x + a*d*x + 2*b*d*x^3))/(8*b) + ((-4*a*b*c + a^2*d)
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

fricas [A] time = 1.16, size = 158, normalized size = 1.82

$$\left[-\frac{\left(4\,abc-a^2d\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(2\,b^2dx^3+\left(4\,b^2c+abd\right)x\right)\sqrt{bx^2+a}}{16\,b^2}, -\frac{\left(4\,abc-a^2d\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2dx^3+\left(4\,b^2c+abd\right)x\right)\sqrt{bx^2+a}}{8\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*a*b*c - a^2*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*(2*b^2*d*x^3 + (4*b^2*c + a*b*d)*x)*sqrt(b*x^2 + a))/b^2, -1/8*(
(4*a*b*c - a^2*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*d*x^
3 + (4*b^2*c + a*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.60, size = 70, normalized size = 0.80

$$\frac{1}{8}\,\sqrt{bx^2+a}\left(2\,dx^2+\frac{4\,b^2c+abd}{b^2}\right)x - \frac{\left(4\,abc-a^2d\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*d*x^2 + (4*b^2*c + a*b*d)/b^2)*x - 1/8*(4*a*b*c - a^
2*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

maple [A] time = 0.00, size = 96, normalized size = 1.10

$$-\frac{a^{2} d \ln \left(\sqrt{b}\, x+\sqrt{b\,x^{2}+a}\right)}{8 b^{\frac{3}{2}}}+\frac{a c \ln \left(\sqrt{b}\, x+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}-\frac{\sqrt{b\,x^{2}+a}\; a d x}{8 b}+\frac{\sqrt{b\,x^{2}+a}\; c x}{2}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} d x}{4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c),x)
```

```
[Out] 1/4*d*x*(b*x^2+a)^(3/2)/b-1/8*d*a/b*x*(b*x^2+a)^(1/2)-1/8*d*a^2/b^(3/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*c*x*(b*x^2+a)^(1/2)+1/2*c*a/b^(1/2)*ln(b^(1/
2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.35, size = 81, normalized size = 0.93

$$\frac{1}{2}\sqrt{bx^{2}+a}\,cx+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}dx}{4\,b}-\frac{\sqrt{bx^{2}+a}\,adx}{8\,b}+\frac{ac\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}-\frac{a^{2}d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*c*x + 1/4*(b*x^2 + a)^(3/2)*d*x/b - 1/8*sqrt(b*x^2 + a)
*a*d*x/b + 1/2*a*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/8*a^2*d*arcsinh(b*x/s
qrt(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2), x)
```

sympy [A] time = 5.68, size = 144, normalized size = 1.66

$$\frac{a^{\frac{3}{2}}dx}{8b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{\sqrt{a}\,cx\sqrt{1+\frac{bx^{2}}{a}}}{2}+\frac{3\sqrt{a}\,dx^{3}}{8\sqrt{1+\frac{bx^{2}}{a}}}-\frac{a^{2}d\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{ac\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{bdx^{5}}{4\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c),x)
```

```
[Out] a**(3/2)*d*x/(8*b*sqrt(1 + b*x**2/a)) + sqrt(a)*c*x*sqrt(1 + b*x**2/a)/2 +
3*sqrt(a)*d*x**3/(8*sqrt(1 + b*x**2/a)) - a**2*d*asinh(sqrt(b)*x/sqrt(a))/(
8*b**(3/2)) + a*c*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b)) + b*d*x**5/(4*sqrt(a
)*sqrt(1 + b*x**2/a))
```

3.48 $\int \sqrt{a+bx^2}\, dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^2}\, dx &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\int \frac{1}{\sqrt{a+bx^2}}\, dx\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\, dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.07

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

IntegrateAlgebraic [A] time = 0.00, size = 48, normalized size = 1.04

$$\frac{1}{2}x\sqrt{a+bx^2}-\frac{a\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 - (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

fricas [A] time = 0.75, size = 94, normalized size = 2.04

$$\left[\frac{2\sqrt{bx^2+a}\,bx+a\sqrt{b}\log\left(-2\,bx^2-2\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b},\frac{\sqrt{bx^2+a}\,bx-a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b, 1/2*(sqrt(b*x^2 + a)*b*x - a*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)))/b]
```

giac [A] time = 0.58, size = 37, normalized size = 0.80

$$\frac{1}{2}\sqrt{bx^2+a}\,x-\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

maple [A] time = 0.00, size = 36, normalized size = 0.78

$$\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*(b*x^2+a)^(1/2)*x
```

maxima [A] time = 1.36, size = 28, normalized size = 0.61

$$\frac{1}{2}\sqrt{bx^2+a}\,x+\frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x + 1/2*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 4.71, size = 35, normalized size = 0.76

$$\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2),x)
```

```
[Out] (x*(a + b*x^2)^(1/2))/2 + (a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2)
)
```

sympy [A] time = 1.86, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

3.49 $\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d}$$

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {402, 217, 206, 377, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*ArcTanh
[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx = \frac{b\int \frac{1}{\sqrt{a+bx^2}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{d}$$
$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d} - \frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d}$$

Mathematica [**A**]   time = 0.04, size = 84, normalized size = 1.02

$$\frac{\sqrt{ad-bc}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d} + \frac{\sqrt{b}\log\left(\sqrt{b}\,\sqrt{a+bx^2}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

```
[Out] (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])
])/(Sqrt[c]*d) + (Sqrt[b]*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/d
```

IntegrateAlgebraic [**A**]   time = 0.22, size = 138, normalized size = 1.68

$$-\frac{\sqrt{ad-bc}\tan^{-1}\left(\frac{\sqrt{b}\,dx^2}{\sqrt{c}\,\sqrt{ad-bc}} - \frac{dx\sqrt{a+bx^2}}{\sqrt{c}\,\sqrt{ad-bc}} + \frac{\sqrt{b}\,\sqrt{c}}{\sqrt{ad-bc}}\right)}{\sqrt{c}\,d} - \frac{\sqrt{b}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

```
[Out] -((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c])/Sqrt[-(b*c) + a*d] + (Sqrt[b
]*d*x^2)/(Sqrt[c]*Sqrt[-(b*c) + a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt
[-(b*c) + a*d])])/(Sqrt[c]*d)) - (Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2
]])/d
```

fricas [**A**]   time = 1.12, size = 596, normalized size = 7.27

$$\left[\frac{2\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2-4(ac^2x+(2bc^2-acd)x^3)\sqrt{bx^2+a}\,\sqrt{\frac{bc-ad}{c}}}{d^2x^4+2cdx^2+c^2}\right)}{4d}, -\frac{4\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2-4(ac^2x+(2bc^2-acd)x^3)\sqrt{bx^2+a}\,\sqrt{\frac{bc-ad}{c}}}{d^2x^4+2cdx^2+c^2}\right)}{4d}, \frac{\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2bc-ad)x^2+ac)\sqrt{bx^2+a}\,\sqrt{-\frac{bc-ad}{c}}}{2((b^2c-abd)x^3+(abc-a^2d)x)}\right)+\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2d}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2bc-ad)x^2+ac)\sqrt{bx^2+a}\,\sqrt{-\frac{bc-ad}{c}}}{2((b^2c-abd)x^3+(abc-a^2d)x)}\right)}{2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + sqrt((b*c
 - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*
c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*
sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/d, -1/4*(4*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x
 + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2
*c*d*x^2 + c^2)))/d, 1/2*(sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^
2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c
 - a^2*d)*x)) + sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/d,
 -1/2*(2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(-(b*c - a*d)/c)
```

```
*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/
((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.04, size = 932, normalized size = 11.37

$$-\frac{a\ln\left(\frac{\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\sqrt{\frac{ad-bc}{d}}}+\frac{a\ln\left(\frac{-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\sqrt{\frac{ad-bc}{d}}}+\frac{bc\ln\left(\frac{\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\sqrt{\frac{ad-bc}{d}}\,d}-\frac{bc\ln\left(\frac{-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\sqrt{\frac{ad-bc}{d}}\,d}+\frac{\sqrt{b}\,\ln\left(\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)b-\frac{\sqrt{-cd}\,b}{d}}{\sqrt{b}}+\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}\right)}{2d}+\frac{\sqrt{b}\,\ln\left(\frac{\left(x-\frac{\sqrt{-cd}}{d}\right)b+\frac{\sqrt{-cd}\,b}{d}}{\sqrt{b}}+\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}\right)}{2d}+\frac{\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{2\sqrt{-cd}}-\frac{\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{2\sqrt{-cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c),x)
```

```
[Out] 1/2/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(1/2)+1/2*b^(1/2)/d*ln((b*(-c*d)^(1/2)/d+(x-(-c*d)^(1/2)/d
)*b)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(1/2))-1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+
2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2
)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d
)^(1/2)/d))*a+1/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*
(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)
^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1
/2)/d))*b*c-1/2/(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/2*b^(1/2)/d*ln((-b*(-c*d)^(1/2)/d+(x+(
-c*d)^(1/2)/d)*b)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c
*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))+1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2
*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((
x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2))/(x+(-c*d)^(1/2)/d))*a-1/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*
d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-
c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))
/(x+(-c*d)^(1/2)/d))*b*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2 + a}}{dx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases} \frac{\sqrt{-b}\,\mathrm{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{c} & \text{if } ((a+b\,c=0 \wedge d=-1) \vee a\,d=b\,c) \wedge b<0 \\ \frac{\sqrt{b}\,\ln\left(2\sqrt{b}\,x+2\sqrt{b\,x^2+a}\right)}{d}+\frac{\mathrm{atan}\left(\frac{x\sqrt{a\,d-b\,c}}{\sqrt{c}\,\sqrt{b\,x^2+a}}\right)\sqrt{a\,d-b\,c}}{\sqrt{c}\,d} & \text{if } a\neq 0 \wedge (((a+b\,c\neq 0 \vee d\neq -1) \wedge a\,d\neq b\,c) \vee \neg b<0) \\ \int \frac{\sqrt{b\,x^2+a}}{d\,x^2+c}\,dx & \text{if } ((((a+b\,c=0 \wedge d=-1) \vee a\,d=b\,c) \wedge b<0) \vee a=0) \wedge (((a+b\,c\neq 0 \vee d\neq -1) \wedge a\,d\neq b\,c) \vee \neg b<0) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2),x)
```

```
[Out] piecewise((a + b*c == 0 & d == -1 | a*d == b*c) & b < 0, ((-b)^(1/2)*asin(x
*(-b/a)^(1/2)))/c, a ~= 0 & ((a + b*c ~= 0 | d ~= -1) & a*d ~= b*c | ~b < 0
), (b^(1/2)*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/d + (atan((x*(a*d - b*c
)^(1/2))/(c^(1/2)*(a + b*x^2)^(1/2)))*(a*d - b*c)^(1/2))/(c^(1/2)*d), ((a +
 b*c == 0 & d == -1 | a*d == b*c) & b < 0 | a == 0) & ((a + b*c ~= 0 | d ~=
 -1) & a*d ~= b*c | ~b < 0), int((a + b*x^2)^(1/2)/(c + d*x^2), x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2}}{c + dx^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2), x)
```

3.50 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2} \, dx$

Optimal. Leaf size=82

$$\frac{a \tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {378, 377, 208}

$$\frac{a \tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[b*c - a*d])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2} \, dx &= \frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}+\frac{a\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)} \, dx}{2c}\\
&= \frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}+\frac{a\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2} \, dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2c}\\
&= \frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}+\frac{a \tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [**B**] time = 0.23, size = 165, normalized size = 2.01

$$\frac{x\sqrt{a+bx^2}\left(\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}+\sqrt{\frac{dx^2}{c}+1}\,\sin^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)\right)}{2c^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(Sqrt[(-(b/a) + d/c)*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))] + Sqrt[1 + (d*x^2)/c]*ArcSin[Sqrt[(-(b/a) + d/c)*x^2]/Sqrt[1 + (d*x
^2)/c]]))/(2*c^2*Sqrt[(-(b/a) + d/c)*x^2]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c])
```

IntegrateAlgebraic [**A**] time = 0.39, size = 145, normalized size = 1.77

$$\frac{a\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\,dx^2}{\sqrt{c}\,\sqrt{ad-bc}}-\frac{dx\sqrt{a+bx^2}}{\sqrt{c}\,\sqrt{ad-bc}}+\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{ad-bc}}\right)}{2c^{3/2}(bc-ad)}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[
b]*Sqrt[c])/Sqrt[-(b*c) + a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[-(b*c) + a*d
]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(2*c^(3/2)*(b*c -
 a*d))
```

fricas [**B**] time = 1.22, size = 369, normalized size = 4.50

$$\left[\frac{4\left(bc^2-acd\right)\sqrt{bx^2+a}\,x+\left(adx^2+ac\right)\sqrt{bc^2-acd}\,\log\left(\frac{\left(8b^2c^2-8abcd+a^2d^2\right)x^4+a^2c^2+2\left(4abc^2-3a^2cd\right)x^2+4\left((2bc-ad)x^3+acx\right)\sqrt{bc^2-acd}\,\sqrt{bx^2+a}}{d^2x^4+2cdx^2+c^2}\right)}{8\left(bc^4-ac^3d+\left(bc^3d-ac^2d^2\right)x^2\right)},\ \frac{2\left(bc^2-acd\right)\sqrt{bx^2+a}\,x-\left(adx^2+ac\right)\sqrt{-bc^2+acd}\,\arctan\left(\frac{\sqrt{-bc^2+acd}\left((2bc-ad)x^2+ac\right)\sqrt{bx^2+a}}{2\left(\left(b^2c^2-abcd\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)}{4\left(bc^4-ac^3d+\left(bc^3d-ac^2d^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x + (a*d*x^2 + a*c)*sqrt(b*c^2 - a*
c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 -
3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x
^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(b*c^4 - a*c^3*d + (b*c^3*d - a*c^2*
d^2)*x^2), 1/4*(2*(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x - (a*d*x^2 + a*c)*sqrt(
-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*s
qrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(b*c^4 -
 a*c^3*d + (b*c^3*d - a*c^2*d^2)*x^2)]
```

giac [**B**] time = 1.64, size = 217, normalized size = 2.65

$$-\frac{a\sqrt{b}\,\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{2\sqrt{-b^2c^2+abcd}\,c}+\frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}c-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}d+a^2\sqrt{b}d}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*a*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)
/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c) + (2*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*b^(3/2)*c - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d +
 a^2*sqrt(b)*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)*c*d)
```

maple [B] time = 0.02, size = 2521, normalized size = 30.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/4/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(
x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-1/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)*(
(x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(
1/2)+1/4/d*b^(3/2)/(a*d-b*c)*ln(((x-(-c*d)^(1/2)/d)*b+(-c*d)^(1/2)*b/d)/b^(
1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c
)/d)^(1/2))+1/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(-c*
d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c
*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/
(x-(-c*d)^(1/2)/d))*a-1/4/d^2*(-c*d)^(1/2)*b^2/(a*d-b*c)/((a*d-b*c)/d)^(1/2
)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(
1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c
)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/4/c*b/(a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2
*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-1/4/c*b^(1/2)/(a*
d-b*c)*ln(((x-(-c*d)^(1/2)/d)*b+(-c*d)^(1/2)*b/d)/b^(1/2)+((x-(-c*d)^(1/2)/
d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*a+1/4/c/(a
*d-b*c)/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)
^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)+1/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)*((x+(-c*d
)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+1/4
/d*b^(3/2)/(a*d-b*c)*ln(((x+(-c*d)^(1/2)/d)*b-(-c*d)^(1/2)*b/d)/b^(1/2)+((x
+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/
2))-1/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2
)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/
2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*
d)^(1/2)/d))*a+1/4/d^2*(-c*d)^(1/2)*b^2/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((-
2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*(
(x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(
1/2))/(x+(-c*d)^(1/2)/d))-1/4/c*b/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d
)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-1/4/c*b^(1/2)/(a*d-b*c)
*ln(((x+(-c*d)^(1/2)/d)*b-(-c*d)^(1/2)*b/d)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b
-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*a+1/4/(-c*d)^(1/
2)/c*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c
)/d)^(1/2)+1/4/c*b^(1/2)/d*ln(((x-(-c*d)^(1/2)/d)*b+(-c*d)^(1/2)*b/d)/b^(1/
2)+((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/
d)^(1/2))-1/4/(-c*d)^(1/2)/c/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*
d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b
+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/
d))*a+1/4/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(
1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(
-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*
b-1/4/(-c*d)^(1/2)/c*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)
/d)*b/d+(a*d-b*c)/d)^(1/2)+1/4/c*b^(1/2)/d*ln(((x+(-c*d)^(1/2)/d)*b-(-c*d)^
(1/2)*b/d)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d
)*b/d+(a*d-b*c)/d)^(1/2))+1/4/(-c*d)^(1/2)/c/((a*d-b*c)/d)^(1/2)*ln((-2*(-c
*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-
c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))
/(x+(-c*d)^(1/2)/d))*a-1/4/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)
^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d
```

```
)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x
+(-c*d)^(1/2)/d))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2 + a}}{\left(dx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2 + a}}{\left(d\,x^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2}}{\left(c + dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2)**2, x)
```

3.51 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3} dx$

Optimal. Leaf size=149

$$\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}+\frac{x\sqrt{a+bx^2}(4bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Rubi [A] time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {382, 378, 377, 208}

$$\frac{x\sqrt{a+bx^2}(4bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}-\frac{dx\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

```
[Out] -(d*x*(a + b*x^2)^(3/2))/(4*c*(b*c - a*d)*(c + d*x^2)^2) + ((4*b*c - 3*a*d)
*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)*(c + d*x^2)) + (a*(4*b*c - 3*a*d)*Ar
cTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*(b*c - a*d
)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3}\,dx = -\frac{dx\left(a+bx^2\right)^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} + \frac{(4bc-3ad)\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} + \frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)\left(c+dx^2\right)} + \frac{(a(4bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} + \frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)\left(c+dx^2\right)} + \frac{(a(4bc-3ad))\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,\right.}{8c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} + \frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)\left(c+dx^2\right)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A]   time = 0.57, size = 176, normalized size = 1.18

$$\frac{x\left(c\left(a^2d\left(5c+3dx^2\right)+ab\left(-4c^2+3cdx^2+3d^2x^4\right)-2b^2cx^2\left(2c+dx^2\right)\right)+\frac{a\left(c+dx^2\right)^2(3ad-4bc)\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)}{\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}}\right)}{8c^3\sqrt{a+bx^2}\left(c+dx^2\right)^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

```
[Out] (x*(c*(-2*b^2*c*x^2*(2*c + d*x^2) + a^2*d*(5*c + 3*d*x^2) + a*b*(-4*c^2 + 3
*c*d*x^2 + 3*d^2*x^4)) + (a*(-4*b*c + 3*a*d)*(c + d*x^2)^2*ArcTanh[Sqrt[((b
*c - a*d)*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))
/(8*c^3*(-(b*c) + a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^2)
```

IntegrateAlgebraic [A]   time = 1.17, size = 154, normalized size = 1.03

$$\frac{\left(4abc-3a^2d\right)\tanh^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,c+\sqrt{b}\,dx^2}{\sqrt{c}\sqrt{bc-ad}}\right)}{8c^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{a+bx^2}\left(-5acdx-3ad^2x^3+4bc^2x+2bcdx^3\right)}{8c^2\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(4*b*c^2*x - 5*a*c*d*x + 2*b*c*d*x^3 - 3*a*d^2*x^3))/(8*c^
2*(b*c - a*d)*(c + d*x^2)^2) + ((4*a*b*c - 3*a^2*d)*ArcTanh[(Sqrt[b]*c + Sq
rt[b]*d*x^2 - d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])])/(8*c^(5/2)*(
b*c - a*d)^(3/2))
```

fricas [B]   time = 1.20, size = 698, normalized size = 4.68

$$\left[\frac{\left(4abc^3-3a^2c^2d+\left(4abcd^2-3a^2d^3\right)x^4+2\left(4abc^2d-3a^2cd^2\right)x^2\right)\sqrt{bc^2-acd}\,\log\left(\frac{\left(8b^2c^2-8abcd+a^2d^2\right)x^4+a^2c^2+2\left(4abc^2-3a^2cd\right)x^2+4\left((2bc-ad)x^3+acx\right)\sqrt{bc^2-acd}\sqrt{bx^2+a}}{d^2x^4+2cdx^2+c^2}\right)+4\left(\left(2b^2c^3d-5abc^2d^2+3a^2cd^3\right)x^3+\left(4b^2c^4-9abc^3d+5a^2c^2d^2\right)x\right)\sqrt{bx^2+a}}{32\left(b^2c^7-2abc^6d+a^2c^5d^2+\left(b^2c^5d^2-2abc^4d^3+a^2c^3d^4\right)x^4+2\left(b^2c^6d-2abc^5d^2+a^2c^4d^3\right)x^2\right)}, -\frac{\left(4abc^3-3a^2c^2d+\left(4abcd^2-3a^2d^3\right)x^4+2\left(4abc^2d-3a^2cd^2\right)x^2\right)\sqrt{-bc^2+acd}\,\arctan\left(\frac{\sqrt{-bc^2+acd}\left((2bc-ad)x^2+ac\right)\sqrt{bx^2+a}}{2\left(\left(b^2c^2-abcd\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)-2\left(\left(2b^2c^3d-5abc^2d^2+3a^2cd^3\right)x^3+\left(4b^2c^4-9abc^3d+5a^2c^2d^2\right)x\right)\sqrt{bx^2+a}}{16\left(b^2c^7-2abc^6d+a^2c^5d^2+\left(b^2c^5d^2-2abc^4d^3+a^2c^3d^4\right)x^4+2\left(b^2c^6d-2abc^5d^2+a^2c^4d^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b*
c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d +
```

```
a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*
x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^
2)) + 4*((2*b^2*c^3*d - 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (4*b^2*c^4 - 9*a
*b*c^3*d + 5*a^2*c^2*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^7 - 2*a*b*c^6*d + a^2*
c^5*d^2 + (b^2*c^5*d^2 - 2*a*b*c^4*d^3 + a^2*c^3*d^4)*x^4 + 2*(b^2*c^6*d -
2*a*b*c^5*d^2 + a^2*c^4*d^3)*x^2), -1/16*((4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b
*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(-b*c^2 +
a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2
 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((2*b^2*c^3*d
- 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (4*b^2*c^4 - 9*a*b*c^3*d + 5*a^2*c^2*d
^2)*x)*sqrt(b*x^2 + a))/(b^2*c^7 - 2*a*b*c^6*d + a^2*c^5*d^2 + (b^2*c^5*d^2
 - 2*a*b*c^4*d^3 + a^2*c^3*d^4)*x^4 + 2*(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^
4*d^3)*x^2)]
```

giac [B] time = 3.75, size = 487, normalized size = 3.27

$$-\frac{\left(4\,ab^{\frac{3}{2}}c - 3\,a^{2}\sqrt{b}d\right)\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{2}d + 2\,bc - ad}{2\,\sqrt{-b^{2}c^{2}+abcd}}\right)}{8\,\sqrt{-b^{2}c^{2}+abcd}\left(bc^{3}-ac^{2}d\right)} - \frac{4\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{6}ab^{\frac{3}{2}}cd^{2} - 3\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{6}a^{2}\sqrt{b}d^{3} - 16\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{4}b^{\frac{7}{2}}c^{3} + 40\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{4}ab^{\frac{5}{2}}c^{2}d - 30\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{4}a^{2}b^{\frac{3}{2}}cd^{2} + 9\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{4}a^{3}\sqrt{b}d^{3} - 16\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{2}a^{2}b^{\frac{5}{2}}c^{2}d + 28\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{2}a^{3}b^{\frac{3}{2}}cd^{2} - 9\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{2}a^{4}\sqrt{b}d^{3} - 2\,a^{4}b^{\frac{3}{2}}cd^{2} + 3\,a^{5}\sqrt{b}d^{3}}{4\left(\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{4}d + 4\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{2}bc - 2\left(\sqrt{b}x - \sqrt{bx^{2}+a}\right)^{2}ad + a^{2}d\right)^{2}\left(bc^{3}d - ac^{2}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*a*b^(3/2)*c - 3*a^2*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
+ a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d
)*(b*c^3 - a*c^2*d)) - 1/4*(4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d
^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^3 - 16*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*b^(7/2)*c^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)
*c^2*d - 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^2 + 9*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^3 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^2
*a^2*b^(5/2)*c^2*d + 28*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^2 -
 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d^3 - 2*a^4*b^(3/2)*c*d^2 +
3*a^5*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2*(b*c
^3*d - a*c^2*d^2))
```

maple [B] time = 0.02, size = 5101, normalized size = 34.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^{2}+a}}{\left(dx^{2}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^{2}+a}}{{\left(d\,x^{2}+c\right)}^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.52 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^4}\,dx$

Optimal. Leaf size=208

$$\frac{a\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}\,(2bc-5ad)(4bc-3ad)}{48c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}\,(4bc-5ad)}{24c^2\left(c+dx^2\right)^2(bc-ad)}+$$

Rubi [A] time = 0.21, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {412, 527, 12, 377, 208}

$$\frac{a\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}\,(2bc-5ad)(4bc-3ad)}{48c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}\,(4bc-5ad)}{24c^2\left(c+dx^2\right)^2(bc-ad)}+\frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(6*c*(c + d*x^2)^3) + ((4*b*c - 5*a*d)*x*Sqrt[a + b*x^2
])/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + ((2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*
Sqrt[a + b*x^2])/(48*c^3*(b*c - a*d)^2*(c + d*x^2)) + (a*(8*b^2*c^2 - 12*a*
b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/
(16*c^(7/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^4}\,dx = \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} - \frac{\int \frac{-5a-4bx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx}{6c}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} - \frac{\int \frac{-a(16bc-15ad)-2b(4bc-5ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx}{24c^2(bc-ad)}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} - \frac{\int -\frac{3a\left(8b^2c\ldots\right.}{\sqrt{a\ldots}}}{48c\ldots}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(a\left(8b^2c^2\ldots\right.\right.}{\ldots}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(a\left(8b^2c^2\ldots\right.\right.}{\ldots}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{a\left(8b^2c^2\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.99, size = 227, normalized size = 1.09

$$\frac{x\sqrt{a+bx^2}\left(\frac{3a\left(c+dx^2\right)^3\left(5a^2d^2-12abcd+8b^2c^2\right)\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\right)}{x^2}+(bc-ad)\left(a^2d^2\left(33c^2+40cdx^2+15d^2x^4\right)-2abcd\left(30c^2+35cdx^2+13d^2x^4\right)+8b^2c^2\left(3c^2+3cdx^2+d^2x^4\right)\right)\right)}{48c^3\left(c+dx^2\right)^3(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((b*c - a*d)*(8*b^2*c^2*(3*c^2 + 3*c*d*x^2 + d^2*x^4) -
2*a*b*c*d*(30*c^2 + 35*c*d*x^2 + 13*d^2*x^4) + a^2*d^2*(33*c^2 + 40*c*d*x^2
 + 15*d^2*x^4)) + (3*a*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Sqrt[((b*c - a*
d)*x^2)/(c*(a + b*x^2))]*(c + d*x^2)^3*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a
 + b*x^2))]])/x^2))/(48*c^3*(b*c - a*d)^3*(c + d*x^2)^3)
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

```
[Out] $Aborted
```

fricas [B] time = 2.59, size = 1220, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a*b^2*c^5 - 12*a^2*b*c^4*d + 5*a^3*c^3*d^2 + (8*a*b^2*c^2*d^3
 - 12*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 3*(8*a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3 +
 5*a^3*c*d^4)*x^4 + 3*(8*a*b^2*c^4*d - 12*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^
2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2
 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2
 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((8*b^3*c^4*d^2
 - 34*a*b^2*c^3*d^3 + 41*a^2*b*c^2*d^4 - 15*a^3*c*d^5)*x^5 + 2*(12*b^3*c^5*
d - 47*a*b^2*c^4*d^2 + 55*a^2*b*c^3*d^3 - 20*a^3*c^2*d^4)*x^3 + 3*(8*b^3*c^
6 - 28*a*b^2*c^5*d + 31*a^2*b*c^4*d^2 - 11*a^3*c^3*d^3)*x)*sqrt(b*x^2 + a))
/(b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3 + (b^3*c^7*d^3 -
 3*a*b^2*c^6*d^4 + 3*a^2*b*c^5*d^5 - a^3*c^4*d^6)*x^6 + 3*(b^3*c^8*d^2 - 3*
a*b^2*c^7*d^3 + 3*a^2*b*c^6*d^4 - a^3*c^5*d^5)*x^4 + 3*(b^3*c^9*d - 3*a*b^2
*c^8*d^2 + 3*a^2*b*c^7*d^3 - a^3*c^6*d^4)*x^2), -1/96*(3*(8*a*b^2*c^5 - 12*
a^2*b*c^4*d + 5*a^3*c^3*d^2 + (8*a*b^2*c^2*d^3 - 12*a^2*b*c*d^4 + 5*a^3*d^5
)*x^6 + 3*(8*a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 3*(8*a*b
^2*c^4*d - 12*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(-b*c^2 + a*c*d)*arct
an(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2
*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((8*b^3*c^4*d^2 - 34*a*b^
2*c^3*d^3 + 41*a^2*b*c^2*d^4 - 15*a^3*c*d^5)*x^5 + 2*(12*b^3*c^5*d - 47*a*b
^2*c^4*d^2 + 55*a^2*b*c^3*d^3 - 20*a^3*c^2*d^4)*x^3 + 3*(8*b^3*c^6 - 28*a*b
^2*c^5*d + 31*a^2*b*c^4*d^2 - 11*a^3*c^3*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^10
 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3 + (b^3*c^7*d^3 - 3*a*b^2*c
^6*d^4 + 3*a^2*b*c^5*d^5 - a^3*c^4*d^6)*x^6 + 3*(b^3*c^8*d^2 - 3*a*b^2*c^7*
d^3 + 3*a^2*b*c^6*d^4 - a^3*c^5*d^5)*x^4 + 3*(b^3*c^9*d - 3*a*b^2*c^8*d^2 +
 3*a^2*b*c^7*d^3 - a^3*c^6*d^4)*x^2)]
```

giac [B] time = 2.87, size = 958, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -1/16*(8*a*b^(5/2)*c^2 - 12*a^2*b^(3/2)*c*d + 5*a^3*sqrt(b)*d^2)*arctan(1/2
*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d)
)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) - 1/24*(
24*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*c^2*d^3 - 36*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a^2*b^(3/2)*c*d^4 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^
3*sqrt(b)*d^5 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c^3*d^2 - 480
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c^2*d^3 + 330*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*
sqrt(b)*d^5 - 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c^5 + 1216*(sqrt
(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d - 2016*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^2*b^(7/2)*c^3*d^2 + 1736*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/
2)*c^2*d^3 - 800*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*c*d^4 + 150*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 - 384*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^2*b^(9/2)*c^4*d + 1392*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/
2)*c^3*d^2 - 1608*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*c^2*d^3 + 780
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 - 150*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*a^6*sqrt(b)*d^5 - 96*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7
/2)*c^3*d^2 + 336*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*c^2*d^3 - 300
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 + 75*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*a^7*sqrt(b)*d^5 - 8*a^6*b^(5/2)*c^2*d^3 + 26*a^7*b^(3/2)*c*d^4
 - 15*a^8*sqrt(b)*d^5)/((b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*((sqrt(b)
*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3)
```

maple [B] time = 0.03, size = 7922, normalized size = 38.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^4,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^2+a}}{\left(d\,x^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^4,x)
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**4,x)
[Out] Timed out
```

3.53 $\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=272

$$\frac{3a^2(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}+\frac{dx\left(a+bx^2\right)^{5/2}\left(5a^2d^2-20abcd+36b^2c^2\right)}{160b^3}+\frac{x\left(a+bx^2\right)^{3}}{}$$

Rubi [A] time = 0.22, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {416, 528, 388, 195, 217, 206}

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(5a^2d^2-20abcd+36b^2c^2\right)}{160b^3}+\frac{x\left(a+bx^2\right)^{3/2}(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)}{128b^3}+\frac{3ax\sqrt{a+bx^2}(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)}{256b^3}+\frac{3a^2(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)(14bc-5ad)}{80b^2}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

```
[Out] (3*a*(4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(25
6*b^3) + ((4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*(a + b*x^2)^(3/
2))/(128*b^3) + (d*(36*b^2*c^2 - 20*a*b*c*d + 5*a^2*d^2)*x*(a + b*x^2)^(5/2
))/(160*b^3) + (d*(14*b*c - 5*a*d)*x*(a + b*x^2)^(5/2)*(c + d*x^2))/(80*b^2
) + (d*x*(a + b*x^2)^(5/2)*(c + d*x^2)^2)/(10*b) + (3*a^2*(4*b*c - a*d)*(8*
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b
^(7/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3\,dx = \frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b} + \frac{\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(c(10bc-ad)+d(14bc- \cdots\right.}{10b}$$

$$= \frac{d(14bc-5ad)x\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{80b^2} + \frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b} + \frac{\int \left(a+b \cdots\right.}{}$$

$$= \frac{d\left(36b^2c^2-20abcd+5a^2d^2\right)x\left(a+bx^2\right)^{5/2}}{160b^3} + \frac{d(14bc-5ad)x\left(a+bx^2\right)^{5/2}\left(c+ \cdots\right.}{80b^2}$$

$$= \frac{(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{128b^3} + \frac{d\left(36b^2c^2-20abcd+5a^2d^2 \cdots\right.}{160b^3}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abcd+ \cdots\right.}{128b^3}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abcd+ \cdots\right.}{128b^3}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abcd+ \cdots\right.}{128b^3}$$

Mathematica [A] time = 5.13, size = 220, normalized size = 0.81

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\left(15a^4d^3-10a^3bd^2\left(9c+dx^2\right)+4a^2b^2d\left(60c^2+15cdx^2+2d^2x^4\right)+16ab^3\left(50c^3+70c^2dx^2+45cd^2x^4+11d^3x^6\right)+32b^4x^2\left(10c^3+20c^2dx^2+15cd^2x^4+4d^3x^6\right)\right)-15a^2(ad-4bc)\left(a^2d^2-2abcd+8b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{1280b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^4*d^3 - 10*a^3*b*d^2*(9*c + d*x^2) + 4*a^2
*b^2*d*(60*c^2 + 15*c*d*x^2 + 2*d^2*x^4) + 32*b^4*x^2*(10*c^3 + 20*c^2*d*x^
2 + 15*c*d^2*x^4 + 4*d^3*x^6) + 16*a*b^3*(50*c^3 + 70*c^2*d*x^2 + 45*c*d^2*
x^4 + 11*d^3*x^6)) - 15*a^2*(-4*b*c + a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(1280*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.41, size = 259, normalized size = 0.95

$$\frac{\sqrt{a+bx^2}\left(15a^4d^3x-90a^3bcd^2x-10a^3bd^3x^3+240a^2b^2c^2dx+60a^2b^2cd^2x^3+8a^2b^2d^3x^5+800ab^3c^3x+1120ab^3c^2dx^3+720ab^3cd^2x^5+176ab^3d^3x^7+320b^4c^3x^3+640b^4c^2dx^5+480b^4cd^2x^7+128b^4d^3x^9\right)}{1280b^3}+\frac{3\left(a^5d^3-6a^4bcd^2+16a^3b^2c^2d-32a^2b^3c^3\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{256b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(800*a*b^3*c^3*x + 240*a^2*b^2*c^2*d*x - 90*a^3*b*c*d^2*x
+ 15*a^4*d^3*x + 320*b^4*c^3*x^3 + 1120*a*b^3*c^2*d*x^3 + 60*a^2*b^2*c*d^2*
```

```
x^3 - 10*a^3*b*d^3*x^3 + 640*b^4*c^2*d*x^5 + 720*a*b^3*c*d^2*x^5 + 8*a^2*b^
2*d^3*x^5 + 480*b^4*c*d^2*x^7 + 176*a*b^3*d^3*x^7 + 128*b^4*d^3*x^9))/(1280
*b^3) + (3*(-32*a^2*b^3*c^3 + 16*a^3*b^2*c^2*d - 6*a^4*b*c*d^2 + a^5*d^3)*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(256*b^(7/2))
```

fricas [A]   time = 1.35, size = 502, normalized size = 1.85

$$\left[-\frac{15\left(32a^2b^3c^3-16a^3b^2c^2d+6a^4bcd^2-a^5d^3\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(128b^5d^3x^9+16\left(30b^5cd^2+11ab^4d^3\right)x^7+8\left(80b^5c^2d+90ab^4cd^2+a^2b^3d^3\right)x^5+10\left(32b^5c^3+112ab^4c^2d+6a^2b^3cd^2-a^3b^2d^3\right)x^3+5\left(160ab^4c^3+48a^2b^3c^2d-18a^3b^2cd^2+3a^4bd^3\right)x\right)\sqrt{bx^2+a}}{2560b^4}, -\frac{15\left(32a^2b^3c^3-16a^3b^2c^2d+6a^4bcd^2-a^5d^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(128b^5d^3x^9+16\left(30b^5cd^2+11ab^4d^3\right)x^7+8\left(80b^5c^2d+90ab^4cd^2+a^2b^3d^3\right)x^5+10\left(32b^5c^3+112ab^4c^2d+6a^2b^3cd^2-a^3b^2d^3\right)x^3+5\left(160ab^4c^3+48a^2b^3c^2d-18a^3b^2cd^2+3a^4bd^3\right)x\right)\sqrt{bx^2+a}}{1280b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*
sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(128*b^5*d^3*x^
9 + 16*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^7 + 8*(80*b^5*c^2*d + 90*a*b^4*c*d^2
 + a^2*b^3*d^3)*x^5 + 10*(32*b^5*c^3 + 112*a*b^4*c^2*d + 6*a^2*b^3*c*d^2 -
a^3*b^2*d^3)*x^3 + 5*(160*a*b^4*c^3 + 48*a^2*b^3*c^2*d - 18*a^3*b^2*c*d^2 +
 3*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/1280*(15*(32*a^2*b^3*c^3 - 16*a^3
*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (128*b^5*d^3*x^9 + 16*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^7 + 8*(80*b^
5*c^2*d + 90*a*b^4*c*d^2 + a^2*b^3*d^3)*x^5 + 10*(32*b^5*c^3 + 112*a*b^4*c^
2*d + 6*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^3 + 5*(160*a*b^4*c^3 + 48*a^2*b^3*c^
2*d - 18*a^3*b^2*c*d^2 + 3*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

giac [A]   time = 0.66, size = 260, normalized size = 0.96

$$\frac{1}{1280}\left(2\left(4\left(2\left(8bd^3x^2+\frac{30b^9cd^2+11ab^8d^3}{b^8}\right)x^2+\frac{80b^9c^2d+90ab^8cd^2+a^2b^7d^3}{b^8}\right)x^2+\frac{5\left(32b^9c^3+112ab^8c^2d+6a^2b^7cd^2-a^3b^6d^3\right)}{b^8}\right)x^2+\frac{5\left(160ab^8c^3+48a^2b^7c^2d-18a^3b^6cd^2+3a^4b^5d^3\right)}{b^8}\right)\sqrt{bx^2+a}\,x-\frac{3\left(32a^2b^3c^3-16a^3b^2c^2d+6a^4bcd^2-a^5d^3\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*b*d^3*x^2 + (30*b^9*c*d^2 + 11*a*b^8*d^3)/b^8)*x^2 + (80
*b^9*c^2*d + 90*a*b^8*c*d^2 + a^2*b^7*d^3)/b^8)*x^2 + 5*(32*b^9*c^3 + 112*a
*b^8*c^2*d + 6*a^2*b^7*c*d^2 - a^3*b^6*d^3)/b^8)*x^2 + 5*(160*a*b^8*c^3 + 4
8*a^2*b^7*c^2*d - 18*a^3*b^6*c*d^2 + 3*a^4*b^5*d^3)/b^8)*sqrt(b*x^2 + a)*x
- 3/256*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*log(a
bs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A]   time = 0.02, size = 393, normalized size = 1.44

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}d^3x^5}{10b}-\frac{3a^5d^3\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{256b^{\frac{7}{2}}}+\frac{9a^4cd^2\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{128b^{\frac{5}{2}}}-\frac{3a^3c^2d\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{16b^{\frac{3}{2}}}+\frac{3a^2c^3\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{8\sqrt{b}}-\frac{3\sqrt{bx^2+a}\,a^4d^3x}{256b^3}+\frac{9\sqrt{bx^2+a}\,a^3cd^2x}{128b^2}-\frac{3\sqrt{bx^2+a}\,a^2c^2dx}{16b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}ad^3x^3}{16b^2}+\frac{3\sqrt{bx^2+a}\,ac^3x}{8}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}cd^2x^3}{8b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}a^3d^3x}{128b^3}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}a^2cd^2x}{64b^2}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}ac^2dx}{8b}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}c^3x}{4}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}a^2d^3x}{32b^3}-\frac{3\left(bx^2+a\right)^{\frac{5}{2}}acd^2x}{16b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}c^2dx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^3,x)
```

```
[Out] 1/10*d^3*x^5*(b*x^2+a)^(5/2)/b-1/16*d^3*a/b^2*x^3*(b*x^2+a)^(5/2)+1/32*d^3*
a^2/b^3*x*(b*x^2+a)^(5/2)-1/128*d^3*a^3/b^3*x*(b*x^2+a)^(3/2)-3/256*d^3*a^4
/b^3*x*(b*x^2+a)^(1/2)-3/256*d^3*a^5/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+
3/8*c*d^2*x^3*(b*x^2+a)^(5/2)/b-3/16*c*d^2*a/b^2*x*(b*x^2+a)^(5/2)+3/64*c*d
^2*a^2/b^2*x*(b*x^2+a)^(3/2)+9/128*c*d^2*a^3/b^2*x*(b*x^2+a)^(1/2)+9/128*c*
d^2*a^4/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*c^2*d*x*(b*x^2+a)^(5/2)/b
-1/8*c^2*d*a/b*x*(b*x^2+a)^(3/2)-3/16*c^2*d*a^2/b*x*(b*x^2+a)^(1/2)-3/16*c^
2*d*a^3/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*c^3*x*(b*x^2+a)^(3/2)+3/8
*c^3*a*x*(b*x^2+a)^(1/2)+3/8*c^3*a^2/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A]   time = 1.50, size = 364, normalized size = 1.34

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}d^3x^5}{10b}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}cd^2x^3}{8b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}ad^3x^3}{16b^2}+\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}c^3x+\frac{3}{8}\sqrt{bx^2+a}\,ac^3x+\frac{\left(bx^2+a\right)^{\frac{5}{2}}c^2dx}{2b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}ac^2dx}{8b}-\frac{3\sqrt{bx^2+a}\,a^2c^2dx}{16b}-\frac{3\left(bx^2+a\right)^{\frac{5}{2}}acd^2x}{16b^2}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}a^2cd^2x}{64b^2}+\frac{9\sqrt{bx^2+a}\,a^3cd^2x}{128b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}a^2d^3x}{32b^3}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}a^3d^3x}{128b^3}-\frac{3\sqrt{bx^2+a}\,a^4d^3x}{256b^3}+\frac{3a^2c^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}}-\frac{3a^3c^2d\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16b^{\frac{3}{2}}}+\frac{9a^4cd^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128b^{\frac{5}{2}}}-\frac{3a^5d^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(5/2)*d^3*x^5/b + 3/8*(b*x^2 + a)^(5/2)*c*d^2*x^3/b - 1/16
*(b*x^2 + a)^(5/2)*a*d^3*x^3/b^2 + 1/4*(b*x^2 + a)^(3/2)*c^3*x + 3/8*sqrt(b
*x^2 + a)*a*c^3*x + 1/2*(b*x^2 + a)^(5/2)*c^2*d*x/b - 1/8*(b*x^2 + a)^(3/2)
*a*c^2*d*x/b - 3/16*sqrt(b*x^2 + a)*a^2*c^2*d*x/b - 3/16*(b*x^2 + a)^(5/2)*
a*c*d^2*x/b^2 + 3/64*(b*x^2 + a)^(3/2)*a^2*c*d^2*x/b^2 + 9/128*sqrt(b*x^2 +
 a)*a^3*c*d^2*x/b^2 + 1/32*(b*x^2 + a)^(5/2)*a^2*d^3*x/b^3 - 1/128*(b*x^2 +
 a)^(3/2)*a^3*d^3*x/b^3 - 3/256*sqrt(b*x^2 + a)*a^4*d^3*x/b^3 + 3/8*a^2*c^3
*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 3/16*a^3*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(
3/2) + 9/128*a^4*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 3/256*a^5*d^3*arcsi
nh(b*x/sqrt(a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2)^3, x)
```

sympy [B] time = 52.75, size = 665, normalized size = 2.44

$$\frac{3a^{\frac{9}{2}}d^3x}{256b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{9a^{\frac{7}{2}}cd^2x}{128b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{7}{2}}d^3x^3}{256b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{\frac{5}{2}}c^2dx}{16b\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{\frac{5}{2}}cd^2x^3}{128b\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{5}{2}}d^3x^5}{640b\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{3}{2}}c^3x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{a^{\frac{3}{2}}c^3x}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{17a^{\frac{3}{2}}c^2dx^3}{16\sqrt{1+\frac{bx^2}{a}}} + \frac{39a^{\frac{3}{2}}cd^2x^5}{64\sqrt{1+\frac{bx^2}{a}}} + \frac{23a^{\frac{3}{2}}d^3x^7}{160\sqrt{1+\frac{bx^2}{a}}} + \frac{3\sqrt{a}\,bc^3x^3}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{11\sqrt{a}\,bc^2dx^5}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{15\sqrt{a}\,bcd^2x^7}{16\sqrt{1+\frac{bx^2}{a}}} + \frac{19\sqrt{a}\,bd^3x^9}{80\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^5d^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{7}{2}}} + \frac{9a^4cd^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}} - \frac{3a^3c^2d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + \frac{3a^2c^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}} + \frac{b^2c^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{b^2c^2dx^7}{2\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{3b^2cd^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{b^2d^3x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**3,x)
```

```
[Out] 3*a**(9/2)*d**3*x/(256*b**3*sqrt(1 + b*x**2/a)) - 9*a**(7/2)*c*d**2*x/(128*
b**2*sqrt(1 + b*x**2/a)) + a**(7/2)*d**3*x**3/(256*b**2*sqrt(1 + b*x**2/a))
 + 3*a**(5/2)*c**2*d*x/(16*b*sqrt(1 + b*x**2/a)) - 3*a**(5/2)*c*d**2*x**3/(
128*b*sqrt(1 + b*x**2/a)) - a**(5/2)*d**3*x**5/(640*b*sqrt(1 + b*x**2/a)) +
 a**(3/2)*c**3*x*sqrt(1 + b*x**2/a)/2 + a**(3/2)*c**3*x/(8*sqrt(1 + b*x**2/
a)) + 17*a**(3/2)*c**2*d*x**3/(16*sqrt(1 + b*x**2/a)) + 39*a**(3/2)*c*d**2*
x**5/(64*sqrt(1 + b*x**2/a)) + 23*a**(3/2)*d**3*x**7/(160*sqrt(1 + b*x**2/a
)) + 3*sqrt(a)*b*c**3*x**3/(8*sqrt(1 + b*x**2/a)) + 11*sqrt(a)*b*c**2*d*x**
5/(8*sqrt(1 + b*x**2/a)) + 15*sqrt(a)*b*c*d**2*x**7/(16*sqrt(1 + b*x**2/a))
 + 19*sqrt(a)*b*d**3*x**9/(80*sqrt(1 + b*x**2/a)) - 3*a**5*d**3*asinh(sqrt(
b)*x/sqrt(a))/(256*b**(7/2)) + 9*a**4*c*d**2*asinh(sqrt(b)*x/sqrt(a))/(128*
b**(5/2)) - 3*a**3*c**2*d*asinh(sqrt(b)*x/sqrt(a))/(16*b**(3/2)) + 3*a**2*c
**3*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + b**2*c**3*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a)) + b**2*c**2*d*x**7/(2*sqrt(a)*sqrt(1 + b*x**2/a)) + 3*b**2*c*
d**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) + b**2*d**3*x**11/(10*sqrt(a)*sqrt
(1 + b*x**2/a))
```

3.54 $\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=196

$$\frac{x\left(a+bx^2\right)^{3/2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{192b^2}+\frac{ax\sqrt{a+bx^2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{128b^2}+\frac{a^2\left(3a^2d^2-16abcd+48b^2c^2\right.}{128b^{5/2}}$$

Rubi [A] time = 0.12, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 388, 195, 217, 206}

$$\frac{x\left(a+bx^2\right)^{3/2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{192b^2}+\frac{ax\sqrt{a+bx^2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{128b^2}+\frac{a^2\left(3a^2d^2-16abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{dx\left(a+bx^2\right)^{5/2}(10bc-3ad)}{48b^2}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

```
[Out] (a*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(128*b^2) + ((4
8*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2))/(192*b^2) + (d*(10
*b*c - 3*a*d)*x*(a + b*x^2)^(5/2))/(48*b^2) + (d*x*(a + b*x^2)^(5/2)*(c + d
*x^2))/(8*b) + (a^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2\,dx &= \frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}+\frac{\int \left(a+bx^2\right)^{3/2}\left(c(8bc-ad)+d(10bc-3ad)x^2\right)dx}{8b} \\
&= \frac{d(10bc-3ad)x\left(a+bx^2\right)^{5/2}}{48b^2}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}-\frac{(ad(10bc-3ad)-6 \\
&= \frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{192b^2}+\frac{d(10bc-3ad)x\left(a+bx^2\right)^{5/2}}{48b^2}+\frac{dx(\\
&= \frac{a\left(48b^2c^2-16abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{128b^2}+\frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x\left(a+b\right.}{192b^2} \\
&= \frac{a\left(48b^2c^2-16abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{128b^2}+\frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x\left(a+b\right.}{192b^2} \\
&= \frac{a\left(48b^2c^2-16abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{128b^2}+\frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x\left(a+b\right.}{192b^2}
\end{aligned}$$

Mathematica [C] time = 2.71, size = 157, normalized size = 0.80

$$\frac{x\sqrt{a+bx^2}\left(6bx^2\left(c+dx^2\right)^2{}_3F_2\left(-\frac{1}{2},\frac{3}{2},2;1,\frac{9}{2};-\frac{bx^2}{a}\right)+12bx^2\left(2c^2+3cdx^2+d^2x^4\right){}_2F_1\left(-\frac{1}{2},\frac{3}{2};\frac{9}{2};-\frac{bx^2}{a}\right)+7a\left(15c^2+10cdx^2+3d^2x^4\right){}_2F_1\left(-\frac{3}{2},\frac{1}{2};\frac{7}{2};-\frac{bx^2}{a}\right)\right)}{105\sqrt{\frac{bx^2}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(7*a*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*Hypergeometric2F1
[-3/2, 1/2, 7/2, -((b*x^2)/a)] + 12*b*x^2*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*Hyp
ergeometric2F1[-1/2, 3/2, 9/2, -((b*x^2)/a)] + 6*b*x^2*(c + d*x^2)^2*Hyperg
eometricPFQ[{-1/2, 3/2, 2}, {1, 9/2}, -((b*x^2)/a)]))/(105*Sqrt[1 + (b*x^2)
/a])
```

IntegrateAlgebraic [A] time = 0.27, size = 173, normalized size = 0.88

$$\frac{\sqrt{a+bx^2}\left(-9a^3d^2x+48a^2bcdx+6a^2bd^2x^3+240ab^2c^2x+224ab^2cdx^3+72ab^2d^2x^5+96b^3c^2x^3+128b^3cdx^5+48b^3d^2x^7\right)}{384b^2}+\frac{\left(-3a^4d^2+16a^3bcd-48a^2b^2c^2\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{128b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(240*a*b^2*c^2*x + 48*a^2*b*c*d*x - 9*a^3*d^2*x + 96*b^3*c
^2*x^3 + 224*a*b^2*c*d*x^3 + 6*a^2*b*d^2*x^3 + 128*b^3*c*d*x^5 + 72*a*b^2*d
^2*x^5 + 48*b^3*d^2*x^7))/(384*b^2) + ((-48*a^2*b^2*c^2 + 16*a^3*b*c*d - 3*
a^4*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(5/2))
```

fricas [A] time = 1.31, size = 344, normalized size = 1.76

$$\left[\frac{3\left(48a^2b^2c^2-16a^3bcd+3a^4d^2\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(48b^4d^2x^7+8\left(16b^4cd+9ab^3d^2\right)x^5+2\left(48b^4c^2+112ab^3cd+3a^2b^2d^2\right)x^3+3\left(80ab^3c^2+16a^2b^2cd-3a^3bd^2\right)x\right)\sqrt{bx^2+a}}{768b^3},-\frac{3\left(48a^2b^2c^2-16a^3bcd+3a^4d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(48b^4d^2x^7+8\left(16b^4cd+9ab^3d^2\right)x^5+2\left(48b^4c^2+112ab^3cd+3a^2b^2d^2\right)x^3+3\left(80ab^3c^2+16a^2b^2cd-3a^3bd^2\right)x\right)\sqrt{bx^2+a}}{384b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(48*b^4*d^2*x^7 + 8*(16*b^4*c*d + 9*
a*b^3*d^2)*x^5 + 2*(48*b^4*c^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)*x^3 + 3*(80
```

```
*a*b^3*c^2 + 16*a^2*b^2*c*d - 3*a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/384*
(3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - (48*b^4*d^2*x^7 + 8*(16*b^4*c*d + 9*a*b^3*d^2)*x^5 + 2*(4
8*b^4*c^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)*x^3 + 3*(80*a*b^3*c^2 + 16*a^2*b
^2*c*d - 3*a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.66, size = 175, normalized size = 0.89

$$\frac{1}{384}\left(2\left(4\left(6\,bd^2x^2+\frac{16\,b^7cd+9\,ab^6d^2}{b^6}\right)x^2+\frac{48\,b^7c^2+112\,ab^6cd+3\,a^2b^5d^2}{b^6}\right)x^2+\frac{3\left(80\,ab^6c^2+16\,a^2b^5cd-3\,a^3b^4d^2\right)}{b^6}\right)\sqrt{bx^2+a}\,x-\frac{\left(48\,a^2b^2c^2-16\,a^3bcd+3\,a^4d^2\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b*d^2*x^2 + (16*b^7*c*d + 9*a*b^6*d^2)/b^6)*x^2 + (48*b^7*c^
2 + 112*a*b^6*c*d + 3*a^2*b^5*d^2)/b^6)*x^2 + 3*(80*a*b^6*c^2 + 16*a^2*b^5*
c*d - 3*a^3*b^4*d^2)/b^6)*sqrt(b*x^2 + a)*x - 1/128*(48*a^2*b^2*c^2 - 16*a^
3*b*c*d + 3*a^4*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 249, normalized size = 1.27

$$\frac{3a^4d^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{128b^{\frac{5}{2}}}-\frac{a^3cd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}+\frac{3a^2c^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}+\frac{3\sqrt{b\,x^2+a}\,a^3d^2x}{128b^2}-\frac{\sqrt{b\,x^2+a}\,a^2cdx}{8b}+\frac{3\sqrt{b\,x^2+a}\,a\,c^2x}{8}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}d^2x^3}{8b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}a^2d^2x}{64b^2}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}acdx}{12b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}c^2x}{4}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}a\,d^2x}{16b^2}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}cdx}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^2,x)
```

```
[Out] 1/8*d^2*x^3*(b*x^2+a)^(5/2)/b-1/16*d^2*a/b^2*x*(b*x^2+a)^(5/2)+1/64*d^2*a^2
/b^2*x*(b*x^2+a)^(3/2)+3/128*d^2*a^3/b^2*x*(b*x^2+a)^(1/2)+3/128*d^2*a^4/b^
(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/3*c*d*x*(b*x^2+a)^(5/2)/b-1/12*c*d*a/
b*x*(b*x^2+a)^(3/2)-1/8*c*d*a^2/b*x*(b*x^2+a)^(1/2)-1/8*c*d*a^3/b^(3/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*c^2*x*(b*x^2+a)^(3/2)+3/8*c^2*a*x*(b*x^2+a)^
(1/2)+3/8*c^2*a^2/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.42, size = 227, normalized size = 1.16

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}d^2x^3}{8\,b}+\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}c^2x+\frac{3}{8}\sqrt{bx^2+a}\,ac^2x+\frac{\left(bx^2+a\right)^{\frac{5}{2}}cdx}{3\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}acdx}{12\,b}-\frac{\sqrt{bx^2+a}\,a^2cdx}{8\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}ad^2x}{16\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}a^2d^2x}{64\,b^2}+\frac{3\sqrt{bx^2+a}\,a^3d^2x}{128\,b^2}+\frac{3\,a^2c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}}-\frac{a^3cd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{3\,a^4d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*d^2*x^3/b + 1/4*(b*x^2 + a)^(3/2)*c^2*x + 3/8*sqrt(b*
x^2 + a)*a*c^2*x + 1/3*(b*x^2 + a)^(5/2)*c*d*x/b - 1/12*(b*x^2 + a)^(3/2)*a
*c*d*x/b - 1/8*sqrt(b*x^2 + a)*a^2*c*d*x/b - 1/16*(b*x^2 + a)^(5/2)*a*d^2*x
/b^2 + 1/64*(b*x^2 + a)^(3/2)*a^2*d^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*a^3*d^2
*x/b^2 + 3/8*a^2*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/8*a^3*c*d*arcsinh(b
*x/sqrt(a*b))/b^(3/2) + 3/128*a^4*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^2+a\right)}^{3/2}\,{\left(d\,x^2+c\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2)^2, x)
```

sympy [B] time = 29.42, size = 440, normalized size = 2.24

$$-\frac{3a^{\frac{7}{2}}d^2x}{128b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{5}{2}}cdx}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{5}{2}}d^2x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}c^2x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}c^2x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{17a^{\frac{3}{2}}cdx^3}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{13a^{\frac{3}{2}}d^2x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}\,bc^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}\,bcdx^5}{12\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}\,bd^2x^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^4d^2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}-\frac{a^3cd\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{3a^2c^2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{b^2c^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^2cdx^7}{3\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^2d^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2,x)
```

```
[Out] -3*a**(7/2)*d**2*x/(128*b**2*sqrt(1 + b*x**2/a)) + a**(5/2)*c*d*x/(8*b*sqrt
(1 + b*x**2/a)) - a**(5/2)*d**2*x**3/(128*b*sqrt(1 + b*x**2/a)) + a**(3/2)*
c**2*x*sqrt(1 + b*x**2/a)/2 + a**(3/2)*c**2*x/(8*sqrt(1 + b*x**2/a)) + 17*a
**(3/2)*c*d*x**3/(24*sqrt(1 + b*x**2/a)) + 13*a**(3/2)*d**2*x**5/(64*sqrt(1
 + b*x**2/a)) + 3*sqrt(a)*b*c**2*x**3/(8*sqrt(1 + b*x**2/a)) + 11*sqrt(a)*b
*c*d*x**5/(12*sqrt(1 + b*x**2/a)) + 5*sqrt(a)*b*d**2*x**7/(16*sqrt(1 + b*x*
*2/a)) + 3*a**4*d**2*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) - a**3*c*d*asi
nh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + 3*a**2*c**2*asinh(sqrt(b)*x/sqrt(a))/(
8*sqrt(b)) + b**2*c**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + b**2*c*d*x**7/
(3*sqrt(a)*sqrt(1 + b*x**2/a)) + b**2*d**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/
a))
```

3.55 $\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\,dx$

Optimal. Leaf size=118

$$\frac{a^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6bc-ad)}{24b}+\frac{ax\sqrt{a+bx^2}\,(6bc-ad)}{16b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}$$

Rubi [A] time = 0.04, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {388, 195, 217, 206}

$$\frac{a^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6bc-ad)}{24b}+\frac{ax\sqrt{a+bx^2}\,(6bc-ad)}{16b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

```
[Out] (a*(6*b*c - a*d)*x*Sqrt[a + b*x^2])/(16*b) + ((6*b*c - a*d)*x*(a + b*x^2)^(
3/2))/(24*b) + (d*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*b*c - a*d)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\,dx &= \frac{dx\left(a+bx^2\right)^{5/2}}{6b}-\frac{(-6bc+ad)\int \left(a+bx^2\right)^{3/2}\,dx}{6b}\\ &= \frac{(6bc-ad)x\left(a+bx^2\right)^{3/2}}{24b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}+\frac{(a(6bc-ad))\int \sqrt{a+bx^2}\,dx}{8b}\\ &= \frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b}+\frac{(6bc-ad)x\left(a+bx^2\right)^{3/2}}{24b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}+\frac{\left(a^2(6bc\right.}{}\\ &= \frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b}+\frac{(6bc-ad)x\left(a+bx^2\right)^{3/2}}{24b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}+\frac{\left(a^2(6bc\right.}{}\\ &= \frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b}+\frac{(6bc-ad)x\left(a+bx^2\right)^{3/2}}{24b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}+\frac{a^2(6bc-}{}\end{aligned}$$

Mathematica [A] time = 0.20, size = 109, normalized size = 0.92

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(3a^2d+2ab\left(15c+7dx^2\right)+4b^2x^2\left(3c+2dx^2\right)\right)-\frac{3a^{3/2}(ad-6bc)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(3*a^2*d + 4*b^2*x^2*(3*c + 2*d*x^2) + 2*a*b*(1
5*c + 7*d*x^2)) - (3*a^(3/2)*(-6*b*c + a*d)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/S
qrt[1 + (b*x^2)/a]))/(48*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.16, size = 102, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\left(3a^2dx+30abcx+14abdx^3+12b^2cx^3+8b^2dx^5\right)}{48b}+\frac{\left(a^3d-6a^2bc\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(30*a*b*c*x + 3*a^2*d*x + 12*b^2*c*x^3 + 14*a*b*d*x^3 + 8*
b^2*d*x^5))/(48*b) + ((-6*a^2*b*c + a^3*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/(16*b^(3/2))
```

fricas [A] time = 0.93, size = 210, normalized size = 1.78

$$\left[-\frac{3\left(6a^2bc-a^3d\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8b^3dx^5+2\left(6b^3c+7ab^2d\right)x^3+3\left(10ab^2c+a^2bd\right)x\right)\sqrt{bx^2+a}}{96b^2},\ -\frac{3\left(6a^2bc-a^3d\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(8b^3dx^5+2\left(6b^3c+7ab^2d\right)x^3+3\left(10ab^2c+a^2bd\right)x\right)\sqrt{bx^2+a}}{48b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*a^2*b*c - a^3*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) - 2*(8*b^3*d*x^5 + 2*(6*b^3*c + 7*a*b^2*d)*x^3 + 3*(10*a*b^2*c +
 a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2, -1/48*(3*(6*a^2*b*c - a^3*d)*sqrt(-b)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*d*x^5 + 2*(6*b^3*c + 7*a*b^2*d)*x
^3 + 3*(10*a*b^2*c + a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.61, size = 103, normalized size = 0.87

$$\frac{1}{48}\left(2\left(4\,bdx^2+\frac{6\,b^5c+7\,ab^4d}{b^4}\right)x^2+\frac{3\left(10\,ab^4c+a^2b^3d\right)}{b^4}\right)\sqrt{bx^2+a}\,x-\frac{\left(6\,a^2bc-a^3d\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*d*x^2 + (6*b^5*c + 7*a*b^4*d)/b^4)*x^2 + 3*(10*a*b^4*c + a^2*b
^3*d)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(6*a^2*b*c - a^3*d)*log(abs(-sqrt(b)*x
+ sqrt(b*x^2 + a)))/b^(3/2)
```

maple [A] time = 0.00, size = 131, normalized size = 1.11

$$-\frac{a^3d\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{3}{2}}}+\frac{3a^2c\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}-\frac{\sqrt{b\,x^2+a}\,a^2dx}{16b}+\frac{3\sqrt{b\,x^2+a}\,acx}{8}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}adx}{24b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}cx}{4}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}dx}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c),x)
```

```
[Out] 1/6*d*x*(b*x^2+a)^(5/2)/b-1/24*d*a/b*x*(b*x^2+a)^(3/2)-1/16*d*a^2/b*x*(b*x^
2+a)^(1/2)-1/16*d*a^3/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*c*x*(b*x^2+
a)^(3/2)+3/8*c*a*x*(b*x^2+a)^(1/2)+3/8*c*a^2/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)
^(1/2))
```

maxima [A] time = 1.37, size = 116, normalized size = 0.98

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}cx+\frac{3}{8}\sqrt{bx^2+a}\,acx+\frac{\left(bx^2+a\right)^{\frac{5}{2}}dx}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}adx}{24\,b}-\frac{\sqrt{bx^2+a}\,a^2dx}{16\,b}+\frac{3\,a^2c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}-\frac{a^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*c*x + 3/8*sqrt(b*x^2 + a)*a*c*x + 1/6*(b*x^2 + a)^(5/
2)*d*x/b - 1/24*(b*x^2 + a)^(3/2)*a*d*x/b - 1/16*sqrt(b*x^2 + a)*a^2*d*x/b
+ 3/8*a^2*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/16*a^3*d*arcsinh(b*x/sqrt(a*
b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2), x)
```

sympy [B] time = 14.71, size = 253, normalized size = 2.14

$$\frac{a^{\frac{5}{2}}dx}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}cx\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}cx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{17a^{\frac{3}{2}}dx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}\,bcx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}\,bdx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{a^3d\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{3a^2c\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{b^2cx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^2dx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c),x)
```

```
[Out] a**(5/2)*d*x/(16*b*sqrt(1 + b*x**2/a)) + a**(3/2)*c*x*sqrt(1 + b*x**2/a)/2
+ a**(3/2)*c*x/(8*sqrt(1 + b*x**2/a)) + 17*a**(3/2)*d*x**3/(48*sqrt(1 + b*x
```

```
**2/a)) + 3*sqrt(a)*b*c*x**3/(8*sqrt(1 + b*x**2/a)) + 11*sqrt(a)*b*d*x**5/(
24*sqrt(1 + b*x**2/a)) - a**3*d*asinh(sqrt(b)*x/sqrt(a))/(16*b**(3/2)) + 3*
a**2*c*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + b**2*c*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a)) + b**2*d*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.56 $\int (a+bx^2)^{3/2}\, dx$

Optimal. Leaf size=65

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^{3/2} dx &= \frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{4}(3a)\int \sqrt{a+bx^2}\, dx\\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}}\, dx\\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\, dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 1.00

$$\frac{1}{8}\sqrt{a+bx^2}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}+5ax+2bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(5*a*x + 2*b*x^3 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/8
```

IntegrateAlgebraic [A] time = 0.00, size = 60, normalized size = 0.92

$$\frac{1}{8}\sqrt{a+bx^2}\left(5ax+2bx^3\right)-\frac{3a^2\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(5*a*x + 2*b*x^3))/8 - (3*a^2*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(8*Sqrt[b])
```

fricas [A] time = 0.70, size = 124, normalized size = 1.91

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,b^2x^3+5\,abx\right)\sqrt{bx^2+a}}{16\,b},-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2x^3+5\,abx\right)\sqrt{bx^2+a}}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.61, size = 49, normalized size = 0.75

$$\frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{bx^2+a}\,x-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/sqrt(b)
```

maple [A] time = 0.00, size = 51, normalized size = 0.78

$$\frac{3a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}+\frac{3\sqrt{b\,x^2+a}\,ax}{8}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(3/2)*x+3/8*(b*x^2+a)^(1/2)*a*x+3/8*a^2/b^(1/2)*ln(b^(1/2)*x+
(b*x^2+a)^(1/2))
```

maxima [A] time = 1.34, size = 43, normalized size = 0.66

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{bx^2+a}\,ax+\frac{3\,a^2\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*x + 3/8*sqrt(b*x^2 + a)*a*x + 3/8*a^2*arcsinh(b*x/sqr
t(a*b))/sqrt(b)
```

mupad [B] time = 4.71, size = 37, normalized size = 0.57

$$\frac{x\left(b\,x^2+a\right)^{3/2}\,{}_2\text{F}_1\left(-\frac{3}{2},\frac{1}{2};\,\frac{3}{2};\,-\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2),x)
```

```
[Out] (x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(3/2)
```

sympy [A] time = 2.91, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{8}+\frac{\sqrt{a}\,bx^3\sqrt{1+\frac{bx^2}{a}}}{4}+\frac{3a^2\,\text{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**2/a)/4 + 3
*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

3.57 $\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx$

Optimal. Leaf size=113

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d^2}-\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2d^2}+\frac{bx\sqrt{a+bx^2}}{2d}$$

Rubi [A] time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {416, 523, 217, 206, 377, 208}

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d^2}-\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2d^2}+\frac{bx\sqrt{a+bx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

```
[Out] (b*x*Sqrt[a + b*x^2])/(2*d) - (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(2*d^2) + ((b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/
(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{c+dx^2}\,dx = \frac{bx\sqrt{a+bx^2}}{2d} + \frac{\int \frac{-a(bc-2ad)-b(2bc-3ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2d}$$

$$= \frac{bx\sqrt{a+bx^2}}{2d} - \frac{(b(2bc-3ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{d^2}$$

$$= \frac{bx\sqrt{a+bx^2}}{2d} - \frac{(b(2bc-3ad))\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2d^2} + \frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\right.}{d^2}$$

$$= \frac{bx\sqrt{a+bx^2}}{2d} - \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2d^2} + \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d^2}$$

Mathematica [A] time = 0.20, size = 110, normalized size = 0.97

$$\frac{\sqrt{b}\,(3ad-2bc)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right) + \frac{2(ad-bc)^{3/2}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}} + bdx\sqrt{a+bx^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

```
[Out] (b*d*x*Sqrt[a + b*x^2] + (2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[-(b*c) + a*d]
*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[c] + Sqrt[b]*(-2*b*c + 3*a*d)*Log[b*x
+ Sqrt[b]*Sqrt[a + b*x^2]])/(2*d^2)
```

IntegrateAlgebraic [A] time = 0.33, size = 148, normalized size = 1.31

$$\frac{\left(2b^{3/2}c-3a\sqrt{b}\,d\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2d^2} + \frac{(bc-ad)\sqrt{ad-bc}\,\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,c+\sqrt{b}\,dx^2}{\sqrt{c}\sqrt{ad-bc}}\right)}{\sqrt{c}\,d^2} + \frac{bx\sqrt{a+bx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

```
[Out] (b*x*Sqrt[a + b*x^2])/(2*d) + ((b*c - a*d)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[
b]*c + Sqrt[b]*d*x^2 - d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/
(Sqrt[c]*d^2) + ((2*b^(3/2)*c - 3*a*Sqrt[b]*d)*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(2*d^2)
```

fricas [A] time = 1.45, size = 721, normalized size = 6.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*d*x - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(b)*x - a) - (b*c - a*d)*sqrt((b*c - a*d)/c)*log(((8*b^2*
```

```
c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 -
4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d
^2*x^4 + 2*c*d*x^2 + c^2)))/d^2, 1/4*(2*sqrt(b*x^2 + a)*b*d*x + 2*(2*b*c -
3*a*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c - a*d)*sqrt((b*c
- a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c
^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*s
qrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/d^2, 1/4*(2*sqrt(b*x^2 +
a)*b*d*x - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2
 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c
- a^2*d)*x)) - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a))/d^2, 1/2*(sqrt(b*x^2 + a)*b*d*x + (2*b*c - 3*a*d)*sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c - a*d)*sqrt(-(b*c - a*d)/c)*arctan(1
/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c -
 a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/d^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.02, size = 1845, normalized size = 16.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c),x)
```

```
[Out] 1/6/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*
b/d+(a*d-b*c)/d)^(3/2)+1/4*b/d*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-
c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+3/4/d*b^(1/2)*ln(((x-(-c*d)^(1/2)/d)
*b+(-c*d)^(1/2)*b/d)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*
d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*a+1/2/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^
2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*a-1/2/(-c*d)^(
1/2)/d*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b
*c)/d)^(1/2)*b*c-1/2/d^2*b^(3/2)*ln(((x-(-c*d)^(1/2)/d)*b+(-c*d)^(1/2)*b/d)
/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d
-b*c)/d)^(1/2))*c-1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(
x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/
d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^
(1/2)/d))*a^2+1/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-
c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2
*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2
)/d))*a*b*c-1/2/(-c*d)^(1/2)/d^2/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-
(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)
^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1
/2)/d))*b^2*c^2-1/6/(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+
(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)+1/4*b/d*((x+(-c*d)^(1/2)/d)^2*b-2*(-
c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+3/4/d*b^(1/2)*ln(((x
+(-c*d)^(1/2)/d)*b-(-c*d)^(1/2)*b/d)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*(-c*
d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*a-1/2/(-c*d)^(1/2)*((x+
(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2
)*a+1/2/(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/
2)/d)*b/d+(a*d-b*c)/d)^(1/2)*b*c-1/2/d^2*b^(3/2)*ln(((x+(-c*d)^(1/2)/d)*b-(
-c*d)^(1/2)*b/d)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(
1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*c+1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((
```

```
-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*
((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^
(1/2))/(x+(-c*d)^(1/2)/d))*a^2-1/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((-2*
(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x
+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/
2))/(x+(-c*d)^(1/2)/d))*a*b*c+1/2/(-c*d)^(1/2)/d^2/((a*d-b*c)/d)^(1/2)*ln((
-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*
((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^
(1/2))/(x+(-c*d)^(1/2)/d))*b^2*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2), x)
```

3.58 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2} dx$

Optimal. Leaf size=131

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}-\frac{x\sqrt{a+bx^2}\,(bc-ad)}{2cd\left(c+dx^2\right)}$$

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {413, 523, 217, 206, 377, 208}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}-\frac{x\sqrt{a+bx^2}\,(bc-ad)}{2cd\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d*(c + d*x^2)) + (b^(3/2)*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/d^2 - (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sq
rt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*d^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^2}\,dx &= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{\int \frac{a(bc+ad)+2b^2cx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2cd}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{b^2\int \frac{1}{\sqrt{a+bx^2}}\,dx}{d^2}-\frac{((bc-ad)(2bc+ad))\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2cd^2}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{((bc-ad)(2bc+ad))\,\text{Subst}\left(\int \frac{\ }{c}\right.}{2cd^2}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{bc-ad}\,(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 142, normalized size = 1.08

$$\frac{\frac{\left(a^2d^2+abcd-2b^2c^2\right)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{c^{3/2}\sqrt{ad-bc}}+2b^{3/2}\log\left(\sqrt{b}\,\sqrt{a+bx^2}+bx\right)-\frac{dx\sqrt{a+bx^2}\,(bc-ad)}{c\left(c+dx^2\right)}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

```
[Out] (-((d*(b*c - a*d)*x*Sqrt[a + b*x^2])/(c*(c + d*x^2))) + ((-2*b^2*c^2 + a*b*
c*d + a^2*d^2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(c
^(3/2)*Sqrt[-(b*c) + a*d]) + 2*b^(3/2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/
(2*d^2)
```

IntegrateAlgebraic [A] time = 0.62, size = 155, normalized size = 1.18

$$-\frac{b^{3/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{d^2}-\frac{\sqrt{ad-bc}\,(ad+2bc)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,c+\sqrt{b}\,dx^2}{\sqrt{c}\,\sqrt{ad-bc}}\right)}{2c^{3/2}d^2}+\frac{x\sqrt{a+bx^2}\,(ad-bc)}{2cd\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

```
[Out] ((-(b*c) + a*d)*x*Sqrt[a + b*x^2])/(2*c*d*(c + d*x^2)) - (Sqrt[-(b*c) + a*d
]*(2*b*c + a*d)*ArcTan[(Sqrt[b]*c + Sqrt[b]*d*x^2 - d*x*Sqrt[a + b*x^2])/(S
qrt[c]*Sqrt[-(b*c) + a*d])])/(2*c^(3/2)*d^2) - (b^(3/2)*Log[-(Sqrt[b]*x) +
Sqrt[a + b*x^2]])/d^2
```

fricas [A] time = 1.36, size = 907, normalized size = 6.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x - 4*(b*c*d*x^2 + b*c^2)*sqrt(b)*
log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*b*c^2 + a*c*d + (2*b*c
*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)
*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*
c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2))
)/(c*d^3*x^2 + c^2*d^2), -1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x + 8*(b*c
*d*x^2 + b*c^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b*c^2 + a*
c*d + (2*b*c*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*
d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (
2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d
*x^2 + c^2)))/(c*d^3*x^2 + c^2*d^2), -1/4*(2*(b*c*d - a*d^2)*sqrt(b*x^2 + a
)*x - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt(-(b*c - a*d)/c)*arctan
(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c
 - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) - 2*(b*c*d*x^2 + b*c^2)*sqrt(b)*log(-2*
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(c*d^3*x^2 + c^2*d^2), -1/4*(2*(b
*c*d - a*d^2)*sqrt(b*x^2 + a)*x + 4*(b*c*d*x^2 + b*c^2)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt(-
(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(
b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/(c*d^3*x^2 + c^2*
d^2)]
```

giac [B] time = 0.68, size = 317, normalized size = 2.42

$$-\frac{b^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\,d^2}+\frac{\left(2\,b^{\frac{5}{2}}c^2-ab^{\frac{3}{2}}cd-a^2\sqrt{b}\,d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}\,cd^2}-\frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{5}{2}}c^2-3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}cd+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^2\sqrt{b}\,d^2+a^2b^{\frac{3}{2}}cd-a^3\sqrt{b}\,d^2}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^2 + 1/2*(2*b^(5/2)*c^2
- a*b^(3/2)*c*d - a^2*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a)
)^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c*
d^2) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*c^2 - 3*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a*b^(3/2)*c*d + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*sqrt(b)*
d^2 + a^2*b^(3/2)*c*d - a^3*sqrt(b)*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*
d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*a*d + a^2*d)*c*d^2)
```

maple [B] time = 0.02, size = 4621, normalized size = 35.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^2,x)
```

```
[Out] -3/8/c*b/(a*d-b*c)*a*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)
/d)*b/d+(a*d-b*c)/d)^(1/2)*x+1/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(
-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-
(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2
))/(x-(-c*d)^(1/2)/d))*a*b-1/4/(-c*d)^(1/2)*c/d^2/((a*d-b*c)/d)^(1/2)*ln((2
*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((
x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1
/2))/(x-(-c*d)^(1/2)/d))*b^2-1/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)*((x-(-c*d)^(1
/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-3/8/c*b
/(a*d-b*c)*a*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+
(a*d-b*c)/d)^(1/2)*x-1/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(
1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^
```

```
(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(
-c*d)^(1/2)/d))*a*b+1/4/(-c*d)^(1/2)*c/d^2/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d
)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*
d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(
x+(-c*d)^(1/2)/d))*b^2+1/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)*((x+(-c*d)^(1/2)/d)
^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)+1/4/c/(a*d-b*
c)/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2
)/d)*b/d+(a*d-b*c)/d)^(5/2)-1/4/(-c*d)^(1/2)/c*((x+(-c*d)^(1/2)/d)^2*b-2*(-
c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*a+1/4/(-c*d)^(1/2)/d*(
(x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(
1/2)*b+1/4/(-c*d)^(1/2)/c*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^
(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*a-1/4/(-c*d)^(1/2)/d*((x-(-c*d)^(1/2)/d)^2*
b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*b+1/4/c/(a*d-b*c
)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)
/d)*b/d+(a*d-b*c)/d)^(5/2)-3/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)/((a*d-b*c)/d)^(
1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/
d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d
-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a^2+3/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)/((
a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2
*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)
/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a^2+3/8/d*b^2/(a*d-b*c)*((x
-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/
2)*x+9/8/d*b^(3/2)/(a*d-b*c)*ln(((x-(-c*d)^(1/2)/d)*b+(-c*d)^(1/2)*b/d)/b^(
1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c
)/d)^(1/2))*a-1/4/d^2*b^(3/2)*ln(((x+(-c*d)^(1/2)/d)*b-(-c*d)^(1/2)*b/d)/b^
(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*
c)/d)^(1/2))+1/12/(-c*d)^(1/2)/c*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-
(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-1/4/d^2*b^(3/2)*ln(((x-(-c*d)^(1/2)/
d)*b+(-c*d)^(1/2)*b/d)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-
c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))-1/12/(-c*d)^(1/2)/c*((x+(-c*d)^(1/2)/
d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-3/4*c/d^2*b
^(5/2)/(a*d-b*c)*ln(((x+(-c*d)^(1/2)/d)*b-(-c*d)^(1/2)*b/d)/b^(1/2)+((x+(-c
*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))-
1/4/c*b/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)
*b/d+(a*d-b*c)/d)^(3/2)*x-3/8/c*b^(1/2)/(a*d-b*c)*a^2*ln(((x+(-c*d)^(1/2)/d
)*b-(-c*d)^(1/2)*b/d)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c
*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))+3/8/d*b^2/(a*d-b*c)*((x+(-c*d)^(1/2)/d
)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-1/4/c*b/(a
*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-
b*c)/d)^(3/2)*x-3/8/c*b^(1/2)/(a*d-b*c)*a^2*ln(((x-(-c*d)^(1/2)/d)*b+(-c*d)
^(1/2)*b/d)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/
d)*b/d+(a*d-b*c)/d)^(1/2))+1/8/c*b/d*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)
*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+3/8/c/d*b^(1/2)*ln(((x+(-c*d)^
(1/2)/d)*b-(-c*d)^(1/2)*b/d)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)
*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*a+1/4/(-c*d)^(1/2)/c/((a*d-b*c)
/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-
b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d
+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a^2+1/8/c*b/d*((x-(-c*d)^(1/2)/d)^
2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+3/8/c/d*b^(1
/2)*ln(((x-(-c*d)^(1/2)/d)*b+(-c*d)^(1/2)*b/d)/b^(1/2)+((x-(-c*d)^(1/2)/d)^
2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*a-1/4/(-c*d)^
(1/2)/c/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*
d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-
c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a^2+9/8/d*b^(3/2)
/(a*d-b*c)*ln(((x+(-c*d)^(1/2)/d)*b-(-c*d)^(1/2)*b/d)/b^(1/2)+((x+(-c*d)^(1
/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*a-3/4/
d^2*(-c*d)^(1/2)*b^2/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-
c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+3/4/d^2*(-c*d)^(1/2)*b^2/(a*d-b*c)*((x
-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/
```

```
2)-3/4*c/d^2*b^(5/2)/(a*d-b*c)*ln(((x-(-c*d)^(1/2)/d)*b+(-c*d)^(1/2)*b/d)/b
^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b
*c)/d)^(1/2))-3/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2*(-
c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*a-3/2/d^2*(-c*d)^(1/2)
*b^2/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/
d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2
)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a+3/4*c/d^
3*(-c*d)^(1/2)*b^3/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*
d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b
+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/
d))+3/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)
*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*a+3/2/d^2*(-c*d)^(1/2)*b^2/(a*d-
b*c)/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-
b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*
d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a-3/4*c/d^3*(-c*d)^
(1/2)*b^3/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)
/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d
)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{\left(c+dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**2, x)
```

3.59 $\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^3} \, dx$

Optimal. Leaf size=113

$$\frac{3a^2 \tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}+\frac{3ax\sqrt{a+bx^2}}{8c^2\left(c+dx^2\right)}+\frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2}$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {378, 377, 208}

$$\frac{3a^2 \tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}+\frac{3ax\sqrt{a+bx^2}}{8c^2\left(c+dx^2\right)}+\frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2)^(3/2))/(4*c*(c + d*x^2)^2) + (3*a*x*Sqrt[a + b*x^2])/(8*c^2*
(c + d*x^2)) + (3*a^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])
])/(8*c^(5/2)*Sqrt[b*c - a*d])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^3}\,dx = \frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2} + \frac{(3a)\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx}{4c}$$

$$= \frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2} + \frac{3ax\sqrt{a+bx^2}}{8c^2\left(c+dx^2\right)} + \frac{\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8c^2}$$

$$= \frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2} + \frac{3ax\sqrt{a+bx^2}}{8c^2\left(c+dx^2\right)} + \frac{\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8c^2}$$

$$= \frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2} + \frac{3ax\sqrt{a+bx^2}}{8c^2\left(c+dx^2\right)} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.69, size = 163, normalized size = 1.44

$$\frac{x\sqrt{a+bx^2}\left(\frac{\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\left(5ac+3adx^2+2bcx^2\right)}{\left(c+dx^2\right)\sqrt{\frac{dx^2}{c}+1}} + \frac{3a\sin^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{\sqrt{\frac{x^2(ad-bc)}{ac}}}\right)}{8c^3\sqrt{\frac{bx^2}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*(5*a*c + 2*b*c*x
^2 + 3*a*d*x^2))/((c + d*x^2)*Sqrt[1 + (d*x^2)/c]) + (3*a*ArcSin[Sqrt[(-(b/
a) + d/c)*x^2]/Sqrt[1 + (d*x^2)/c]])/Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]))/(8*
c^3*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [B] time = 3.19, size = 1323, normalized size = 11.71

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

```
[Out] (3*a^2)/(8*c^2*d*x*(-(Sqrt[b]*x) + Sqrt[a + b*x^2])) + (-1/8*a^4/(c*d^2) -
(11*a^3*b*x^2)/(8*c*d^2) - (3*a^2*b^2*x^4)/(2*c*d^2) + (5*a^3*Sqrt[b]*x*Sqr
t[a + b*x^2])/(8*c*d^2) + (3*a^2*b^(3/2)*x^3*Sqrt[a + b*x^2])/(2*c*d^2))/(x
^3*(-(Sqrt[b]*x) + Sqrt[a + b*x^2])*(a^2 + 8*a*b*x^2 + 8*b^2*x^4 - 4*a*Sqrt
[b]*x*Sqrt[a + b*x^2] - 8*b^(3/2)*x^3*Sqrt[a + b*x^2])) + (-1/4*a^4/d - (7*
a^3*b*x^2)/(2*d) - (41*a^2*b^2*x^4)/(4*d) - (11*a*b^3*x^6)/d - (4*b^4*x^8)/
d + (5*a^3*Sqrt[b]*x*Sqrt[a + b*x^2])/(4*d) + (25*a^2*b^(3/2)*x^3*Sqrt[a +
b*x^2])/(4*d) + (9*a*b^(5/2)*x^5*Sqrt[a + b*x^2])/d + (4*b^(7/2)*x^7*Sqrt[a
 + b*x^2])/d)/(x*(c + d*x^2)^2*(-(Sqrt[b]*x) + Sqrt[a + b*x^2])*(a + 2*b*x^
2 - 2*Sqrt[b]*x*Sqrt[a + b*x^2])^2) + (a^5/(8*d^2) + (23*a^4*b*x^2)/(8*d^2)
 + (27*a^3*b^2*x^4)/(4*d^2) - (8*a^2*b^3*x^6)/d^2 - (28*a*b^4*x^8)/d^2 - (1
6*b^5*x^10)/d^2 - (7*a^4*Sqrt[b]*x*Sqrt[a + b*x^2])/(8*d^2) - (21*a^3*b^(3/
2)*x^3*Sqrt[a + b*x^2])/(4*d^2) + (20*a*b^(7/2)*x^7*Sqrt[a + b*x^2])/d^2 +
(16*b^(9/2)*x^9*Sqrt[a + b*x^2])/d^2)/(x^3*(c + d*x^2)*(-(Sqrt[b]*x) + Sqrt
```

```
[a + b*x^2])*(a + 2*b*x^2 - 2*Sqrt[b]*x*Sqrt[a + b*x^2])*(a^2 + 8*a*b*x^2 +
 8*b^2*x^4 - 4*a*Sqrt[b]*x*Sqrt[a + b*x^2] - 8*b^(3/2)*x^3*Sqrt[a + b*x^2])
) - (3*a^2*ArcTan[(Sqrt[b]*Sqrt[c])/Sqrt[-(b*c) + a*d] + (Sqrt[b]*d*x^2)/(S
qrt[c]*Sqrt[-(b*c) + a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a
*d])])/(8*c^(5/2)*Sqrt[-(b*c) + a*d]) - (4*b^2*ArcTan[(Sqrt[b]*Sqrt[c])/Sqr
t[-(b*c) + a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[-(b*c) + a*d]) - (d*x*Sqrt[
a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*d^2*Sqrt[-(b*c) + a*d])
 + (3*a*b*ArcTan[(Sqrt[b]*Sqrt[c])/Sqrt[-(b*c) + a*d] + (Sqrt[b]*d*x^2)/(Sq
rt[c]*Sqrt[-(b*c) + a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a*
d])])/(c^(3/2)*d*Sqrt[-(b*c) + a*d]) - (4*b^2*ArcTanh[(Sqrt[b]*Sqrt[c])/Sqr
t[b*c - a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[b*c - a*d]) - (d*x*Sqrt[a + b*
x^2])/(Sqrt[c]*Sqrt[b*c - a*d])])/(Sqrt[c]*d^2*Sqrt[b*c - a*d]) + (3*a*b*Ar
cTanh[(Sqrt[b]*Sqrt[c])/Sqrt[b*c - a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[b*c
 - a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])])/(c^(3/2)*d*Sqr
t[b*c - a*d])
```

fricas [B] time = 1.21, size = 526, normalized size = 4.65

$$\left[\frac{3\left(a^{2}d^{2}x^{4}+2\,a^{2}cdx^{2}+a^{2}c^{2}\right)\sqrt{bc^{2}-acd}\,\log\left(\frac{\left(8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}\right)x^{4}+a^{2}c^{2}+2\left(4\,abc^{2}-3\,a^{2}cd\right)x^{2}+4\left(\left(2\,bc-ad\right)x^{3}+acx\right)\sqrt{bc^{2}-acd}\,\sqrt{bx^{2}+a}}{d^{2}x^{4}+2\,cdx^{2}+c^{2}}\right)+4\left(\left(2\,b^{2}c^{3}+abc^{2}d-3\,a^{2}cd^{2}\right)x^{3}+5\left(abc^{3}-a^{2}c^{2}d\right)x\right)\sqrt{bx^{2}+a}}{32\left(bc^{6}-ac^{5}d+\left(bc^{4}d^{2}-ac^{3}d^{3}\right)x^{4}+2\left(bc^{5}d-ac^{4}d^{2}\right)x^{2}\right)},-\frac{3\left(a^{2}d^{2}x^{4}+2\,a^{2}cdx^{2}+a^{2}c^{2}\right)\sqrt{-bc^{2}+acd}\,\arctan\left(\frac{\sqrt{-bc^{2}+acd}\left(\left(2\,bc-ad\right)x^{2}+ac\right)\sqrt{bx^{2}+a}}{2\left(\left(b^{2}c^{2}-abcd\right)x^{3}+\left(abc^{2}-a^{2}cd\right)x\right)}\right)-2\left(\left(2\,b^{2}c^{3}+abc^{2}d-3\,a^{2}cd^{2}\right)x^{3}+5\left(abc^{3}-a^{2}c^{2}d\right)x\right)\sqrt{bx^{2}+a}}{16\left(bc^{6}-ac^{5}d+\left(bc^{4}d^{2}-ac^{3}d^{3}\right)x^{4}+2\left(bc^{5}d-ac^{4}d^{2}\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a^2*d^2*x^4 + 2*a^2*c*d*x^2 + a^2*c^2)*sqrt(b*c^2 - a*c*d)*log(((
8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*
x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d
^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3 +
 5*(a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(b*c^6 - a*c^5*d + (b*c^4*d^2
- a*c^3*d^3)*x^4 + 2*(b*c^5*d - a*c^4*d^2)*x^2), -1/16*(3*(a^2*d^2*x^4 + 2*
a^2*c*d*x^2 + a^2*c^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)
*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*
c^2 - a^2*c*d)*x)) - 2*((2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3 + 5*(a*b*
c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(b*c^6 - a*c^5*d + (b*c^4*d^2 - a*c^3*
d^3)*x^4 + 2*(b*c^5*d - a*c^4*d^2)*x^2)]
```

giac [B] time = 3.72, size = 451, normalized size = 3.99

$$-\frac{3\,a^{2}\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}d+2\,bc-ad}{2\sqrt{-b^{2}c^{2}+abcd}}\right)}{8\sqrt{-b^{2}c^{2}+abcd}\,c^{2}}+\frac{8\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}b^{\frac{5}{2}}c^{2}d-3\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}a^{2}\sqrt{b}d^{3}+16\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}b^{\frac{7}{2}}c^{3}+8\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}ab^{\frac{5}{2}}c^{2}d-18\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}a^{2}b^{\frac{3}{2}}cd^{2}+9\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}a^{3}\sqrt{b}d^{3}+8\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}a^{2}b^{\frac{5}{2}}c^{2}d+16\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}a^{3}b^{\frac{3}{2}}cd^{2}-9\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}a^{4}\sqrt{b}d^{3}+2\,a^{4}b^{\frac{3}{2}}cd^{2}+3\,a^{5}\sqrt{b}d^{3}}{4\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}d+4\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}bc-2\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}ad+a^{2}d\right)^{2}c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -3/8*a^2*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*
d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c^2) + 1/4*(8*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*a^2*sqrt(b)*d^3 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3 + 8*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d - 18*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*a^2*b^(3/2)*c*d^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^3
+ 8*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*d + 16*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a^3*b^(3/2)*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*s
qrt(b)*d^3 + 2*a^4*b^(3/2)*c*d^2 + 3*a^5*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b
*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a*d + a^2*d)^2*c^2*d^2)
```

maple [B] time = 0.02, size = 9059, normalized size = 80.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{3}{2}}}{\left(dx^2 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2 + a\right)^{3/2}}{\left(d\,x^2 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^{\frac{3}{2}}}{\left(c + dx^2\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**3, x)
```

3.60 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4} dx$

Optimal. Leaf size=199

$$\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{3/2}}+\frac{ax\sqrt{a+bx^2}(6bc-5ad)}{16c^3(c+dx^2)(bc-ad)}+\frac{x(a+bx^2)^{3/2}(6bc-5ad)}{24c^2(c+dx^2)^2(bc-ad)}-\frac{dx(a+bx^2)^{5/2}}{6c(c+dx^2)^3(bc-ad)}$$

Rubi [A] time = 0.12, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {382, 378, 377, 208}

$$\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{3/2}}+\frac{x(a+bx^2)^{3/2}(6bc-5ad)}{24c^2(c+dx^2)^2(bc-ad)}+\frac{ax\sqrt{a+bx^2}(6bc-5ad)}{16c^3(c+dx^2)(bc-ad)}-\frac{dx(a+bx^2)^{5/2}}{6c(c+dx^2)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```

```
[Out] -(d*x*(a + b*x^2)^(5/2))/(6*c*(b*c - a*d)*(c + d*x^2)^3) + ((6*b*c - 5*a*d)
*x*(a + b*x^2)^(3/2))/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + (a*(6*b*c - 5*a*
d)*x*Sqrt[a + b*x^2])/(16*c^3*(b*c - a*d)*(c + d*x^2)) + (a^2*(6*b*c - 5*a*
d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*(b*c
 - a*d)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^4}\,dx = -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3} + \frac{(6bc-5ad)\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^3}dx}{6c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3} + \frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(a(6bc-5ad))\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}dx}{8c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3} + \frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)\left(c+dx^2\right)} + \frac{\left(a^2(6b\right.}{}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3} + \frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)\left(c+dx^2\right)} + \frac{\left(a^2(6b\right.}{}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3} + \frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)\left(c+dx^2\right)} + \frac{a^2(6bc}{}$$

Mathematica [A] time = 0.82, size = 247, normalized size = 1.24

$$\frac{ax\left(\frac{bx^2}{a}+1\right)\left(\frac{3a^2\left(c+dx^2\right)^3(5ad-6bc)\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)}{\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}}+c\left(a^3d\left(33c^2+40cdx^2+15d^2x^4\right)+a^2b\left(-30c^3+11c^2dx^2+32cd^2x^4+15d^3x^6\right)-2ab^2cx^2\left(21c^2+13cdx^2+4d^2x^4\right)-4b^3c^2x^4\left(3c+dx^2\right)\right)\right)}{48c^4\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```

```
[Out] (a*x*(1 + (b*x^2)/a)*(c*(-4*b^3*c^2*x^4*(3*c + d*x^2) - 2*a*b^2*c*x^2*(21*c
^2 + 13*c*d*x^2 + 4*d^2*x^4) + a^3*d*(33*c^2 + 40*c*d*x^2 + 15*d^2*x^4) + a
^2*b*(-30*c^3 + 11*c^2*d*x^2 + 32*c*d^2*x^4 + 15*d^3*x^6)) + (3*a^2*(-6*b*c
 + 5*a*d)*(c + d*x^2)^3*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/S
qrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(48*c^4*(-(b*c) + a*d)*(a + b*x^2)
^(3/2)*(c + d*x^2)^3)
```

IntegrateAlgebraic [B] time = 43.50, size = 1786, normalized size = 8.97

result too large to display

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```

```
[Out] (-33*a^6*c^2*d^3*x - 40*a^6*c*d^4*x^3 - 15*a^6*d^5*x^5 + Sqrt[b]*Sqrt[a + b
*x^2]*(-15*a^5*c^3*d^2 + 153*a^5*c^2*d^3*x^2 + 195*a^5*c*d^4*x^4 + 75*a^5*d
^5*x^6) + b*(120*a^5*c^3*d^2*x - 335*a^5*c^2*d^3*x^3 - 482*a^5*c*d^4*x^5 -
195*a^5*d^5*x^7) + b^(11/2)*Sqrt[a + b*x^2]*(128*c^5*x^6 + 384*c^4*d*x^8) +
 b^(9/2)*Sqrt[a + b*x^2]*(192*a*c^5*x^4 + 832*a*c^4*d*x^6 - 384*a*c^3*d^2*x
^8) + b^(3/2)*Sqrt[a + b*x^2]*(8*a^4*c^4*d - 426*a^4*c^3*d^2*x^2 + 336*a^4*
c^2*d^3*x^4 + 670*a^4*c*d^4*x^6 + 300*a^4*d^5*x^8) + b^6*(-128*c^5*x^7 - 38
4*c^4*d*x^9) + b^5*(-256*a*c^5*x^5 - 1024*a*c^4*d*x^7 + 384*a*c^3*d^2*x^9)
+ b^2*(-48*a^4*c^4*d*x + 1008*a^4*c^3*d^2*x^3 - 190*a^4*c^2*d^3*x^5 - 826*a
^4*c*d^4*x^7 - 420*a^4*d^5*x^9) + b^(7/2)*Sqrt[a + b*x^2]*(72*a^2*c^5*x^2 +
 600*a^2*c^4*d*x^4 - 1488*a^2*c^3*d^2*x^6 - 720*a^2*c^2*d^3*x^8 - 288*a^2*c
*d^4*x^10) + b^(5/2)*Sqrt[a + b*x^2]*(4*a^3*c^5 + 156*a^3*c^4*d*x^2 - 1488*
a^3*c^3*d^2*x^4 - 472*a^3*c^2*d^3*x^6 + 240*a^3*c*d^4*x^8 + 240*a^3*d^5*x^1
```

```
0) + b^4*(-152*a^2*c^5*x^3 - 968*a^2*c^4*d*x^5 + 1680*a^2*c^3*d^2*x^7 + 720
*a^2*c^2*d^3*x^9 + 288*a^2*c*d^4*x^11) + b^3*(-24*a^3*c^5*x - 376*a^3*c^4*d
*x^3 + 2184*a^3*c^3*d^2*x^5 + 832*a^3*c^2*d^3*x^7 - 96*a^3*c*d^4*x^9 - 240*
a^3*d^5*x^11))/(288*a^4*Sqrt[b]*c^3*d^3*x*(c + d*x^2)^3 - 1536*b^(9/2)*c^4*
d^2*x^7*(c + d*x^2)^3 - 48*a^4*c^3*d^3*Sqrt[a + b*x^2]*(c + d*x^2)^3 + 1536
*b^4*c^4*d^2*x^6*Sqrt[a + b*x^2]*(c + d*x^2)^3 + 48*b*c^3*d^2*Sqrt[a + b*x^
2]*(c + d*x^2)^3*(a^3*c - 18*a^3*d*x^2) + 48*b^(3/2)*c^3*d^2*(c + d*x^2)^3*
(-6*a^3*c*x + 38*a^3*d*x^3) + 48*b^2*c^3*d^2*Sqrt[a + b*x^2]*(c + d*x^2)^3*
(18*a^2*c*x^2 - 48*a^2*d*x^4) + 48*b^(5/2)*c^3*d^2*(c + d*x^2)^3*(-38*a^2*c
*x^3 + 64*a^2*d*x^5) + 48*b^3*c^3*d^2*Sqrt[a + b*x^2]*(c + d*x^2)^3*(48*a*c
*x^4 - 32*a*d*x^6) + 48*b^(7/2)*c^3*d^2*(c + d*x^2)^3*(-64*a*c*x^5 + 32*a*d
*x^7)) + ((-21*a^2*b^2)/(4*c^(3/2)*(b*c - a*d)^(5/2)) - (39*a*b^3)/(2*Sqrt[
c]*d*(b*c - a*d)^(5/2)))*ArcTanh[(Sqrt[b]*c + Sqrt[b]*d*x^2 - d*x*Sqrt[a +
b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])] + ((15*b^4*Sqrt[c])/(d^2*(b*c - a*d)^(5/
2)) + (39*a*b^3)/(2*Sqrt[c]*d*(b*c - a*d)^(5/2)))*ArcTanh[(Sqrt[b]*c + Sqrt
[b]*d*x^2 - d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])] - (15*b^4*Sqrt[
c]*ArcTanh[(Sqrt[b]*Sqrt[c])/Sqrt[b*c - a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqr
t[b*c - a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])]])/(d^2*(b*c
 - a*d)^(5/2)) + (21*a^3*b*d*ArcTanh[(Sqrt[b]*Sqrt[c])/Sqrt[b*c - a*d] + (S
qrt[b]*d*x^2)/(Sqrt[c]*Sqrt[b*c - a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sq
rt[b*c - a*d])]])/(4*c^(5/2)*(b*c - a*d)^(5/2)) + (45*a^2*b*ArcTanh[(Sqrt[b]
*Sqrt[c])/Sqrt[b*c - a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[b*c - a*d]) - (d*
x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])]])/(8*c^(5/2)*(b*c - a*d)^(3/2)
) - (5*a^3*d*ArcTanh[(Sqrt[b]*Sqrt[c])/Sqrt[b*c - a*d] + (Sqrt[b]*d*x^2)/(S
qrt[c]*Sqrt[b*c - a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])]])
/(16*c^(7/2)*(b*c - a*d)^(3/2))
```

fricas [B] time = 1.89, size = 972, normalized size = 4.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(6*a^2*b*c^4 - 5*a^3*c^3*d + (6*a^2*b*c*d^3 - 5*a^3*d^4)*x^6 + 3*
(6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2*b*c^3*d - 5*a^3*c^2*d^2)*x^2
)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2
+ 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2
- a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((4*b^3*c^4*d +
4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^5 + 2*(6*b^3*c^5 + 5*a
*b^2*c^4*d - 31*a^2*b*c^3*d^2 + 20*a^3*c^2*d^3)*x^3 + 3*(10*a*b^2*c^5 - 21*
a^2*b*c^4*d + 11*a^3*c^3*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^9 - 2*a*b*c^8*d +
a^2*c^7*d^2 + (b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x^6 + 3*(b^2*c^7*
d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4)*x^4 + 3*(b^2*c^8*d - 2*a*b*c^7*d^2 + a^2
*c^6*d^3)*x^2), -1/96*(3*(6*a^2*b*c^4 - 5*a^3*c^3*d + (6*a^2*b*c*d^3 - 5*a^
3*d^4)*x^6 + 3*(6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2*b*c^3*d - 5*a
^3*c^2*d^2)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b
*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 -
a^2*c*d)*x)) - 2*((4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^
3*c*d^4)*x^5 + 2*(6*b^3*c^5 + 5*a*b^2*c^4*d - 31*a^2*b*c^3*d^2 + 20*a^3*c^2
*d^3)*x^3 + 3*(10*a*b^2*c^5 - 21*a^2*b*c^4*d + 11*a^3*c^3*d^2)*x)*sqrt(b*x^
2 + a))/(b^2*c^9 - 2*a*b*c^8*d + a^2*c^7*d^2 + (b^2*c^6*d^3 - 2*a*b*c^5*d^4
 + a^2*c^4*d^5)*x^6 + 3*(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4)*x^4 + 3
*(b^2*c^8*d - 2*a*b*c^7*d^2 + a^2*c^6*d^3)*x^2)]
```

giac [B] time = 2.89, size = 919, normalized size = 4.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -1/16*(6*a^2*b^(3/2)*c - 5*a^3*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x
^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^4 - a*c^3*d)*sq
rt(-b^2*c^2 + a*b*c*d)) - 1/24*(18*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(
3/2)*c*d^4 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*d^5 - 96*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c^4*d + 96*(sqrt(b)*x - sqrt(b*x^2 + a))
^8*a*b^(7/2)*c^3*d^2 + 180*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c^2*
d^3 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 + 75*(sqrt(b)*x
 - sqrt(b*x^2 + a))^8*a^4*sqrt(b)*d^5 - 128*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*b^(11/2)*c^5 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d + 720*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2)*c^3*d^2 - 968*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a^3*b^(5/2)*c^2*d^3 + 620*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*
b^(3/2)*c*d^4 - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 - 96*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(9/2)*c^4*d - 288*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*a^3*b^(7/2)*c^3*d^2 + 864*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^
(5/2)*c^2*d^3 - 600*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 + 150
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*sqrt(b)*d^5 - 48*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*a^4*b^(7/2)*c^3*d^2 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(
5/2)*c^2*d^3 + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 - 75*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*sqrt(b)*d^5 - 4*a^6*b^(5/2)*c^2*d^3 - 8*
a^7*b^(3/2)*c*d^4 + 15*a^8*sqrt(b)*d^5)/((b*c^4*d^2 - a*c^3*d^3)*((sqrt(b)*
x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3)
```

maple [B] time = 0.03, size = 13766, normalized size = 69.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^4,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{\left(d\,x^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^4,x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

3.61 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5} \, dx$

Optimal. Leaf size=300

$$\frac{a^2(35a^2d^2-80abcd+48b^2c^2)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}\,(-35a^2d^2+24abcd+8b^2c^2)}{192c^3d\left(c+dx^2\right)^2(bc-ad)}+\frac{x\sqrt{a+bx^2}\,(10\ldots}{3\ldots}$$

Rubi [A] time = 0.37, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {413, 527, 12, 377, 208}

$$\frac{x\sqrt{a+bx^2}\,(-170a^2bcd^2+105a^3d^3+40ab^2c^2d+16b^3c^3)}{384c^4d\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}\,(-35a^2d^2+24abcd+8b^2c^2)}{192c^3d\left(c+dx^2\right)^2(bc-ad)}+\frac{a^2(35a^2d^2-80abcd+48b^2c^2)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}\,(7ad+2bc)}{48c^2d\left(c+dx^2\right)^3}-\frac{x\sqrt{a+bx^2}\,(bc-ad)}{8cd\left(c+dx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

```
[Out] -((b*c - a*d)*x*Sqrt[a + b*x^2])/(8*c*d*(c + d*x^2)^4) + ((2*b*c + 7*a*d)*x
*Sqrt[a + b*x^2])/(48*c^2*d*(c + d*x^2)^3) + ((8*b^2*c^2 + 24*a*b*c*d - 35*
a^2*d^2)*x*Sqrt[a + b*x^2])/(192*c^3*d*(b*c - a*d)*(c + d*x^2)^2) + ((16*b^
3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sqrt[a + b*x^2])/
(384*c^4*d*(b*c - a*d)^2*(c + d*x^2)) + (a^2*(48*b^2*c^2 - 80*a*b*c*d + 35*
a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128*c^(9/
2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^5}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{\int \frac{a(bc+7ad)+2b(bc+3ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^4}\,dx}{8cd}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}+\frac{\int \frac{a(bc-ad)(4bc+35ad)+4b(bc-ad)(2bc+7ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx}{48c^2d(bc-ad)}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}+\frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2}+$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}+\frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2}+$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}+\frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2}+$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}+\frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2}+$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}+\frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2}+$$

Mathematica [A] time = 1.38, size = 362, normalized size = 1.21

$$\frac{ax\left(\frac{bx^2}{a}+1\right)\left(\frac{3a^2\left(c+dx^2\right)^4\left(35a^2d^2-80abcd+48b^2c^2\right)\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)}{\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}}+c\left(a^4d^2\left(279c^3+511c^2dx^2+385cd^2x^4+105d^3x^6\right)+a^3bd\left(-528c^4-563c^3dx^2-117c^2d^2x^4+215cd^3x^6+105d^4x^8\right)+2a^2b^2c\left(120c^4-160c^3dx^2-345c^2d^2x^4-294cd^3x^6-85d^4x^8\right)+8ab^3c^2x^2\left(42c^3+34c^2dx^2+21cd^2x^4+5d^3x^6\right)+16b^4c^3x^4\left(6c^2+4cdx^2+d^2x^4\right)\right)\right)}{384c^5\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^4(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

```
[Out] (a*x*(1 + (b*x^2)/a)*(c*(16*b^4*c^3*x^4*(6*c^2 + 4*c*d*x^2 + d^2*x^4) + 8*a
*b^3*c^2*x^2*(42*c^3 + 34*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^6) + a^4*d^2*(
279*c^3 + 511*c^2*d*x^2 + 385*c*d^2*x^4 + 105*d^3*x^6) + 2*a^2*b^2*c*(120*c
^4 - 160*c^3*d*x^2 - 345*c^2*d^2*x^4 - 294*c*d^3*x^6 - 85*d^4*x^8) + a^3*b*
d*(-528*c^4 - 563*c^3*d*x^2 - 117*c^2*d^2*x^4 + 215*c*d^3*x^6 + 105*d^4*x^8
)) + (3*a^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*(c + d*x^2)^4*ArcTanh[Sq
rt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^
2))]))/(384*c^5*(b*c - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^2)^4)
```

IntegrateAlgebraic [B] time = 43.48, size = 2592, normalized size = 8.64

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

```
[Out] (Sqrt[a + b*x^2]*(279*a^7*c^3*d^4*x + 511*a^7*c^2*d^5*x^3 + 385*a^7*c*d^6*x
^5 + 105*a^7*d^7*x^7) + Sqrt[b]*(105*a^7*c^4*d^3 - 1812*a^7*c^3*d^4*x^2 - 3
458*a^7*c^2*d^5*x^4 - 2660*a^7*c*d^6*x^6 - 735*a^7*d^7*x^8) + b^7*Sqrt[a +
b*x^2]*(-2048*c^7*x^7 - 8192*c^6*d*x^9) + b^6*Sqrt[a + b*x^2]*(-3072*a*c^7*
x^5 - 17408*a*c^6*d*x^7 + 16384*a*c^5*d^2*x^9) + b*Sqrt[a + b*x^2]*(-1368*a
^6*c^4*d^3*x + 4726*a^6*c^3*d^4*x^3 + 10684*a^6*c^2*d^5*x^5 + 8790*a^6*c*d^
6*x^7 + 2520*a^6*d^7*x^9) + b^(15/2)*(2048*c^7*x^8 + 8192*c^6*d*x^10) + b^(
13/2)*(4096*a*c^7*x^6 + 21504*a*c^6*d*x^8 - 16384*a*c^5*d^2*x^10) + b^(3/2)
*(-170*a^6*c^5*d^2 + 6904*a^6*c^4*d^3*x^2 - 5396*a^6*c^3*d^4*x^4 - 20464*a^
6*c^2*d^5*x^6 - 19250*a^6*c*d^6*x^8 - 5880*a^6*d^7*x^10) + b^2*Sqrt[a + b*x
^2]*(1600*a^5*c^5*d^2*x - 19648*a^5*c^4*d^3*x^3 - 7592*a^5*c^3*d^4*x^5 + 16
744*a^5*c^2*d^5*x^7 + 23920*a^5*c*d^6*x^9 + 8400*a^5*d^7*x^11) + b^(5/2)*(4
0*a^5*c^6*d - 7200*a^5*c^5*d^2*x^2 + 40080*a^5*c^4*d^3*x^4 + 31712*a^5*c^3*
d^4*x^6 - 4968*a^5*c^2*d^5*x^8 - 28000*a^5*c*d^6*x^10 - 11760*a^5*d^7*x^12)
 + b^5*Sqrt[a + b*x^2]*(-1280*a^2*c^7*x^3 - 12800*a^2*c^6*d*x^5 + 60160*a^2
*c^5*d^2*x^7 + 31744*a^2*c^4*d^3*x^9 + 32256*a^2*c^3*d^4*x^11 + 9216*a^2*c^
2*d^5*x^13) + b^4*Sqrt[a + b*x^2]*(-128*a^3*c^7*x - 3712*a^3*c^6*d*x^3 + 60
544*a^3*c^5*d^2*x^5 - 2432*a^3*c^4*d^3*x^7 - 25856*a^3*c^3*d^4*x^9 - 42240*
a^3*c^2*d^5*x^11 - 15360*a^3*c*d^6*x^13) + b^3*Sqrt[a + b*x^2]*(-320*a^4*c^
6*d*x + 20096*a^4*c^5*d^2*x^3 - 45440*a^4*c^4*d^3*x^5 - 58736*a^4*c^3*d^4*x
^7 - 35264*a^4*c^2*d^5*x^9 + 4320*a^4*c*d^6*x^11 + 6720*a^4*d^7*x^13) + b^(
11/2)*(2560*a^2*c^7*x^4 + 20480*a^2*c^6*d*x^6 - 68352*a^2*c^5*d^2*x^8 - 317
44*a^2*c^4*d^3*x^10 - 32256*a^2*c^3*d^4*x^12 - 9216*a^2*c^2*d^5*x^14) + b^(
9/2)*(512*a^3*c^7*x^2 + 8448*a^3*c^6*d*x^4 - 88576*a^3*c^5*d^2*x^6 - 13440*
a^3*c^4*d^3*x^8 + 9728*a^3*c^3*d^4*x^10 + 37632*a^3*c^2*d^5*x^12 + 15360*a^
3*c*d^6*x^14) + b^(7/2)*(16*a^4*c^7 + 1344*a^4*c^6*d*x^2 - 43872*a^4*c^5*d^
2*x^4 + 50624*a^4*c^4*d^3*x^6 + 75696*a^4*c^3*d^4*x^8 + 57536*a^4*c^2*d^5*x
^10 + 3360*a^4*c*d^6*x^12 - 6720*a^4*d^7*x^14))/(384*a^6*c^4*d^4*(c + d*x^2
)^4 + 49152*b^6*c^6*d^2*x^8*(c + d*x^2)^4 - 3072*a^5*Sqrt[b]*c^4*d^4*x*Sqrt
[a + b*x^2]*(c + d*x^2)^4 - 49152*b^(11/2)*c^6*d^2*x^7*Sqrt[a + b*x^2]*(c +
 d*x^2)^4 + 384*b*c^4*d^2*(c + d*x^2)^4*(-2*a^5*c*d + 32*a^5*d^2*x^2) + 384
*b^(3/2)*c^4*d^2*Sqrt[a + b*x^2]*(c + d*x^2)^4*(16*a^4*c*d*x - 80*a^4*d^2*x
^3) + 384*b^2*c^4*d^2*(c + d*x^2)^4*(a^4*c^2 - 64*a^4*c*d*x^2 + 160*a^4*d^2
*x^4) + 384*b^(5/2)*c^4*d^2*Sqrt[a + b*x^2]*(c + d*x^2)^4*(-8*a^3*c^2*x + 1
60*a^3*c*d*x^3 - 192*a^3*d^2*x^5) + 384*b^3*c^4*d^2*(c + d*x^2)^4*(32*a^3*c
^2*x^2 - 320*a^3*c*d*x^4 + 256*a^3*d^2*x^6) + 384*b^(9/2)*c^4*d^2*Sqrt[a +
b*x^2]*(c + d*x^2)^4*(-192*a*c^2*x^5 + 256*a*c*d*x^7) + 384*b^(7/2)*c^4*d^2
*Sqrt[a + b*x^2]*(c + d*x^2)^4*(-80*a^2*c^2*x^3 + 384*a^2*c*d*x^5 - 128*a^2
*d^2*x^7) + 384*b^5*c^4*d^2*(c + d*x^2)^4*(256*a*c^2*x^6 - 256*a*c*d*x^8) +
 384*b^4*c^4*d^2*(c + d*x^2)^4*(160*a^2*c^2*x^4 - 512*a^2*c*d*x^6 + 128*a^2
*d^2*x^8)) + (2*b^4*ArcTan[(Sqrt[b]*Sqrt[c])/Sqrt[-(b*c) + a*d] + (Sqrt[b]*
d*x^2)/(Sqrt[c]*Sqrt[-(b*c) + a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-
(b*c) + a*d])]])/(Sqrt[c]*d^2*(b*c - a*d)^2*Sqrt[-(b*c) + a*d]) - (2*a*b^3*A
rcTan[(Sqrt[b]*Sqrt[c])/Sqrt[-(b*c) + a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[
-(b*c) + a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a*d])]])/(c^(3
/2)*d*(b*c - a*d)^2*Sqrt[-(b*c) + a*d]) + (3*a^2*b^2*ArcTanh[(Sqrt[b]*Sqrt[
c])/Sqrt[b*c - a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[b*c - a*d]) - (d*x*Sqrt
[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])]])/(8*c^(5/2)*(b*c - a*d)^(5/2)) + (2
*b^4*ArcTanh[(Sqrt[b]*Sqrt[c])/Sqrt[b*c - a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*S
qrt[b*c - a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])]])/(Sqrt[c
]*d^2*(b*c - a*d)^(5/2)) - (2*a*b^3*ArcTanh[(Sqrt[b]*Sqrt[c])/Sqrt[b*c - a*
d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[b*c - a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqr
t[c]*Sqrt[b*c - a*d])]])/(c^(3/2)*d*(b*c - a*d)^(5/2)) - (5*a^3*b*d*ArcTanh[
(Sqrt[b]*Sqrt[c])/Sqrt[b*c - a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[b*c - a*d
]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])]])/(8*c^(7/2)*(b*c - a*
d)^(5/2)) + (35*a^4*d^2*ArcTanh[(Sqrt[b]*Sqrt[c])/Sqrt[b*c - a*d] + (Sqrt[b
]*d*x^2)/(Sqrt[c]*Sqrt[b*c - a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*
c - a*d])]])/(128*c^(9/2)*(b*c - a*d)^(5/2))
```

fricas [B] time = 5.36, size = 1604, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(48*a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35*a^4*c^4*d^2 + (48*a^2*b^2*
c^2*d^4 - 80*a^3*b*c*d^5 + 35*a^4*d^6)*x^8 + 4*(48*a^2*b^2*c^3*d^3 - 80*a^3
*b*c^2*d^4 + 35*a^4*c*d^5)*x^6 + 6*(48*a^2*b^2*c^4*d^2 - 80*a^3*b*c^3*d^3 +
 35*a^4*c^2*d^4)*x^4 + 4*(48*a^2*b^2*c^5*d - 80*a^3*b*c^4*d^2 + 35*a^4*c^3*
d^3)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 +
a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqr
t(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((16*b^4
*c^5*d^2 + 24*a*b^3*c^4*d^3 - 210*a^2*b^2*c^3*d^4 + 275*a^3*b*c^2*d^5 - 105
*a^4*c*d^6)*x^7 + (64*b^4*c^6*d + 88*a*b^3*c^5*d^2 - 780*a^2*b^2*c^4*d^3 +
1013*a^3*b*c^3*d^4 - 385*a^4*c^2*d^5)*x^5 + (96*b^4*c^7 + 112*a*b^3*c^6*d -
 1050*a^2*b^2*c^5*d^2 + 1353*a^3*b*c^4*d^3 - 511*a^4*c^3*d^4)*x^3 + 3*(80*a
*b^3*c^7 - 256*a^2*b^2*c^6*d + 269*a^3*b*c^5*d^2 - 93*a^4*c^4*d^3)*x)*sqrt(
b*x^2 + a))/(b^3*c^12 - 3*a*b^2*c^11*d + 3*a^2*b*c^10*d^2 - a^3*c^9*d^3 + (
b^3*c^8*d^4 - 3*a*b^2*c^7*d^5 + 3*a^2*b*c^6*d^6 - a^3*c^5*d^7)*x^8 + 4*(b^3
*c^9*d^3 - 3*a*b^2*c^8*d^4 + 3*a^2*b*c^7*d^5 - a^3*c^6*d^6)*x^6 + 6*(b^3*c^
10*d^2 - 3*a*b^2*c^9*d^3 + 3*a^2*b*c^8*d^4 - a^3*c^7*d^5)*x^4 + 4*(b^3*c^11
*d - 3*a*b^2*c^10*d^2 + 3*a^2*b*c^9*d^3 - a^3*c^8*d^4)*x^2), -1/768*(3*(48*
a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35*a^4*c^4*d^2 + (48*a^2*b^2*c^2*d^4 - 80*a^
3*b*c*d^5 + 35*a^4*d^6)*x^8 + 4*(48*a^2*b^2*c^3*d^3 - 80*a^3*b*c^2*d^4 + 35
*a^4*c*d^5)*x^6 + 6*(48*a^2*b^2*c^4*d^2 - 80*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4
)*x^4 + 4*(48*a^2*b^2*c^5*d - 80*a^3*b*c^4*d^2 + 35*a^4*c^3*d^3)*x^2)*sqrt(
-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*s
qrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*
b^4*c^5*d^2 + 24*a*b^3*c^4*d^3 - 210*a^2*b^2*c^3*d^4 + 275*a^3*b*c^2*d^5 -
105*a^4*c*d^6)*x^7 + (64*b^4*c^6*d + 88*a*b^3*c^5*d^2 - 780*a^2*b^2*c^4*d^3
 + 1013*a^3*b*c^3*d^4 - 385*a^4*c^2*d^5)*x^5 + (96*b^4*c^7 + 112*a*b^3*c^6*
d - 1050*a^2*b^2*c^5*d^2 + 1353*a^3*b*c^4*d^3 - 511*a^4*c^3*d^4)*x^3 + 3*(8
0*a*b^3*c^7 - 256*a^2*b^2*c^6*d + 269*a^3*b*c^5*d^2 - 93*a^4*c^4*d^3)*x)*sq
rt(b*x^2 + a))/(b^3*c^12 - 3*a*b^2*c^11*d + 3*a^2*b*c^10*d^2 - a^3*c^9*d^3
+ (b^3*c^8*d^4 - 3*a*b^2*c^7*d^5 + 3*a^2*b*c^6*d^6 - a^3*c^5*d^7)*x^8 + 4*(
b^3*c^9*d^3 - 3*a*b^2*c^8*d^4 + 3*a^2*b*c^7*d^5 - a^3*c^6*d^6)*x^6 + 6*(b^3
*c^10*d^2 - 3*a*b^2*c^9*d^3 + 3*a^2*b*c^8*d^4 - a^3*c^7*d^5)*x^4 + 4*(b^3*c
^11*d - 3*a*b^2*c^10*d^2 + 3*a^2*b*c^9*d^3 - a^3*c^8*d^4)*x^2)]
```

giac [B] time = 9.60, size = 1557, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/128*(48*a^2*b^(5/2)*c^2 - 80*a^3*b^(3/2)*c*d + 35*a^4*sqrt(b)*d^2)*arcta
n(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b
*c*d))/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) - 1
/192*(144*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2*b^(5/2)*c^2*d^5 - 240*(sqrt(
b)*x - sqrt(b*x^2 + a))^14*a^3*b^(3/2)*c*d^6 + 105*(sqrt(b)*x - sqrt(b*x^2
+ a))^14*a^4*sqrt(b)*d^7 + 2016*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(7/2
)*c^3*d^4 - 4368*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(5/2)*c^2*d^5 + 315
0*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(3/2)*c*d^6 - 735*(sqrt(b)*x - sqr
t(b*x^2 + a))^12*a^5*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^
(13/2)*c^6*d + 4096*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(11/2)*c^5*d^2 + 7
936*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2)*c^4*d^3 - 26624*(sqrt(b)*x
 - sqrt(b*x^2 + a))^10*a^3*b^(7/2)*c^3*d^4 + 26944*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^4*b^(5/2)*c^2*d^5 - 12320*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^5*b
^(3/2)*c*d^6 + 2205*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^6*sqrt(b)*d^7 - 2048
```

```
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(15/2)*c^7 - 1024*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*a*b^(13/2)*c^6*d + 27392*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(1
1/2)*c^5*d^2 - 65920*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2)*c^4*d^3 +
81680*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*b^(7/2)*c^3*d^4 - 58840*(sqrt(b)*
x - sqrt(b*x^2 + a))^8*a^5*b^(5/2)*c^2*d^5 + 22750*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^6*b^(3/2)*c*d^6 - 3675*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*sqrt(b
)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(13/2)*c^6*d - 8192*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(11/2)*c^5*d^2 + 47104*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*a^4*b^(9/2)*c^4*d^3 - 74240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a
^5*b^(7/2)*c^3*d^4 + 56416*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(5/2)*c^2*
d^5 - 22400*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^7*b^(3/2)*c*d^6 + 3675*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*a^8*sqrt(b)*d^7 - 1536*(sqrt(b)*x - sqrt(b*x^2 +
a))^4*a^4*b^(11/2)*c^5*d^2 - 2304*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/
2)*c^4*d^3 + 17696*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(7/2)*c^3*d^4 - 23
152*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(5/2)*c^2*d^5 + 11690*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^8*b^(3/2)*c*d^6 - 2205*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*a^9*sqrt(b)*d^7 - 256*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2)*c^4*d^
3 - 512*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*b^(7/2)*c^3*d^4 + 2896*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*a^8*b^(5/2)*c^2*d^5 - 2800*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^9*b^(3/2)*c*d^6 + 735*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*sqrt(b
)*d^7 - 16*a^8*b^(7/2)*c^3*d^4 - 40*a^9*b^(5/2)*c^2*d^5 + 170*a^10*b^(3/2)*
c*d^6 - 105*a^11*sqrt(b)*d^7)/((b^2*c^6*d^2 - 2*a*b*c^5*d^3 + a^2*c^4*d^4)*
((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c
- 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^4)
```

maple [**B**] time = 0.05, size = 18791, normalized size = 62.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^5,x)
```

```
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{3}{2}}}{\left(dx^2 + c\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^5, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2 + a\right)^{3/2}}{\left(d\,x^2 + c\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^5,x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^5, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**5,x)
```

```
[Out] Timed out
```

3.62 $\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=349

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd+152b^2c^2\right)}{960b^3}+\frac{x\left(a+bx^2\right)^{5/2}\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)}{1920b^3}+\frac{ax}{}$$

Rubi [A] time = 0.25, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {416, 528, 388, 195, 217, 206}

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd+152b^2c^2\right)}{960b^3}+\frac{x\left(a+bx^2\right)^{5/2}\left(36a^2bcd^2-5a^3d^3-120ab^2c^2d+320b^3c^3\right)}{1920b^3}+\frac{ax\left(a+bx^2\right)^{3/2}\left(36a^2bcd^2-5a^3d^3-120ab^2c^2d+320b^3c^3\right)}{1536b^3}+\frac{a^2x\sqrt{a+bx^2}\left(36a^2bcd^2-5a^3d^3-120ab^2c^2d+320b^3c^3\right)}{1024b^3}+\frac{a^3\left(36a^2bcd^2-5a^3d^3-120ab^2c^2d+320b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)(16bc-5ad)}{120b^2}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

```
[Out] (a^2*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a
+ b*x^2])/(1024*b^3) + (a*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 -
 5*a^3*d^3)*x*(a + b*x^2)^(3/2))/(1536*b^3) + ((320*b^3*c^3 - 120*a*b^2*c^2
*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*(a + b*x^2)^(5/2))/(1920*b^3) + (d*(152*
b^2*c^2 - 68*a*b*c*d + 15*a^2*d^2)*x*(a + b*x^2)^(7/2))/(960*b^3) + (d*(16*
b*c - 5*a*d)*x*(a + b*x^2)^(7/2)*(c + d*x^2))/(120*b^2) + (d*x*(a + b*x^2)^
(7/2)*(c + d*x^2)^2)/(12*b) + (a^3*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*
b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(7/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
```

```
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3\,dx = \frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b} + \frac{\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)\left(c(12bc-ad)+d(16bc-5a\ldots\right.}{12b}$$

$$= \frac{d(16bc-5ad)x\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{120b^2} + \frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b} + \frac{\int \left(a+bx^2\right)^{\ldots}}{\ldots}$$

$$= \frac{d\left(152b^2c^2-68abcd+15a^2d^2\right)x\left(a+bx^2\right)^{7/2}}{960b^3} + \frac{d(16bc-5ad)x\left(a+bx^2\right)^{7/2}\left(c+\ldots\right.}{120b^2}$$

$$= \frac{\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\left(a+bx^2\right)^{5/2}}{1920b^3} + \frac{d\left(152b^2c^2-68abcd\ldots\right.}{9\ldots}$$

$$= \frac{a\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\left(a+bx^2\right)^{3/2}}{1536b^3} + \frac{\left(320b^3c^3-120ab^2\ldots\right.}{\ldots}$$

$$= \frac{a^2\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{1024b^3} + \frac{a\left(320b^3c^3-120ab^2\ldots\right.}{\ldots}$$

$$= \frac{a^2\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{1024b^3} + \frac{a\left(320b^3c^3-120ab^2\ldots\right.}{\ldots}$$

$$= \frac{a^2\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{1024b^3} + \frac{a\left(320b^3c^3-120ab^2\ldots\right.}{\ldots}$$

Mathematica [A]   time = 5.18, size = 270, normalized size = 0.77

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(75a^5d^3-10a^4bd^2\left(54c+5dx^2\right)+40a^3b^2d\left(45c^2+9cdx^2+d^2x^4\right)+48a^2b^3\left(220c^3+295c^2dx^2+186cd^2x^4+45d^3x^6\right)+64ab^4x^2\left(130c^3+255c^2dx^2+189cd^2x^4+50d^3x^6\right)+128b^5x^4\left(20c^3+45c^2dx^2+36cd^2x^4+10d^3x^6\right)\right)-15a^3\left(5a^3d^3-36a^2bcd^2+120ab^2c^2d-320b^3c^3\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{15360b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(75*a^5*d^3 - 10*a^4*b*d^2*(54*c + 5*d*x^2) + 40
*a^3*b^2*d*(45*c^2 + 9*c*d*x^2 + d^2*x^4) + 128*b^5*x^4*(20*c^3 + 45*c^2*d*
x^2 + 36*c*d^2*x^4 + 10*d^3*x^6) + 48*a^2*b^3*(220*c^3 + 295*c^2*d*x^2 + 18
6*c*d^2*x^4 + 45*d^3*x^6) + 64*a*b^4*x^2*(130*c^3 + 255*c^2*d*x^2 + 189*c*d
^2*x^4 + 50*d^3*x^6)) - 15*a^3*(-320*b^3*c^3 + 120*a*b^2*c^2*d - 36*a^2*b*c
*d^2 + 5*a^3*d^3)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(15360*b^(7/2))
```

IntegrateAlgebraic [A]   time = 0.54, size = 318, normalized size = 0.91

$$\frac{\left(5a^6d^3-36a^5bcd^2+120a^4b^2c^2d-320a^3b^3c^3\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{1024b^{7/2}}+\frac{\sqrt{a+bx^2}\left(75a^5d^3x-540a^4bcd^2x-50a^4bd^3x^3+1800a^3b^2c^2dx+360a^3b^2cd^2x^3+40a^3b^2d^3x^5+10560a^2b^3c^3x+14160a^2b^3c^2dx^3+8928a^2b^3cd^2x^5+2160a^2b^3d^3x^7+8320ab^4c^3x^3+16320ab^4c^2dx^5+12096ab^4cd^2x^7+3200ab^4d^3x^9+2560b^5c^3x^5+5760b^5c^2dx^7+4608b^5cd^2x^9+1280b^5d^3x^{11}\right)}{15360b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(10560*a^2*b^3*c^3*x + 1800*a^3*b^2*c^2*d*x - 540*a^4*b*c*
d^2*x + 75*a^5*d^3*x + 8320*a*b^4*c^3*x^3 + 14160*a^2*b^3*c^2*d*x^3 + 360*a
^3*b^2*c*d^2*x^3 - 50*a^4*b*d^3*x^3 + 2560*b^5*c^3*x^5 + 16320*a*b^4*c^2*d*
x^5 + 8928*a^2*b^3*c*d^2*x^5 + 40*a^3*b^2*d^3*x^5 + 5760*b^5*c^2*d*x^7 + 12
096*a*b^4*c*d^2*x^7 + 2160*a^2*b^3*d^3*x^7 + 4608*b^5*c*d^2*x^9 + 3200*a*b^
4*d^3*x^9 + 1280*b^5*d^3*x^11))/(15360*b^3) + ((-320*a^3*b^3*c^3 + 120*a^4*
b^2*c^2*d - 36*a^5*b*c*d^2 + 5*a^6*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
)/(1024*b^(7/2))
```

fricas [**A**] time = 2.03, size = 608, normalized size = 1.74

$$\left[-\frac{15\left(320a^3b^3c^3-120a^4b^2c^2d+36a^5bcd^2-5a^6d^3\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(1280b^6d^3x^{11}+128\left(36b^6cd^2+25ab^5d^3\right)x^9+144\left(40b^6c^2d+84ab^5cd^2+15a^2b^4d^3\right)x^7+8\left(320b^6c^3+2040ab^5c^2d+1116a^2b^4cd^2+5a^3b^3d^3\right)x^5+10\left(832ab^5c^3+1416a^2b^4c^2d+36a^3b^3cd^2-5a^4b^2d^3\right)x^3+15\left(704a^2b^4c^3+120a^3b^3c^2d-36a^4b^2cd^2+5a^5bd^3\right)x\right)\sqrt{bx^2+a}}{30720b^4}, -\frac{15\left(320a^3b^3c^3-120a^4b^2c^2d+36a^5bcd^2-5a^6d^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(1280b^6d^3x^{11}+128\left(36b^6cd^2+25ab^5d^3\right)x^9+144\left(40b^6c^2d+84ab^5cd^2+15a^2b^4d^3\right)x^7+8\left(320b^6c^3+2040ab^5c^2d+1116a^2b^4cd^2+5a^3b^3d^3\right)x^5+10\left(832ab^5c^3+1416a^2b^4c^2d+36a^3b^3cd^2-5a^4b^2d^3\right)x^3+15\left(704a^2b^4c^3+120a^3b^3c^2d-36a^4b^2cd^2+5a^5bd^3\right)x\right)\sqrt{bx^2+a}}{15360b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(320*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6
*d^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1280*b^6
*d^3*x^11 + 128*(36*b^6*c*d^2 + 25*a*b^5*d^3)*x^9 + 144*(40*b^6*c^2*d + 84*
a*b^5*c*d^2 + 15*a^2*b^4*d^3)*x^7 + 8*(320*b^6*c^3 + 2040*a*b^5*c^2*d + 111
6*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)*x^5 + 10*(832*a*b^5*c^3 + 1416*a^2*b^4*c^2
*d + 36*a^3*b^3*c*d^2 - 5*a^4*b^2*d^3)*x^3 + 15*(704*a^2*b^4*c^3 + 120*a^3*
b^3*c^2*d - 36*a^4*b^2*c*d^2 + 5*a^5*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/153
60*(15*(320*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^3)*s
qrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (1280*b^6*d^3*x^11 + 128*(36*b
^6*c*d^2 + 25*a*b^5*d^3)*x^9 + 144*(40*b^6*c^2*d + 84*a*b^5*c*d^2 + 15*a^2*
b^4*d^3)*x^7 + 8*(320*b^6*c^3 + 2040*a*b^5*c^2*d + 1116*a^2*b^4*c*d^2 + 5*a
^3*b^3*d^3)*x^5 + 10*(832*a*b^5*c^3 + 1416*a^2*b^4*c^2*d + 36*a^3*b^3*c*d^2
 - 5*a^4*b^2*d^3)*x^3 + 15*(704*a^2*b^4*c^3 + 120*a^3*b^3*c^2*d - 36*a^4*b^
2*c*d^2 + 5*a^5*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

giac [**A**] time = 0.68, size = 321, normalized size = 0.92

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10b^2d^3x^2+\frac{36b^{12}cd^2+25ab^{11}d^3}{b^{10}}\right)x^2+\frac{9\left(40b^{12}c^2d+84ab^{11}cd^2+15a^2b^{10}d^3\right)}{b^{10}}\right)x^2+\frac{320b^{12}c^3+2040ab^{11}c^2d+1116a^2b^{10}cd^2+5a^3b^9d^3}{b^{10}}\right)x^2+\frac{5\left(832ab^{11}c^3+1416a^2b^{10}c^2d+36a^3b^9cd^2-5a^4b^8d^3\right)}{b^{10}}\right)x^2+\frac{15\left(704a^2b^{10}c^3+120a^3b^9c^2d-36a^4b^8cd^2+5a^5b^7d^3\right)}{b^{10}}\right)\sqrt{bx^2+a}\,x-\frac{\left(320a^3b^3c^3-120a^4b^2c^2d+36a^5bcd^2-5a^6d^3\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{1024b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d^3*x^2 + (36*b^12*c*d^2 + 25*a*b^11*d^3)/b^10)
*x^2 + 9*(40*b^12*c^2*d + 84*a*b^11*c*d^2 + 15*a^2*b^10*d^3)/b^10)*x^2 + (3
20*b^12*c^3 + 2040*a*b^11*c^2*d + 1116*a^2*b^10*c*d^2 + 5*a^3*b^9*d^3)/b^10
)*x^2 + 5*(832*a*b^11*c^3 + 1416*a^2*b^10*c^2*d + 36*a^3*b^9*c*d^2 - 5*a^4*
b^8*d^3)/b^10)*x^2 + 15*(704*a^2*b^10*c^3 + 120*a^3*b^9*c^2*d - 36*a^4*b^8*
c*d^2 + 5*a^5*b^7*d^3)/b^10)*sqrt(b*x^2 + a)*x - 1/1024*(320*a^3*b^3*c^3 -
120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^3)*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(7/2)
```

maple [**A**] time = 0.02, size = 476, normalized size = 1.36

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}d^3x^5}{12b}-\frac{5a^6d^3\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{1024b^{\frac{7}{2}}}+\frac{9a^5cd^2\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{256b^{\frac{5}{2}}}-\frac{15a^4c^2d\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{128b^{\frac{3}{2}}}+\frac{5a^3c^3\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{16\sqrt{b}}-\frac{5\sqrt{bx^2+a}\,a^5d^3x}{1024b^3}+\frac{9\sqrt{bx^2+a}\,a^4cd^2x}{256b^2}-\frac{15\sqrt{bx^2+a}\,a^3c^2dx}{128b}+\frac{5\sqrt{bx^2+a}\,a^2c^3x}{16}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}a^4d^3x}{1536b^3}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}a^3cd^2x}{128b^2}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}a^2c^2dx}{64b}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}ad^3x^3}{24b^2}+\frac{5\left(bx^2+a\right)^{\frac{3}{2}}ac^3x}{24}+\frac{3\left(bx^2+a\right)^{\frac{7}{2}}cd^2x^3}{10b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}a^3d^3x}{384b^3}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}a^2cd^2x}{160b^2}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}ac^2dx}{16b}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}c^3x}{6}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}a^2d^3x}{64b^3}-\frac{9\left(bx^2+a\right)^{\frac{7}{2}}acd^2x}{80b^2}+\frac{3\left(bx^2+a\right)^{\frac{7}{2}}c^2dx}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(d*x^2+c)^3,x)
```

```
[Out] 1/12*d^3*x^5*(b*x^2+a)^(7/2)/b-1/24*d^3*a/b^2*x^3*(b*x^2+a)^(7/2)+1/64*d^3*
a^2/b^3*x*(b*x^2+a)^(7/2)-1/384*d^3*a^3/b^3*x*(b*x^2+a)^(5/2)-5/1536*d^3*a^
4/b^3*x*(b*x^2+a)^(3/2)-5/1024*d^3*a^5/b^3*x*(b*x^2+a)^(1/2)-5/1024*d^3*a^6
/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+3/10*c*d^2*x^3*(b*x^2+a)^(7/2)/b-9/8
```

```
0*c*d^2*a/b^2*x*(b*x^2+a)^(7/2)+3/160*c*d^2*a^2/b^2*x*(b*x^2+a)^(5/2)+3/128
*c*d^2*a^3/b^2*x*(b*x^2+a)^(3/2)+9/256*c*d^2*a^4/b^2*x*(b*x^2+a)^(1/2)+9/25
6*c*d^2*a^5/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+3/8*c^2*d*x*(b*x^2+a)^(7/
2)/b-1/16*c^2*d*a/b*x*(b*x^2+a)^(5/2)-5/64*c^2*d*a^2/b*x*(b*x^2+a)^(3/2)-15
/128*c^2*d*a^3/b*x*(b*x^2+a)^(1/2)-15/128*c^2*d*a^4/b^(3/2)*ln(b^(1/2)*x+(b
*x^2+a)^(1/2))+1/6*c^3*x*(b*x^2+a)^(5/2)+5/24*c^3*a*x*(b*x^2+a)^(3/2)+5/16*
c^3*a^2*x*(b*x^2+a)^(1/2)+5/16*c^3*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)
)
```

maxima [A] time = 1.47, size = 447, normalized size = 1.28

$$\frac{(bx^2+a)^{\frac{7}{2}}d^3x^5}{12b}+\frac{3(bx^2+a)^{\frac{7}{2}}cd^2x^3}{10b}-\frac{(bx^2+a)^{\frac{7}{2}}ad^3x^3}{24b^2}+\frac{1}{6}(bx^2+a)^{\frac{5}{2}}c^3x+\frac{5}{24}(bx^2+a)^{\frac{3}{2}}ac^3x+\frac{5}{16}\sqrt{bx^2+a}\,a^2c^3x+\frac{3(bx^2+a)^{\frac{7}{2}}c^2dx}{8b}-\frac{(bx^2+a)^{\frac{5}{2}}ac^2dx}{16b}-\frac{5(bx^2+a)^{\frac{3}{2}}a^2c^2dx}{64b}-\frac{15\sqrt{bx^2+a}\,a^3c^2dx}{128b}-\frac{9(bx^2+a)^{\frac{7}{2}}acd^2x}{80b^2}+\frac{3(bx^2+a)^{\frac{5}{2}}a^2cd^2x}{160b^2}+\frac{3(bx^2+a)^{\frac{3}{2}}a^3cd^2x}{128b^2}+\frac{9\sqrt{bx^2+a}\,a^4cd^2x}{256b^2}+\frac{(bx^2+a)^{\frac{7}{2}}a^2d^3x}{64b^3}-\frac{(bx^2+a)^{\frac{5}{2}}a^3d^3x}{384b^3}-\frac{5(bx^2+a)^{\frac{3}{2}}a^4d^3x}{1536b^3}-\frac{5\sqrt{bx^2+a}\,a^5d^3x}{1024b^3}+\frac{5a^3c^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}-\frac{15a^4c^2d\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128b^{\frac{3}{2}}}+\frac{9a^5cd^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256b^{\frac{5}{2}}}-\frac{5a^6d^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{1024b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^(7/2)*d^3*x^5/b + 3/10*(b*x^2 + a)^(7/2)*c*d^2*x^3/b - 1/2
4*(b*x^2 + a)^(7/2)*a*d^3*x^3/b^2 + 1/6*(b*x^2 + a)^(5/2)*c^3*x + 5/24*(b*x
^2 + a)^(3/2)*a*c^3*x + 5/16*sqrt(b*x^2 + a)*a^2*c^3*x + 3/8*(b*x^2 + a)^(7
/2)*c^2*d*x/b - 1/16*(b*x^2 + a)^(5/2)*a*c^2*d*x/b - 5/64*(b*x^2 + a)^(3/2)
*a^2*c^2*d*x/b - 15/128*sqrt(b*x^2 + a)*a^3*c^2*d*x/b - 9/80*(b*x^2 + a)^(7
/2)*a*c*d^2*x/b^2 + 3/160*(b*x^2 + a)^(5/2)*a^2*c*d^2*x/b^2 + 3/128*(b*x^2
+ a)^(3/2)*a^3*c*d^2*x/b^2 + 9/256*sqrt(b*x^2 + a)*a^4*c*d^2*x/b^2 + 1/64*(
b*x^2 + a)^(7/2)*a^2*d^3*x/b^3 - 1/384*(b*x^2 + a)^(5/2)*a^3*d^3*x/b^3 - 5/
1536*(b*x^2 + a)^(3/2)*a^4*d^3*x/b^3 - 5/1024*sqrt(b*x^2 + a)*a^5*d^3*x/b^3
 + 5/16*a^3*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 15/128*a^4*c^2*d*arcsinh(b
*x/sqrt(a*b))/b^(3/2) + 9/256*a^5*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/
1024*a^6*d^3*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^2+a\right)^{5/2}\,\left(d\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)*(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(5/2)*(c + d*x^2)^3, x)
```

sympy [B] time = 102.67, size = 796, normalized size = 2.28

$$\frac{5a^{\frac{11}{2}}d^3x}{1024b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{9a^{\frac{9}{2}}cd^2x}{256b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{9}{2}}d^3x^3}{3072b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^{\frac{7}{2}}c^2dx}{128b\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{7}{2}}cd^2x^3}{256b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{7}{2}}d^3x^5}{1536b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{5}{2}}c^3x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3a^{\frac{5}{2}}c^3x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{133a^{\frac{5}{2}}c^2dx^3}{128\sqrt{1+\frac{bx^2}{a}}}+\frac{387a^{\frac{5}{2}}cd^2x^5}{640\sqrt{1+\frac{bx^2}{a}}}+\frac{55a^{\frac{5}{2}}d^3x^7}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^{\frac{3}{2}}bc^3x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{127a^{\frac{3}{2}}bc^2dx^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{219a^{\frac{3}{2}}bcd^2x^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{67a^{\frac{3}{2}}bd^3x^9}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{17\sqrt{a}\,b^2c^3x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{23\sqrt{a}\,b^2c^2dx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{87\sqrt{a}\,b^2cd^2x^9}{80\sqrt{1+\frac{bx^2}{a}}}+\frac{7\sqrt{a}\,b^2d^3x^{11}}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^6d^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1024b^{\frac{7}{2}}}+\frac{9a^5cd^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}-\frac{15a^4c^2d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{5a^3c^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{b^3c^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{3b^3c^2dx^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{3b^3cd^2x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^3d^3x^{13}}{12\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c)**3,x)
```

```
[Out] 5*a**(11/2)*d**3*x/(1024*b**3*sqrt(1 + b*x**2/a)) - 9*a**(9/2)*c*d**2*x/(25
6*b**2*sqrt(1 + b*x**2/a)) + 5*a**(9/2)*d**3*x**3/(3072*b**2*sqrt(1 + b*x**
2/a)) + 15*a**(7/2)*c**2*d*x/(128*b*sqrt(1 + b*x**2/a)) - 3*a**(7/2)*c*d**2
*x**3/(256*b*sqrt(1 + b*x**2/a)) - a**(7/2)*d**3*x**5/(1536*b*sqrt(1 + b*x*
*2/a)) + a**(5/2)*c**3*x*sqrt(1 + b*x**2/a)/2 + 3*a**(5/2)*c**3*x/(16*sqrt(
1 + b*x**2/a)) + 133*a**(5/2)*c**2*d*x**3/(128*sqrt(1 + b*x**2/a)) + 387*a*
*(5/2)*c*d**2*x**5/(640*sqrt(1 + b*x**2/a)) + 55*a**(5/2)*d**3*x**7/(384*sq
rt(1 + b*x**2/a)) + 35*a**(3/2)*b*c**3*x**3/(48*sqrt(1 + b*x**2/a)) + 127*a
**(3/2)*b*c**2*d*x**5/(64*sqrt(1 + b*x**2/a)) + 219*a**(3/2)*b*c*d**2*x**7/
(160*sqrt(1 + b*x**2/a)) + 67*a**(3/2)*b*d**3*x**9/(192*sqrt(1 + b*x**2/a))
 + 17*sqrt(a)*b**2*c**3*x**5/(24*sqrt(1 + b*x**2/a)) + 23*sqrt(a)*b**2*c**2
*d*x**7/(16*sqrt(1 + b*x**2/a)) + 87*sqrt(a)*b**2*c*d**2*x**9/(80*sqrt(1 +
b*x**2/a)) + 7*sqrt(a)*b**2*d**3*x**11/(24*sqrt(1 + b*x**2/a)) - 5*a**6*d**
```

```
3*asinh(sqrt(b)*x/sqrt(a))/(1024*b**(7/2)) + 9*a**5*c*d**2*asinh(sqrt(b)*x/
sqrt(a))/(256*b**(5/2)) - 15*a**4*c**2*d*asinh(sqrt(b)*x/sqrt(a))/(128*b**(
3/2)) + 5*a**3*c**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + b**3*c**3*x**7/
(6*sqrt(a)*sqrt(1 + b*x**2/a)) + 3*b**3*c**2*d*x**9/(8*sqrt(a)*sqrt(1 + b*x
**2/a)) + 3*b**3*c*d**2*x**11/(10*sqrt(a)*sqrt(1 + b*x**2/a)) + b**3*d**3*x
**13/(12*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.63 $\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=241

$$\frac{x\left(a+bx^2\right)^{5/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{480b^2}+\frac{ax\left(a+bx^2\right)^{3/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{384b^2}+\frac{a^2x\sqrt{a+bx^2}\left(3a^2d^2-20\right.}{256b^2}$$

Rubi [A] time = 0.15, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 388, 195, 217, 206}

$$\frac{x\left(a+bx^2\right)^{5/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{480b^2}+\frac{ax\left(a+bx^2\right)^{3/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{384b^2}+\frac{a^2x\sqrt{a+bx^2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{256b^2}+\frac{a^3\left(3a^2d^2-20abcd+80b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{3dx\left(a+bx^2\right)^{7/2}(4bc-ad)}{80b^2}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

```
[Out] (a^2*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(256*b^2) + (
a*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2))/(384*b^2) + ((
80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(5/2))/(480*b^2) + (3*d*
(4*b*c - a*d)*x*(a + b*x^2)^(7/2))/(80*b^2) + (d*x*(a + b*x^2)^(7/2)*(c + d
*x^2))/(10*b) + (a^3*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2\,dx &= \frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}+\frac{\int \left(a+bx^2\right)^{5/2}\left(c(10bc-ad)+3d(4bc-ad)x^2\right)dx}{10b}\\
&= \frac{3d(4bc-ad)x\left(a+bx^2\right)^{7/2}}{80b^2}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}-\frac{(3ad(4bc-ad)-8bc
\\
&= \frac{\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a+bx^2\right)^{5/2}}{480b^2}+\frac{3d(4bc-ad)x\left(a+bx^2\right)^{7/2}}{80b^2}+\frac{dx\left(a
\\
&= \frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{384b^2}+\frac{\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a
\\
&= \frac{a^2\left(80b^2c^2-20abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{256b^2}+\frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a
\\
&= \frac{a^2\left(80b^2c^2-20abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{256b^2}+\frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a
\\
&= \frac{a^2\left(80b^2c^2-20abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{256b^2}+\frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a
\end{aligned}
$$

Mathematica [C] time = 2.80, size = 158, normalized size = 0.66

$$
\frac{ax\sqrt{a+bx^2}\left(10bx^2\left(c+dx^2\right)^2{}_3F_2\left(-\frac{3}{2},\frac{3}{2},2;1,\frac{9}{2};-\frac{bx^2}{a}\right)+20bx^2\left(2c^2+3cdx^2+d^2x^4\right){}_2F_1\left(-\frac{3}{2},\frac{3}{2};\frac{9}{2};-\frac{bx^2}{a}\right)+7a\left(15c^2+10cdx^2+3d^2x^4\right){}_2F_1\left(-\frac{5}{2},\frac{1}{2};\frac{7}{2};-\frac{bx^2}{a}\right)\right)}{105\sqrt{\frac{bx^2}{a}+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

```
[Out] (a*x*Sqrt[a + b*x^2]*(7*a*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*Hypergeometric2
F1[-5/2, 1/2, 7/2, -((b*x^2)/a)] + 20*b*x^2*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*H
ypergeometric2F1[-3/2, 3/2, 9/2, -((b*x^2)/a)] + 10*b*x^2*(c + d*x^2)^2*Hyp
ergeometricPFQ[{-3/2, 3/2, 2}, {1, 9/2}, -((b*x^2)/a)]))/(105*Sqrt[1 + (b*x
^2)/a])
```

IntegrateAlgebraic [A] time = 0.38, size = 214, normalized size = 0.89

$$
\frac{\left(-3a^5d^2+20a^4bcd-80a^3b^2c^2\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{256b^{5/2}}+\frac{\sqrt{a+bx^2}\left(-45a^4d^2x+300a^3bcdx+30a^3bd^2x^3+2640a^2b^2c^2x+2360a^2b^2cdx^3+744a^2b^2d^2x^5+2080ab^3c^2x^3+2720ab^3cdx^5+1008ab^3d^2x^7+640b^4c^2x^5+960b^4cdx^7+384b^4d^2x^9\right)}{3840b^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(2640*a^2*b^2*c^2*x + 300*a^3*b*c*d*x - 45*a^4*d^2*x + 208
0*a*b^3*c^2*x^3 + 2360*a^2*b^2*c*d*x^3 + 30*a^3*b*d^2*x^3 + 640*b^4*c^2*x^5
 + 2720*a*b^3*c*d*x^5 + 744*a^2*b^2*d^2*x^5 + 960*b^4*c*d*x^7 + 1008*a*b^3*
d^2*x^7 + 384*b^4*d^2*x^9))/(3840*b^2) + ((-80*a^3*b^2*c^2 + 20*a^4*b*c*d -
 3*a^5*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(256*b^(5/2))
```

fricas [A] time = 1.79, size = 420, normalized size = 1.74

$$
\left[\frac{15\left(80a^3b^2c^2-20a^4bcd+3a^5d^2\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(384b^5d^2x^9+48\left(20b^5cd+21ab^4d^2\right)x^7+8\left(80b^5c^2+340ab^4cd+93a^2b^3d^2\right)x^5+10\left(208ab^4c^2+236a^2b^3cd+3a^3b^2d^2\right)x^3+15\left(176a^2b^3c^2+20a^3b^2cd-3a^4bd^2\right)x\right)\sqrt{bx^2+a}}{7680b^3},-\frac{15\left(80a^3b^2c^2-20a^4bcd+3a^5d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(384b^5d^2x^9+48\left(20b^5cd+21ab^4d^2\right)x^7+8\left(80b^5c^2+340ab^4cd+93a^2b^3d^2\right)x^5+10\left(208ab^4c^2+236a^2b^3cd+3a^3b^2d^2\right)x^3+15\left(176a^2b^3c^2+20a^3b^2cd-3a^4bd^2\right)x\right)\sqrt{bx^2+a}}{3840b^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

[Out] [1/7680*(15*(80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(384*b^5*d^2*x^9 + 48*(20*b^5*c*d + 21*a*b^4*d^2)*x^7 + 8*(80*b^5*c^2 + 340*a*b^4*c*d + 93*a^2*b^3*d^2)*x^5 + 10*(208*a*b^4*c^2 + 236*a^2*b^3*c*d + 3*a^3*b^2*d^2)*x^3 + 15*(176*a^2*b^3*c^2 + 20*a^3*b^2*c*d - 3*a^4*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/3840*(15*(80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*b^5*d^2*x^9 + 48*(20*b^5*c*d + 21*a*b^4*d^2)*x^7 + 8*(80*b^5*c^2 + 340*a*b^4*c*d + 93*a^2*b^3*d^2)*x^5 + 10*(208*a*b^4*c^2 + 236*a^2*b^3*c*d + 3*a^3*b^2*d^2)*x^3 + 15*(176*a^2*b^3*c^2 + 20*a^3*b^2*c*d - 3*a^4*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]

giac [A] time = 0.66, size = 221, normalized size = 0.92

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2d^2x^2+\frac{20\,b^{10}cd+21\,ab^9d^2}{b^8}\right)x^2+\frac{80\,b^{10}c^2+340\,ab^9cd+93\,a^2b^8d^2}{b^8}\right)x^2+\frac{5\left(208\,ab^9c^2+236\,a^2b^8cd+3\,a^3b^7d^2\right)}{b^8}\right)x^2+\frac{15\left(176\,a^2b^8c^2+20\,a^3b^7cd-3\,a^4b^6d^2\right)}{b^8}\right)\sqrt{bx^2+a}\,x-\frac{\left(80\,a^3b^2c^2-20\,a^4bcd+3\,a^5d^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="giac")

[Out] 1/3840*(2*(4*(6*(8*b^2*d^2*x^2 + (20*b^10*c*d + 21*a*b^9*d^2)/b^8)*x^2 + (80*b^10*c^2 + 340*a*b^9*c*d + 93*a^2*b^8*d^2)/b^8)*x^2 + 5*(208*a*b^9*c^2 + 236*a^2*b^8*c*d + 3*a^3*b^7*d^2)/b^8)*x^2 + 15*(176*a^2*b^8*c^2 + 20*a^3*b^7*c*d - 3*a^4*b^6*d^2)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)

maple [A] time = 0.01, size = 308, normalized size = 1.28

$$\frac{3a^5d^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{256b^{\frac{5}{2}}}-\frac{5a^4cd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{64b^{\frac{3}{2}}}+\frac{5a^3c^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}+\frac{3\sqrt{b\,x^2+a}\,a^4d^2x}{256b^2}-\frac{5\sqrt{b\,x^2+a}\,a^3cdx}{64b}+\frac{5\sqrt{b\,x^2+a}\,a^2c^2x}{16}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}a^3d^2x}{128b^2}-\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}a^2cdx}{96b}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}a\,c^2x}{24}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}d^2x^3}{10b}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}a^2d^2x}{160b^2}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}acdx}{24b}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}c^2x}{6}-\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}a\,d^2x}{80b^2}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}cdx}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(5/2)*(d*x^2+c)^2,x)

[Out] 1/10*d^2*x^3*(b*x^2+a)^(7/2)/b-3/80*d^2*a/b^2*x*(b*x^2+a)^(7/2)+1/160*d^2*a^2/b^2*x*(b*x^2+a)^(5/2)+1/128*d^2*a^3/b^2*x*(b*x^2+a)^(3/2)+3/256*d^2*a^4/b^2*x*(b*x^2+a)^(1/2)+3/256*d^2*a^5/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*c*d*x*(b*x^2+a)^(7/2)/b-1/24*c*d*a/b*x*(b*x^2+a)^(5/2)-5/96*c*d*a^2/b*x*(b*x^2+a)^(3/2)-5/64*c*d*a^3/b*x*(b*x^2+a)^(1/2)-5/64*c*d*a^4/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/6*c^2*x*(b*x^2+a)^(5/2)+5/24*c^2*a*x*(b*x^2+a)^(3/2)+5/16*c^2*a^2*x*(b*x^2+a)^(1/2)+5/16*c^2*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))

maxima [A] time = 1.39, size = 286, normalized size = 1.19

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}d^2x^3}{10\,b}+\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}c^2x+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ac^2x+\frac{5}{16}\sqrt{bx^2+a}\,a^2c^2x+\frac{\left(bx^2+a\right)^{\frac{7}{2}}cdx}{4\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}acdx}{24\,b}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}a^2cdx}{96\,b}-\frac{5\sqrt{bx^2+a}\,a^3cdx}{64\,b}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}}ad^2x}{80\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}a^2d^2x}{160\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}a^3d^2x}{128\,b^2}+\frac{3\sqrt{bx^2+a}\,a^4d^2x}{256\,b^2}+\frac{5\,a^3c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}-\frac{5\,a^4cd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{64\,b^{\frac{3}{2}}}+\frac{3\,a^5d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="maxima")

[Out] 1/10*(b*x^2 + a)^(7/2)*d^2*x^3/b + 1/6*(b*x^2 + a)^(5/2)*c^2*x + 5/24*(b*x^2 + a)^(3/2)*a*c^2*x + 5/16*sqrt(b*x^2 + a)*a^2*c^2*x + 1/4*(b*x^2 + a)^(7/2)*c*d*x/b - 1/24*(b*x^2 + a)^(5/2)*a*c*d*x/b - 5/96*(b*x^2 + a)^(3/2)*a^2*c*d*x/b - 5/64*sqrt(b*x^2 + a)*a^3*c*d*x/b - 3/80*(b*x^2 + a)^(7/2)*a*d^2*x/b^2 + 1/160*(b*x^2 + a)^(5/2)*a^2*d^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*a^3*d^2*x/b^2 + 3/256*sqrt(b*x^2 + a)*a^4*d^2*x/b^2 + 5/16*a^3*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/64*a^4*c*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/256*a^5*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^2+a\right)}^{5/2}\,{\left(d\,x^2+c\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)*(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(5/2)*(c + d*x^2)^2, x)
```

sympy [B] time = 58.63, size = 537, normalized size = 2.23

$$-\frac{3a^{\frac{9}{2}}d^2x}{256b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{7}{2}}cdx}{64b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{7}{2}}d^2x^3}{256b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{5}{2}}c^2x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3a^{\frac{5}{2}}c^2x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{133a^{\frac{5}{2}}cdx^3}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{129a^{\frac{5}{2}}d^2x^5}{640\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^{\frac{3}{2}}bc^2x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{127a^{\frac{3}{2}}bcdx^5}{96\sqrt{1+\frac{bx^2}{a}}}+\frac{73a^{\frac{3}{2}}bd^2x^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{17\sqrt{a}\,b^2c^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{23\sqrt{a}\,b^2cdx^7}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{29\sqrt{a}\,b^2d^2x^9}{80\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^5d^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}-\frac{5a^4cd\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}+\frac{5a^3c^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{b^3c^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^3cdx^9}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^3d^2x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c)**2,x)
```

```
[Out] -3*a**(9/2)*d**2*x/(256*b**2*sqrt(1 + b*x**2/a)) + 5*a**(7/2)*c*d*x/(64*b*s
qrt(1 + b*x**2/a)) - a**(7/2)*d**2*x**3/(256*b*sqrt(1 + b*x**2/a)) + a**(5/
2)*c**2*x*sqrt(1 + b*x**2/a)/2 + 3*a**(5/2)*c**2*x/(16*sqrt(1 + b*x**2/a))
+ 133*a**(5/2)*c*d*x**3/(192*sqrt(1 + b*x**2/a)) + 129*a**(5/2)*d**2*x**5/(
640*sqrt(1 + b*x**2/a)) + 35*a**(3/2)*b*c**2*x**3/(48*sqrt(1 + b*x**2/a)) +
 127*a**(3/2)*b*c*d*x**5/(96*sqrt(1 + b*x**2/a)) + 73*a**(3/2)*b*d**2*x**7/
(160*sqrt(1 + b*x**2/a)) + 17*sqrt(a)*b**2*c**2*x**5/(24*sqrt(1 + b*x**2/a)
) + 23*sqrt(a)*b**2*c*d*x**7/(24*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**2*d**2
*x**9/(80*sqrt(1 + b*x**2/a)) + 3*a**5*d**2*asinh(sqrt(b)*x/sqrt(a))/(256*b
**(5/2)) - 5*a**4*c*d*asinh(sqrt(b)*x/sqrt(a))/(64*b**(3/2)) + 5*a**3*c**2*
asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + b**3*c**2*x**7/(6*sqrt(a)*sqrt(1 +
b*x**2/a)) + b**3*c*d*x**9/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + b**3*d**2*x**11
/(10*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.64 $\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right) dx$

Optimal. Leaf size=149

$$\frac{5a^3(8bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8bc-ad)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8bc-ad)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8bc-ad)}{192b}$$

Rubi [A] time = 0.05, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {388, 195, 217, 206}

$$\frac{5a^3(8bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8bc-ad)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8bc-ad)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8bc-ad)}{192b}+\frac{dx\left(a+bx^2\right)^{7/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

```
[Out] (5*a^2*(8*b*c - a*d)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*b*c - a*d)*x*(a +
 b*x^2)^(3/2))/(192*b) + ((8*b*c - a*d)*x*(a + b*x^2)^(5/2))/(48*b) + (d*x*
(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(128*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right) dx = \frac{dx\left(a+bx^2\right)^{7/2}}{8b} - \frac{(-8bc+ad)\int \left(a+bx^2\right)^{5/2} dx}{8b}$$
$$= \frac{(8bc-ad)x\left(a+bx^2\right)^{5/2}}{48b} + \frac{dx\left(a+bx^2\right)^{7/2}}{8b} + \frac{(5a(8bc-ad))\int \left(a+bx^2\right)^{3/2} dx}{48b}$$
$$= \frac{5a(8bc-ad)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8bc-ad)x\left(a+bx^2\right)^{5/2}}{48b} + \frac{dx\left(a+bx^2\right)^{7/2}}{8b} + \frac{\left(5a^2\right.}{}$$
$$= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc-ad)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8bc-ad)x\left(a+bx^2\right)^{5}}{48b}$$
$$= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc-ad)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8bc-ad)x\left(a+bx^2\right)^{5}}{48b}$$
$$= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc-ad)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8bc-ad)x\left(a+bx^2\right)^{5}}{48b}$$

Mathematica [A] time = 0.23, size = 130, normalized size = 0.87

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(15a^3d+2a^2b\left(132c+59dx^2\right)+8ab^2x^2\left(26c+17dx^2\right)+16b^3x^4\left(4c+3dx^2\right)\right)-\frac{15a^{5/2}(ad-8bc)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{384b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(15*a^3*d + 16*b^3*x^4*(4*c + 3*d*x^2) + 8*a*b^
2*x^2*(26*c + 17*d*x^2) + 2*a^2*b*(132*c + 59*d*x^2)) - (15*a^(5/2)*(-8*b*c
 + a*d)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(384*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.23, size = 126, normalized size = 0.85

$$\frac{5\left(a^4d-8a^3bc\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{128b^{3/2}}+\frac{\sqrt{a+bx^2}\left(15a^3dx+264a^2bcx+118a^2bdx^3+208ab^2cx^3+136ab^2dx^5+64b^3cx^5+48b^3dx^7\right)}{384b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(264*a^2*b*c*x + 15*a^3*d*x + 208*a*b^2*c*x^3 + 118*a^2*b*
d*x^3 + 64*b^3*c*x^5 + 136*a*b^2*d*x^5 + 48*b^3*d*x^7))/(384*b) + (5*(-8*a^
3*b*c + a^4*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(3/2))
```

fricas [A] time = 1.93, size = 260, normalized size = 1.74

$$\left[-\frac{15\left(8a^3bc-a^4d\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}\,x-a\right)-2\left(48b^4dx^7+8\left(8b^4c+17ab^3d\right)x^5+2\left(104ab^3c+59a^2b^2d\right)x^3+3\left(88a^2b^2c+5a^3bd\right)x\right)\sqrt{bx^2+a}}{768b^2},\ -\frac{15\left(8a^3bc-a^4d\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(48b^4dx^7+8\left(8b^4c+17ab^3d\right)x^5+2\left(104ab^3c+59a^2b^2d\right)x^3+3\left(88a^2b^2c+5a^3bd\right)x\right)\sqrt{bx^2+a}}{384b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(8*a^3*b*c - a^4*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - 2*(48*b^4*d*x^7 + 8*(8*b^4*c + 17*a*b^3*d)*x^5 + 2*(104*a*b^
3*c + 59*a^2*b^2*d)*x^3 + 3*(88*a^2*b^2*c + 5*a^3*b*d)*x)*sqrt(b*x^2 + a))/
b^2, -1/384*(15*(8*a^3*b*c - a^4*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) - (48*b^4*d*x^7 + 8*(8*b^4*c + 17*a*b^3*d)*x^5 + 2*(104*a*b^3*c + 59*a
^2*b^2*d)*x^3 + 3*(88*a^2*b^2*c + 5*a^3*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.63, size = 135, normalized size = 0.91

$$\frac{1}{384}\left(2\left(4\left(6\,b^{2}dx^{2}+\frac{8\,b^{8}c+17\,ab^{7}d}{b^{6}}\right)x^{2}+\frac{104\,ab^{7}c+59\,a^{2}b^{6}d}{b^{6}}\right)x^{2}+\frac{3\left(88\,a^{2}b^{6}c+5\,a^{3}b^{5}d\right)}{b^{6}}\right)\sqrt{bx^{2}+a}\,x-\frac{5\left(8\,a^{3}bc-a^{4}d\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^{2}+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*d*x^2 + (8*b^8*c + 17*a*b^7*d)/b^6)*x^2 + (104*a*b^7*c +
 59*a^2*b^6*d)/b^6)*x^2 + 3*(88*a^2*b^6*c + 5*a^3*b^5*d)/b^6)*sqrt(b*x^2 +
a)*x - 5/128*(8*a^3*b*c - a^4*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
3/2)
```

maple [A] time = 0.01, size = 166, normalized size = 1.11

$$-\frac{5a^{4}d\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{3}{2}}}+\frac{5a^{3}c\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16\sqrt{b}}-\frac{5\sqrt{b\,x^{2}+a}\,a^{3}dx}{128b}+\frac{5\sqrt{b\,x^{2}+a}\,a^{2}cx}{16}-\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}dx}{192b}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}acx}{24}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}adx}{48b}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}cx}{6}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}dx}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(d*x^2+c),x)
```

```
[Out] 1/8*d*x*(b*x^2+a)^(7/2)/b-1/48*d*a/b*x*(b*x^2+a)^(5/2)-5/192*d*a^2/b*x*(b*x
^2+a)^(3/2)-5/128*d*a^3/b*x*(b*x^2+a)^(1/2)-5/128*d*a^4/b^(3/2)*ln(b^(1/2)*
x+(b*x^2+a)^(1/2))+1/6*c*x*(b*x^2+a)^(5/2)+5/24*c*a*x*(b*x^2+a)^(3/2)+5/16*
c*a^2*x*(b*x^2+a)^(1/2)+5/16*c*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.40, size = 151, normalized size = 1.01

$$\frac{1}{6}\left(bx^{2}+a\right)^{\frac{5}{2}}cx+\frac{5}{24}\left(bx^{2}+a\right)^{\frac{3}{2}}acx+\frac{5}{16}\sqrt{bx^{2}+a}\,a^{2}cx+\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}dx}{8\,b}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}adx}{48\,b}-\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}dx}{192\,b}-\frac{5\sqrt{bx^{2}+a}\,a^{3}dx}{128\,b}+\frac{5\,a^{3}c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}-\frac{5\,a^{4}d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*c*x + 5/24*(b*x^2 + a)^(3/2)*a*c*x + 5/16*sqrt(b*x^2
+ a)*a^2*c*x + 1/8*(b*x^2 + a)^(7/2)*d*x/b - 1/48*(b*x^2 + a)^(5/2)*a*d*x/b
 - 5/192*(b*x^2 + a)^(3/2)*a^2*d*x/b - 5/128*sqrt(b*x^2 + a)*a^3*d*x/b + 5/
16*a^3*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/128*a^4*d*arcsinh(b*x/sqrt(a*b)
)/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^{5/2}\,\left(d\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)*(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(5/2)*(c + d*x^2), x)
```

sympy [B] time = 29.53, size = 316, normalized size = 2.12

$$\frac{5a^{\frac{7}{2}}dx}{128b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{a^{\frac{5}{2}}cx\sqrt{1+\frac{bx^{2}}{a}}}{2}+\frac{3a^{\frac{5}{2}}cx}{16\sqrt{1+\frac{bx^{2}}{a}}}+\frac{133a^{\frac{5}{2}}dx^{3}}{384\sqrt{1+\frac{bx^{2}}{a}}}+\frac{35a^{\frac{3}{2}}bcx^{3}}{48\sqrt{1+\frac{bx^{2}}{a}}}+\frac{127a^{\frac{3}{2}}bdx^{5}}{192\sqrt{1+\frac{bx^{2}}{a}}}+\frac{17\sqrt{a}\,b^{2}cx^{5}}{24\sqrt{1+\frac{bx^{2}}{a}}}+\frac{23\sqrt{a}\,b^{2}dx^{7}}{48\sqrt{1+\frac{bx^{2}}{a}}}-\frac{5a^{4}d\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{5a^{3}c\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{b^{3}cx^{7}}{6\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{b^{3}dx^{9}}{8\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c),x)
```

```
[Out] 5*a**(7/2)*d*x/(128*b*sqrt(1 + b*x**2/a)) + a**(5/2)*c*x*sqrt(1 + b*x**2/a)
/2 + 3*a**(5/2)*c*x/(16*sqrt(1 + b*x**2/a)) + 133*a**(5/2)*d*x**3/(384*sqrt
```

```
(1 + b*x**2/a)) + 35*a**(3/2)*b*c*x**3/(48*sqrt(1 + b*x**2/a)) + 127*a**(3/
2)*b*d*x**5/(192*sqrt(1 + b*x**2/a)) + 17*sqrt(a)*b**2*c*x**5/(24*sqrt(1 +
b*x**2/a)) + 23*sqrt(a)*b**2*d*x**7/(48*sqrt(1 + b*x**2/a)) - 5*a**4*d*asin
h(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + 5*a**3*c*asinh(sqrt(b)*x/sqrt(a))/(16
*sqrt(b)) + b**3*c*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + b**3*d*x**9/(8*sqr
t(a)*sqrt(1 + b*x**2/a))
```

3.65 $\int \left(a+bx^2\right)^{5/2} \, dx$

Optimal. Leaf size=84

$$\frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*(a + b*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^{5/2} \, dx &= \frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{6}(5a)\int \left(a+bx^2\right)^{3/2} \, dx \\
&= \frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \sqrt{a+bx^2} \, dx \\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{a+bx^2}} \, dx \\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \right. \\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 76, normalized size = 0.90

$$\frac{1}{48}\sqrt{a+bx^2}\left(\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}+33a^2x+26abx^3+8b^2x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5 + (15*a^(5/2)*ArcSinh[(
Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/48
```

IntegrateAlgebraic [A] time = 0.00, size = 71, normalized size = 0.85

$$\frac{1}{48}\sqrt{a+bx^2}\left(33a^2x+26abx^3+8b^2x^5\right)-\frac{5a^3\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5))/48 - (5*a^3*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

fricas [A] time = 1.71, size = 146, normalized size = 1.74

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx\right)\sqrt{bx^2+a}}{96\,b},-\frac{15\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx\right)\sqrt{bx^2+a}}{48\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
8*b^3*x^5 + 26*a*b^2*x^3 + 33*a^2*b*x)*sqrt(b*x^2 + a))/b, -1/48*(15*a^3*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 26*a*b^2*x^3 + 33*
a^2*b*x)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.61, size = 63, normalized size = 0.75

$$-\frac{5\,a^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2*(4*b^2*x
^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.00, size = 66, normalized size = 0.79

$$\frac{5a^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}+\frac{5\sqrt{b\,x^2+a}\,a^2x}{16}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}ax}{24}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2),x)
```

```
[Out] 1/6*(b*x^2+a)^(5/2)*x+5/24*(b*x^2+a)^(3/2)*a*x+5/16*(b*x^2+a)^(1/2)*a^2*x+5
/16*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.33, size = 58, normalized size = 0.69

$$\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}x+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ax+\frac{5}{16}\sqrt{bx^2+a}\,a^2x+\frac{5\,a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*x + 5/24*(b*x^2 + a)^(3/2)*a*x + 5/16*sqrt(b*x^2 + a)
*a^2*x + 5/16*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 4.69, size = 37, normalized size = 0.44

$$\frac{x\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2),x)
```

```
[Out] (x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(5/2)
```

sympy [A] time = 4.28, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}\,b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 + b*x**2/a)
/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*asinh(sqrt(b)*x/sqrt(
a))/(16*sqrt(b))
```

3.66 $\int \frac{\left(a+bx^2\right)^{5/2}}{c+dx^2}\,dx$

Optimal. Leaf size=157

$$\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8d^3}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d^3}-\frac{bx\sqrt{a+bx^2}\,(4bc-7ad)}{8d^2}+\frac{b\ldots}{\ldots}$$

Rubi [A] time = 0.20, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {416, 528, 523, 217, 206, 377, 208}

$$\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8d^3}-\frac{bx\sqrt{a+bx^2}\,(4bc-7ad)}{8d^2}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d^3}+\frac{bx\left(a+bx^2\right)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

```
[Out] -(b*(4*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/(8*d^2) + (b*x*(a + b*x^2)^(3/2))/(4
*d) + (Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(8*d^3) - ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(S
qrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{c+dx^2}\,dx = \frac{bx\left(a+bx^2\right)^{3/2}}{4d} + \frac{\int \frac{\sqrt{a+bx^2}\left(-a(bc-4ad)-b(4bc-7ad)x^2\right)}{c+dx^2}\,dx}{4d}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)^{3/2}}{4d} + \frac{\int \frac{a\left(4b^2c^2-9abcd+8a^2d^2\right)+b\left(8b^2c^2-20abcd+15a^2d^2\right)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8d^2}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)^{3/2}}{4d} - \frac{(bc-ad)^3\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{d^3} + \frac{\left(b\left(8b^2c^2-20\right.\right.}{}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)^{3/2}}{4d} - \frac{(bc-ad)^3\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{d^3}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)^{3/2}}{4d} + \frac{\sqrt{b}\left(8b^2c^2-20abcd+15a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx}}\right.}{8d^3}$$

Mathematica [A] time = 0.12, size = 140, normalized size = 0.89

$$\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+bdx\sqrt{a+bx^2}\left(9ad-4bc+2bdx^2\right)+\frac{8(ad-bc)^{5/2}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}}}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

```
[Out] (b*d*x*Sqrt[a + b*x^2]*(-4*b*c + 9*a*d + 2*b*d*x^2) + (8*(-(b*c) + a*d)^(5/
2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[c] + Sqrt
[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]
])/(8*d^3)
```

IntegrateAlgebraic [A] time = 0.45, size = 199, normalized size = 1.27

$$\frac{\left(-15a^2\sqrt{b}\,d^2+20ab^{3/2}cd-8b^{5/2}c^2\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8d^3} - \frac{\sqrt{ad-bc}\left(a^2d^2-2abcd+b^2c^2\right)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,c+\sqrt{b}\,dx^2}{\sqrt{c}\sqrt{ad-bc}}\right)}{\sqrt{c}\,d^3} + \frac{\sqrt{a+bx^2}\left(9abdx-4b^2cx+2b^2dx^3\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-4*b^2*c*x + 9*a*b*d*x + 2*b^2*d*x^3))/(8*d^2) - (Sqrt[-(
b*c) + a*d]*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*c + Sqrt[b]*d*x
```

```
^2 - d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*d^3) + ((
-8*b^(5/2)*c^2 + 20*a*b^(3/2)*c*d - 15*a^2*Sqrt[b]*d^2)*Log[-(Sqrt[b]*x) +
Sqrt[a + b*x^2]])/(8*d^3)
```

fricas [A] time = 3.40, size = 935, normalized size = 5.96

$$\left[\frac{(8b^2c^2-20abcd+15a^2d^2)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+4(b^2c^2-2abcd+a^2d^2)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2-4(ac^2x+(2bc^2-acd)x^3)\sqrt{bx^2+a}\sqrt{\frac{bc-ad}{c}}}{d^2x^4+2cdx^2+c^2}\right)+2\left(2b^2d^2x^3-(4b^2cd-9abd^2)x\right)\sqrt{bx^2+a}}{16d^3}, -\frac{(8b^2c^2-20abcd+15a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-2(b^2c^2-2abcd+a^2d^2)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2-4(ac^2x+(2bc^2-acd)x^3)\sqrt{bx^2+a}\sqrt{\frac{bc-ad}{c}}}{d^2x^4+2cdx^2+c^2}\right)-\left(2b^2d^2x^3-(4b^2cd-9abd^2)x\right)\sqrt{bx^2+a}}{8d^3}, \frac{8(b^2c^2-2abcd+a^2d^2)\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2((b^2c-abd)x^3+(abc-a^2d)x)}\right)+(8b^2c^2-20abcd+15a^2d^2)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2b^2d^2x^3-(4b^2cd-9abd^2)x\right)\sqrt{bx^2+a}}{16d^3}, -\frac{(8b^2c^2-20abcd+15a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-4(b^2c^2-2abcd+a^2d^2)\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2((b^2c-abd)x^3+(abc-a^2d)x)}\right)-\left(2b^2d^2x^3-(4b^2cd-9abd^2)x\right)\sqrt{bx^2+a}}{8d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/16*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(b)*x - a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a
*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2
- 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt
((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 2*(2*b^2*d^2*x^3 - (4*b^2*c
*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, -1/8*((8*b^2*c^2 - 20*a*b*c*d + 15
*a^2*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 2*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x
^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*
d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) -
 (2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, 1/16*(8*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a
*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 +
(a*b*c - a^2*d)*x)) + (8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(-2*
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*b^2*d^2*x^3 - (4*b^2*c*d -
9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, -1/8*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*
d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 4*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*
x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) -
(2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.02, size = 3053, normalized size = 19.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c),x)
```

```
[Out] -1/(-c*d)^(1/2)/d*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)
*b/d+(a*d-b*c)/d)^(1/2)*a*b*c+1/2/(-c*d)^(1/2)/d^3/((a*d-b*c)/d)^(1/2)*ln((
2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*(
(x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(
1/2))/(x-(-c*d)^(1/2)/d))*b^3*c^3+1/(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)^2*b-
2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*a*b*c-1/2/(-c*d)^(
1/2)/d^3/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(
a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+
(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*b^3*c^3+3/2/(-c
*d)^(1/2)/d^2/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/
```

```
d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2
)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a*b^2*c^2+
3/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d
)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^
(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a^2*b*
c-3/2/(-c*d)^(1/2)/d^2/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/
2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c
*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a*
b^2*c^2-3/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d
)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-
2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d
))*a^2*b*c+1/8*b/d*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d
)*b/d+(a*d-b*c)/d)^(3/2)*x+1/8*b/d*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(
x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*x+15/16/d*b^(1/2)*ln(((x-(-c*d)^(1
/2)/d)*b+(-c*d)^(1/2)*b/d)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(
x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*a^2+1/2/d^3*b^(5/2)*ln(((x-(-c*d)
^(1/2)/d)*b+(-c*d)^(1/2)*b/d)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2
)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*c^2-1/2/(-c*d)^(1/2)/((a*d-b*c
)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-
b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d
+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a^3+15/16/d*b^(1/2)*ln(((x+(-c*d)^
(1/2)/d)*b-(-c*d)^(1/2)*b/d)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)
*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*a^2+1/2/d^3*b^(5/2)*ln(((x+(-c*
d)^(1/2)/d)*b-(-c*d)^(1/2)*b/d)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1
/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*c^2+1/2/(-c*d)^(1/2)/((a*d-b
*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a
*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*
b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a^3-1/10/(-c*d)^(1/2)*((x+(-c*d
)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(5/2)+1/1
0/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/
d+(a*d-b*c)/d)^(5/2)-1/2/(-c*d)^(1/2)/d^2*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^
(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*b^2*c^2-1/6/(-c*d)^(1/2)*((
x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3
/2)*a-1/2/(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/
2)/d)*b/d+(a*d-b*c)/d)^(1/2)*a^2+1/6/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2
*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*a+1/2/(-c*d)^(1/2)*
((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^
(1/2)*a^2-1/6/(-c*d)^(1/2)/d*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*
d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*b*c+7/16*b/d*a*((x-(-c*d)^(1/2)/d)^2*b+2
*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-1/4/d^2*b^2*((x-(
-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)
*x*c-5/4/d^2*b^(3/2)*ln(((x-(-c*d)^(1/2)/d)*b+(-c*d)^(1/2)*b/d)/b^(1/2)+((x
-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/
2))*c*a+1/2/(-c*d)^(1/2)/d^2*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*
d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*b^2*c^2+7/16*b/d*a*((x+(-c*d)^(1/2)/d)^2
*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+1/6/(-c*d)^(1
/2)/d*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*
c)/d)^(3/2)*b*c-1/4/d^2*b^2*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d
)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x*c-5/4/d^2*b^(3/2)*ln(((x+(-c*d)^(1/2)/d
)*b-(-c*d)^(1/2)*b/d)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c
*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))*c*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{5/2}}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2), x)
```

3.67 $\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=175

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2d^3}+\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}d^3}+\frac{bx\sqrt{a+bx^2}\,(2bc-ad)}{2cd^2}-\frac{x\left(a+bx\right.}{2cd\,(}$$

Rubi [A] time = 0.23, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {413, 528, 523, 217, 206, 377, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2d^3}+\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}d^3}+\frac{bx\sqrt{a+bx^2}\,(2bc-ad)}{2cd^2}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d^2) - ((b*c - a*d)*x*(a + b*x^2)^
(3/2))/(2*c*d*(c + d*x^2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(2*d^3) + ((b*c - a*d)^(3/2)*(4*b*c + a*d)*ArcTanh[(Sqrt[b
*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*d^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^2}\,dx = -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{2cd\left(c+dx^2\right)} + \frac{\int \frac{\sqrt{a+bx^2}\left(a(bc+ad)+2b(2bc-ad)x^2\right)}{c+dx^2}\,dx}{2cd}$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{2cd\left(c+dx^2\right)} + \frac{\int \frac{-2a\left(2b^2c^2-2abcd-a^2d^2\right)-2b^2c(4bc-5ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{4cd^2}$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{2cd\left(c+dx^2\right)} - \frac{\left(b^2(4bc-5ad)\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2d^3} + \frac{\left((bc-\right.}{}$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{2cd\left(c+dx^2\right)} - \frac{\left(b^2(4bc-5ad)\right)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x,\right.}{2d^3}$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{2cd\left(c+dx^2\right)} - \frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^3} + \frac{(b}{}$$

Mathematica [A] time = 0.16, size = 144, normalized size = 0.82

$$\frac{-\left(b^{3/2}(4bc-5ad)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\right)+dx\sqrt{a+bx^2}\left(\frac{(bc-ad)^2}{c\left(c+dx^2\right)}+b^2\right)+\frac{(ad-bc)^{3/2}(ad+4bc)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

```
[Out] (d*x*Sqrt[a + b*x^2]*(b^2 + (b*c - a*d)^2/(c*(c + d*x^2))) + ((-(b*c) + a*d
)^(3/2)*(4*b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2
])])/c^(3/2) - b^(3/2)*(4*b*c - 5*a*d)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/
(2*d^3)
```

IntegrateAlgebraic [A] time = 0.81, size = 210, normalized size = 1.20

$$\frac{\sqrt{a+bx^2}\left(a^2d^2x-2abcdx+2b^2c^2x+b^2cdx^3\right)}{2cd^2\left(c+dx^2\right)}+\frac{\sqrt{ad-bc}\left(-a^2d^2-3abcd+4b^2c^2\right)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}c+\sqrt{b}dx^2}{\sqrt{c}\sqrt{ad-bc}}\right)}{2c^{3/2}d^3}+\frac{\left(4b^{5/2}c-5ab^{3/2}d\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(2*b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x + b^2*c*d*x^3))/(2*
c*d^2*(c + d*x^2)) + (Sqrt[-(b*c) + a*d]*(4*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*
ArcTan[(Sqrt[b]*c + Sqrt[b]*d*x^2 - d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*
c) + a*d])])/(2*c^(3/2)*d^3) + ((4*b^(5/2)*c - 5*a*b^(3/2)*d)*Log[-(Sqrt[b]
*x) + Sqrt[a + b*x^2]])/(2*d^3)
```

fricas [A] time = 2.68, size = 1236, normalized size = 7.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(b
)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (4*b^2*c^3 - 3*a*b*c^2*
d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt((b*c - a*d)
/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3
*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b
*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(b^2*c*d^2*x^3 + (2*b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), 1/8*(
4*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - (4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*
b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2
 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(
a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*
x^4 + 2*c*d*x^2 + c^2)) + 4*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), -1/4*((4*b^2*c^3 - 3*a*b
*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-(b*c
- a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c -
 a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) + (4*b^2*c^3 - 5*a*b*c^
2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) - 2*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), 1/4*(2*(4*b^2*c^3 - 5*a*b*c^
2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^
2 + a)) - (4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2
 - a^2*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*
sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)
*x)) + 2*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x
^2 + a))/(c*d^4*x^2 + c^2*d^3)]
```

giac [B] time = 0.69, size = 405, normalized size = 2.31

$$\frac{\sqrt{bx^2+a}\,b^2x}{2\,d^2}+\frac{\left(4\,b^{\frac{5}{2}}c-5\,ab^{\frac{3}{2}}d\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{4\,d^3}-\frac{\left(4\,b^{\frac{7}{2}}c^3-7\,ab^{\frac{5}{2}}c^2d+2\,a^2b^{\frac{3}{2}}cd^2+a^3\sqrt{b}\,d^3\right)\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}\,cd^3}+\frac{2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2b^{\frac{7}{2}}c^3-5\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{5}{2}}c^2d+4\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{3}{2}}cd^2-\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^3\sqrt{b}\,d^3+a^2b^{\frac{5}{2}}c^2d-2\,a^3b^{\frac{3}{2}}cd^2+a^4\sqrt{b}\,d^3}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^2*x/d^2 + 1/4*(4*b^(5/2)*c - 5*a*b^(3/2)*d)*log((sqrt
(b)*x - sqrt(b*x^2 + a))^2)/d^3 - 1/2*(4*b^(7/2)*c^3 - 7*a*b^(5/2)*c^2*d +
2*a^2*b^(3/2)*c*d^2 + a^3*sqrt(b)*d^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
+ a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d
)*c*d^3) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(7/2)*c^3 - 5*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a*b^(5/2)*c^2*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*
b^(3/2)*c*d^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*d^3 + a^2*b^(5/
2)*c^2*d - 2*a^3*b^(3/2)*c*d^2 + a^4*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b*x^2
 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*d + a^2*d)*c*d^3)
```

maple [B] time = 0.02, size = 7345, normalized size = 41.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{2}}}{\left(dx^2 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2 + a\right)^{5/2}}{\left(d\,x^2 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^{\frac{5}{2}}}{\left(c + dx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**2, x)
```

3.68 $\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^3} dx$

Optimal. Leaf size=194

$$-\frac{\sqrt{bc-ad}\left(3a^2d^2+4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}d^3}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{8c^2d^2\left(c+dx^2\right)}-$$

Rubi [A] time = 0.19, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{413, 526, 523, 217, 206, 377, 208\}$

$$-\frac{\sqrt{bc-ad}\left(3a^2d^2+4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}d^3}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{8c^2d^2\left(c+dx^2\right)}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2)^(3/2))/(4*c*d*(c + d*x^2)^2) - ((b*c - a*d)*(4*
b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*c^2*d^2*(c + d*x^2)) + (b^(5/2)*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d^3 - (Sqrt[b*c - a*d]*(8*b^2*c^2 + 4*a*b*c*d
 + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^
(5/2)*d^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^3}\,dx = -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}+\frac{\int \frac{\sqrt{a+bx^2}\left(a(bc+3ad)+4b^2cx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}-\frac{\int \frac{-a\left(4b^2c^2+ad(bc+3ad)\right)-8b^3c^2x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}}{8c^2d^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}+\frac{b^3\int \frac{1}{\sqrt{a+bx^2}}\,dx}{d^3}-\frac{\left((bc-ad\right.}{}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}+\frac{b^3\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{}{\sqrt{a}}\right.}{d^3}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{\sqrt{b}}{}$$

Mathematica [A] time = 0.20, size = 184, normalized size = 0.95

$$\frac{\frac{\left(3a^3d^3+a^2bcd^2+4ab^2c^2d-8b^3c^3\right)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{5/2}\sqrt{ad-bc}}+8b^{5/2}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\frac{dx\sqrt{a+bx^2}(ad-bc)\left(ad\left(5c+3dx^2\right)+2bc\left(2c+3dx^2\right)\right)}{c^2\left(c+dx^2\right)^2}}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

```
[Out] ((d*(-(b*c) + a*d)*x*Sqrt[a + b*x^2]*(2*b*c*(2*c + 3*d*x^2) + a*d*(5*c + 3*
d*x^2)))/(c^2*(c + d*x^2)^2) + ((-8*b^3*c^3 + 4*a*b^2*c^2*d + a^2*b*c*d^2 +
 3*a^3*d^3)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(c^(5
/2)*Sqrt[-(b*c) + a*d]) + 8*b^(5/2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*
d^3)
```

IntegrateAlgebraic [A] time = 1.91, size = 311, normalized size = 1.60

$$-\frac{3\sqrt{ad-bc}\left(a^2d^2-8abcd+16b^2c^2\right)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}c+\sqrt{b}dx^2}{\sqrt{c}\sqrt{ad-bc}}\right)}{8c^{5/2}d^3}+\frac{\sqrt{a+bx^2}\left(5a^2cd^2x+3a^2d^3x^3-abc^2dx+3abcd^2x^3-4b^2c^3x-6b^2c^2dx^3\right)}{8c^2d^2\left(c+dx^2\right)^2}-\frac{b^{5/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{d^3}+\frac{\sqrt{bc-ad}\left(10b^2c-7abd\right)\tanh^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}c+\sqrt{b}dx^2}{\sqrt{c}\sqrt{bc-ad}}\right)}{2c^{3/2}d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-4*b^2*c^3*x - a*b*c^2*d*x + 5*a^2*c*d^2*x - 6*b^2*c^2*d*
x^3 + 3*a*b*c*d^2*x^3 + 3*a^2*d^3*x^3))/(8*c^2*d^2*(c + d*x^2)^2) - (3*Sqrt
[-(b*c) + a*d]*(16*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*c + Sqrt[
b]*d*x^2 - d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a*d])]])/(8*c^(5/2)*d
^3) + (Sqrt[b*c - a*d]*(10*b^2*c - 7*a*b*d)*ArcTanh[(Sqrt[b]*c + Sqrt[b]*d*
x^2 - d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])]])/(2*c^(3/2)*d^3) - (b
^(5/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/d^3
```

fricas [B] time = 3.10, size = 1517, normalized size = 7.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b)*log(-2*b*x^
2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2
*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a
*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*
c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x +
 (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c
*d*x^2 + c^2)) - 4*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^3 + (4*b^2*c^
3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/(c^2*d^5*x^4 + 2*c^3*d
^4*x^2 + c^4*d^3), -1/32*(32*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^2*c^4 + 4*a*b*c^3*d + 3*
a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*
d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*
c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^
4 + 2*c*d*x^2 + c^2)) + 4*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^3 + (4
*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/(c^2*d^5*x^4 +
2*c^3*d^4*x^2 + c^4*d^3), 1/16*((8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2 +
(8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*
d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2
+ a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c -
 a^2*d)*x)) + 8*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b)*log(-
2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(3*(2*b^2*c^2*d^2 - a*b*c*d^
3 - a^2*d^4)*x^3 + (4*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2
+ a))/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3), -1/16*(16*(b^2*c^2*d^2*x^4 +
 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (
8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*
a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-(b*
c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c
 - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) + 2*(3*(2*b^2*c^2*d^2
 - a*b*c*d^3 - a^2*d^4)*x^3 + (4*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*
sqrt(b*x^2 + a))/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3)]
```

giac [B] time = 0.73, size = 659, normalized size = 3.40

$$-\frac{b^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2d^3}+\frac{\left(8b^{\frac{7}{2}}c^3-4ab^{\frac{5}{2}}c^2d-a^2b^{\frac{3}{2}}cd^2-3a^3\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{8\sqrt{-b^2c^2+abcd}\,c^2d^3}-\frac{16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^{\frac{7}{2}}c^3d-20\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{5}{2}}c^2d^2+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2b^{\frac{3}{2}}cd^3+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^3\sqrt{b}d^4+48\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4b^{\frac{9}{2}}c^4-72\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4ab^{\frac{7}{2}}c^3d+18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{5}{2}}c^2d^2+15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^3b^{\frac{3}{2}}cd^3-9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^4\sqrt{b}d^4+32\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{7}{2}}c^3d-28\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^3b^{\frac{5}{2}}c^2d^2-13\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^4b^{\frac{3}{2}}cd^3+9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^5\sqrt{b}d^4+6a^4b^{\frac{5}{2}}c^2d^2-3a^5b^{\frac{3}{2}}cd^3-3a^6\sqrt{b}d^4}{4\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)^2c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^3 + 1/8*(8*b^(7/2)*c^3
- 4*a*b^(5/2)*c^2*d - a^2*b^(3/2)*c*d^2 - 3*a^3*sqrt(b)*d^3)*arctan(1/2*((s
qrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(s
qrt(-b^2*c^2 + a*b*c*d)*c^2*d^3) - 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
b^(7/2)*c^3*d - 20*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2)*c^2*d^2 + (sqr
```

```
t(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(3/2)*c*d^3 + 3*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^3*sqrt(b)*d^4 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(9/2)*c^4 - 7
2*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c^3*d + 18*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*a^2*b^(5/2)*c^2*d^2 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^
(3/2)*c*d^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*sqrt(b)*d^4 + 32*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a^2*b^(7/2)*c^3*d - 28*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*a^3*b^(5/2)*c^2*d^2 - 13*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(3/2)*
c*d^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*sqrt(b)*d^4 + 6*a^4*b^(5/2)*c
^2*d^2 - 3*a^5*b^(3/2)*c*d^3 - 3*a^6*sqrt(b)*d^4)/(((sqrt(b)*x - sqrt(b*x^2
 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*d + a^2*d)^2*c^2*d^3)
```

maple [B] time = 0.03, size = 14133, normalized size = 72.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^3,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{2}}}{\left(dx^2 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2 + a\right)^{5/2}}{\left(d\,x^2 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^3,x)
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^{\frac{5}{2}}}{\left(c + dx^2\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**3, x)
```

3.69 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4} dx$

Optimal. Leaf size=144

$$\frac{5a^3\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3(c+dx^2)}+\frac{5ax(a+bx^2)^{3/2}}{24c^2(c+dx^2)^2}+\frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3}$$

Rubi [A] time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {378, 377, 208}

$$\frac{5a^2x\sqrt{a+bx^2}}{16c^3(c+dx^2)}+\frac{5a^3\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}+\frac{5ax(a+bx^2)^{3/2}}{24c^2(c+dx^2)^2}+\frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

```
[Out] (x*(a + b*x^2)^(5/2))/(6*c*(c + d*x^2)^3) + (5*a*x*(a + b*x^2)^(3/2))/(24*c
^2*(c + d*x^2)^2) + (5*a^2*x*Sqrt[a + b*x^2])/(16*c^3*(c + d*x^2)) + (5*a^3
*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*Sqrt[b
*c - a*d])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^4}\,dx = \frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3} + \frac{(5a)\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^3}\,dx}{6c}$$

$$= \frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3} + \frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2} + \frac{\left(5a^2\right)\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx}{8c^2}$$

$$= \frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3} + \frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2} + \frac{5a^2x\sqrt{a+bx^2}}{16c^3\left(c+dx^2\right)} + \frac{\left(5a^3\right)\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{16c^3}$$

$$= \frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3} + \frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2} + \frac{5a^2x\sqrt{a+bx^2}}{16c^3\left(c+dx^2\right)} + \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{\ }{\sqrt{a+}}\right.}{16c^3}$$

$$= \frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3} + \frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2} + \frac{5a^2x\sqrt{a+bx^2}}{16c^3\left(c+dx^2\right)} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.81, size = 201, normalized size = 1.40

$$\frac{x\sqrt{a+bx^2}\left(\frac{\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\left(a^2\left(33c^2+40cdx^2+15d^2x^4\right)+2abcx^2\left(13c+5dx^2\right)+8b^2c^2x^4\right)}{\left(c+dx^2\right)^2\sqrt{\frac{dx^2}{c}+1}}+\frac{15a^2\sin^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{\sqrt{\frac{x^2(ad-bc)}{ac}}}\right)}{48c^4\sqrt{\frac{bx^2}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*(8*b^2*c^2*x^4 +
 2*a*b*c*x^2*(13*c + 5*d*x^2) + a^2*(33*c^2 + 40*c*d*x^2 + 15*d^2*x^4)))/((
c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]) + (15*a^2*ArcSin[Sqrt[(-(b/a) + d/c)*x^2]
/Sqrt[1 + (d*x^2)/c]])/Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]))/(48*c^4*Sqrt[1 +
(b*x^2)/a])
```

IntegrateAlgebraic [B] time = 20.71, size = 1180, normalized size = 8.19

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

```
[Out] (b^3*Sqrt[a + b*x^2]*(-48*a^2*c^5*x - 464*a^2*c^4*d*x^3 - 1584*a^2*c^3*d^2*
x^5) + Sqrt[a + b*x^2]*(33*a^5*c^2*d^3*x + 40*a^5*c*d^4*x^3 + 15*a^5*d^5*x^
5) + b^(7/2)*(144*a^2*c^5*x^2 + 912*a^2*c^4*d*x^4 + 2352*a^2*c^3*d^2*x^6) +
 Sqrt[b]*(15*a^5*c^3*d^2 - 153*a^5*c^2*d^3*x^2 - 195*a^5*c*d^4*x^4 - 75*a^5
*d^5*x^6) + b^4*Sqrt[a + b*x^2]*(-256*a*c^5*x^3 - 1088*a*c^4*d*x^5 - 1728*a
*c^3*d^2*x^7) + b*Sqrt[a + b*x^2]*(-90*a^4*c^3*d^2*x + 350*a^4*c^2*d^3*x^3
+ 460*a^4*c*d^4*x^5 + 180*a^4*d^5*x^7) + b^(9/2)*(384*a*c^5*x^4 + 1472*a*c^
4*d*x^6 + 2112*a*c^3*d^2*x^8) + b^(3/2)*(10*a^4*c^4*d + 300*a^4*c^3*d^2*x^2
 - 570*a^4*c^2*d^3*x^4 - 760*a^4*c*d^4*x^6 - 300*a^4*d^5*x^8) + b^5*Sqrt[a
```

```
+ b*x^2]*(-256*c^5*x^5 - 768*c^4*d*x^7 - 768*c^3*d^2*x^9) + b^2*Sqrt[a + b*
x^2]*(-60*a^3*c^4*d*x - 660*a^3*c^3*d^2*x^3 + 440*a^3*c^2*d^3*x^5 + 600*a^3
*c*d^4*x^7 + 240*a^3*d^5*x^9) + b^(11/2)*(256*c^5*x^6 + 768*c^4*d*x^8 + 768
*c^3*d^2*x^10) + b^(5/2)*(8*a^3*c^5 + 204*a^3*c^4*d*x^2 + 1284*a^3*c^3*d^2*
x^4 - 440*a^3*c^2*d^3*x^6 - 600*a^3*c*d^4*x^8 - 240*a^3*d^5*x^10))/(48*a^3*
c^3*d^3*(c + d*x^2)^3 + 864*a^2*b*c^3*d^3*x^2*(c + d*x^2)^3 + 2304*a*b^2*c^
3*d^3*x^4*(c + d*x^2)^3 + 1536*b^3*c^3*d^3*x^6*(c + d*x^2)^3 - 288*a^2*Sqrt
[b]*c^3*d^3*x*Sqrt[a + b*x^2]*(c + d*x^2)^3 - 1536*a*b^(3/2)*c^3*d^3*x^3*Sq
rt[a + b*x^2]*(c + d*x^2)^3 - 1536*b^(5/2)*c^3*d^3*x^5*Sqrt[a + b*x^2]*(c +
 d*x^2)^3) - (b^3*ArcTan[(Sqrt[b]*Sqrt[c])/Sqrt[-(b*c) + a*d] + (Sqrt[b]*d*
x^2)/(Sqrt[c]*Sqrt[-(b*c) + a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b
*c) + a*d])])/(Sqrt[c]*d^3*Sqrt[-(b*c) + a*d]) + (5*a^3*ArcTanh[(Sqrt[b]*Sq
rt[c])/Sqrt[b*c - a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[b*c - a*d]) - (d*x*S
qrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])])/(16*c^(7/2)*Sqrt[b*c - a*d]) -
(b^3*ArcTanh[(Sqrt[b]*Sqrt[c])/Sqrt[b*c - a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*S
qrt[b*c - a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])])/(Sqrt[c
]*d^3*Sqrt[b*c - a*d])
```

fricas [B] time = 1.51, size = 706, normalized size = 4.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(15*(a^3*d^3*x^6 + 3*a^3*c*d^2*x^4 + 3*a^3*c^2*d*x^2 + a^3*c^3)*sqrt
(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4
*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((8*b^3*c^4 + 2*a*b^2*
c^3*d + 5*a^2*b*c^2*d^2 - 15*a^3*c*d^3)*x^5 + 2*(13*a*b^2*c^4 + 7*a^2*b*c^3
*d - 20*a^3*c^2*d^2)*x^3 + 33*(a^2*b*c^4 - a^3*c^3*d)*x)*sqrt(b*x^2 + a))/(
b*c^8 - a*c^7*d + (b*c^5*d^3 - a*c^4*d^4)*x^6 + 3*(b*c^6*d^2 - a*c^5*d^3)*x
^4 + 3*(b*c^7*d - a*c^6*d^2)*x^2), -1/96*(15*(a^3*d^3*x^6 + 3*a^3*c*d^2*x^4
 + 3*a^3*c^2*d*x^2 + a^3*c^3)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 +
 a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3
+ (a*b*c^2 - a^2*c*d)*x)) - 2*((8*b^3*c^4 + 2*a*b^2*c^3*d + 5*a^2*b*c^2*d^2
 - 15*a^3*c*d^3)*x^5 + 2*(13*a*b^2*c^4 + 7*a^2*b*c^3*d - 20*a^3*c^2*d^2)*x^
3 + 33*(a^2*b*c^4 - a^3*c^3*d)*x)*sqrt(b*x^2 + a))/(b*c^8 - a*c^7*d + (b*c^
5*d^3 - a*c^4*d^4)*x^6 + 3*(b*c^6*d^2 - a*c^5*d^3)*x^4 + 3*(b*c^7*d - a*c^6
*d^2)*x^2)]
```

giac [B] time = 2.86, size = 846, normalized size = 5.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -5/16*a^3*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a
*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c^3) + 1/24*(48*(sq
rt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*c^3*d^2 - 15*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^3*sqrt(b)*d^5 + 192*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c^4*
d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c^3*d^2 - 150*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 + 75*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
a^4*sqrt(b)*d^5 + 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c^5 - 64*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d + 288*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*a^2*b^(7/2)*c^3*d^2 - 440*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/
2)*c^2*d^3 + 440*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*c*d^4 - 150*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 + 192*(sqrt(b)*x - sqrt(b*x^2
```

```
 + a))^4*a^2*b^(9/2)*c^4*d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/2)
*c^3*d^2 + 360*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*c^2*d^3 - 420*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 + 150*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*a^6*sqrt(b)*d^5 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7/2)
*c^3*d^2 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*c^2*d^3 + 120*(sq
rt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 - 75*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^7*sqrt(b)*d^5 + 8*a^6*b^(5/2)*c^2*d^3 + 10*a^7*b^(3/2)*c*d^4 + 1
5*a^8*sqrt(b)*d^5)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3*c^3*
d^3)
```

maple [B] time = 0.04, size = 21220, normalized size = 147.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^4,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{2}}}{\left(dx^2 + c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2 + a\right)^{5/2}}{\left(d\,x^2 + c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^4,x)
```

```
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

3.70 $\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^5} dx$

Optimal. Leaf size=249

$$\frac{5a^3(8bc-7ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8bc-7ad)}{128c^4\left(c+dx^2\right)(bc-ad)}+\frac{5ax\left(a+bx^2\right)^{3/2}(8bc-7ad)}{192c^3\left(c+dx^2\right)^2(bc-ad)}+\frac{x\left(a+bx^2\right)^{5/2}(8bc}{48c^2\left(c+dx^2\right)^3(b}$$

Rubi [A] time = 0.14, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {382, 378, 377, 208}

$$\frac{5a^2x\sqrt{a+bx^2}(8bc-7ad)}{128c^4\left(c+dx^2\right)(bc-ad)}+\frac{5a^3(8bc-7ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{3/2}}+\frac{x\left(a+bx^2\right)^{5/2}(8bc-7ad)}{48c^2\left(c+dx^2\right)^3(bc-ad)}+\frac{5ax\left(a+bx^2\right)^{3/2}(8bc-7ad)}{192c^3\left(c+dx^2\right)^2(bc-ad)}-\frac{dx\left(a+bx^2\right)^{7/2}}{8c\left(c+dx^2\right)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

```
[Out] -(d*x*(a + b*x^2)^(7/2))/(8*c*(b*c - a*d)*(c + d*x^2)^4) + ((8*b*c - 7*a*d)
*x*(a + b*x^2)^(5/2))/(48*c^2*(b*c - a*d)*(c + d*x^2)^3) + (5*a*(8*b*c - 7*
a*d)*x*(a + b*x^2)^(3/2))/(192*c^3*(b*c - a*d)*(c + d*x^2)^2) + (5*a^2*(8*b
*c - 7*a*d)*x*Sqrt[a + b*x^2])/(128*c^4*(b*c - a*d)*(c + d*x^2)) + (5*a^3*(
8*b*c - 7*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128
*c^(9/2)*(b*c - a*d)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^5}\,dx = -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^4}\,dx}{8c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x\left(a+bx^2\right)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{(5a(8bc-7ad))\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^3}\,dx}{48c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x\left(a+bx^2\right)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{5a(8bc-7ad)x\left(a+bx^2\right)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} + \frac{(5a}{}$$

$$= -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x\left(a+bx^2\right)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{5a(8bc-7ad)x\left(a+bx^2\right)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} + \frac{5a^2}{12}$$

$$= -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x\left(a+bx^2\right)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{5a(8bc-7ad)x\left(a+bx^2\right)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} + \frac{5a^2}{12}$$

$$= -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x\left(a+bx^2\right)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{5a(8bc-7ad)x\left(a+bx^2\right)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} + \frac{5a^2}{12}$$

Mathematica [A] time = 1.09, size = 306, normalized size = 1.23

$$\frac{x\left(\frac{15a^3\left(c+dx^2\right)^4(7ad-8bc)\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)}{\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}} - c\left(-a^4d\left(279c^3+511c^2dx^2+385cd^2x^4+105d^3x^6\right)+a^3b\left(264c^4-21c^3dx^2-323c^2d^2x^4-335cd^3x^6-105d^4x^8\right)+2a^2b^2cx^2\left(236c^3+173c^2dx^2+106cd^2x^4+25d^3x^6\right)+8ab^3c^2x^4\left(34c^2+13cdx^2+3d^2x^4\right)+16b^4c^3x^6\left(4c+dx^2\right)\right)\right)}{384c^5\sqrt{a+bx^2}\left(c+dx^2\right)^4(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

```
[Out] (x*(-(c*(16*b^4*c^3*x^6*(4*c + d*x^2) + 8*a*b^3*c^2*x^4*(34*c^2 + 13*c*d*x^
2 + 3*d^2*x^4) + 2*a^2*b^2*c*x^2*(236*c^3 + 173*c^2*d*x^2 + 106*c*d^2*x^4 +
 25*d^3*x^6) - a^4*d*(279*c^3 + 511*c^2*d*x^2 + 385*c*d^2*x^4 + 105*d^3*x^6
) + a^3*b*(264*c^4 - 21*c^3*d*x^2 - 323*c^2*d^2*x^4 - 335*c*d^3*x^6 - 105*d
^4*x^8))) + (15*a^3*(-8*b*c + 7*a*d)*(c + d*x^2)^4*ArcTanh[Sqrt[((b*c - a*d
)*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(384*c^
5*(-(b*c) + a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^4)
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

```
[Out] $Aborted
```

fricas [B] time = 2.68, size = 1258, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] [1/1536*(15*(8*a^3*b*c^5 - 7*a^4*c^4*d + (8*a^3*b*c*d^4 - 7*a^4*d^5)*x^8 +
4*(8*a^3*b*c^2*d^3 - 7*a^4*c*d^4)*x^6 + 6*(8*a^3*b*c^3*d^2 - 7*a^4*c^2*d^3)
*x^4 + 4*(8*a^3*b*c^4*d - 7*a^4*c^3*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b
^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2
 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*
x^4 + 2*c*d*x^2 + c^2)) + 4*((16*b^4*c^5*d + 8*a*b^3*c^4*d^2 + 26*a^2*b^2*c
^3*d^3 - 155*a^3*b*c^2*d^4 + 105*a^4*c*d^5)*x^7 + (64*b^4*c^6 + 24*a*b^3*c^
5*d + 100*a^2*b^2*c^4*d^2 - 573*a^3*b*c^3*d^3 + 385*a^4*c^2*d^4)*x^5 + (208
*a*b^3*c^6 + 50*a^2*b^2*c^5*d - 769*a^3*b*c^4*d^2 + 511*a^4*c^3*d^3)*x^3 +
3*(88*a^2*b^2*c^6 - 181*a^3*b*c^5*d + 93*a^4*c^4*d^2)*x)*sqrt(b*x^2 + a))/(
b^2*c^11 - 2*a*b*c^10*d + a^2*c^9*d^2 + (b^2*c^7*d^4 - 2*a*b*c^6*d^5 + a^2*
c^5*d^6)*x^8 + 4*(b^2*c^8*d^3 - 2*a*b*c^7*d^4 + a^2*c^6*d^5)*x^6 + 6*(b^2*c
^9*d^2 - 2*a*b*c^8*d^3 + a^2*c^7*d^4)*x^4 + 4*(b^2*c^10*d - 2*a*b*c^9*d^2 +
 a^2*c^8*d^3)*x^2), -1/768*(15*(8*a^3*b*c^5 - 7*a^4*c^4*d + (8*a^3*b*c*d^4
- 7*a^4*d^5)*x^8 + 4*(8*a^3*b*c^2*d^3 - 7*a^4*c*d^4)*x^6 + 6*(8*a^3*b*c^3*d
^2 - 7*a^4*c^2*d^3)*x^4 + 4*(8*a^3*b*c^4*d - 7*a^4*c^3*d^2)*x^2)*sqrt(-b*c^
2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b
*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*b^4*c
^5*d + 8*a*b^3*c^4*d^2 + 26*a^2*b^2*c^3*d^3 - 155*a^3*b*c^2*d^4 + 105*a^4*c
*d^5)*x^7 + (64*b^4*c^6 + 24*a*b^3*c^5*d + 100*a^2*b^2*c^4*d^2 - 573*a^3*b*
c^3*d^3 + 385*a^4*c^2*d^4)*x^5 + (208*a*b^3*c^6 + 50*a^2*b^2*c^5*d - 769*a^
3*b*c^4*d^2 + 511*a^4*c^3*d^3)*x^3 + 3*(88*a^2*b^2*c^6 - 181*a^3*b*c^5*d +
93*a^4*c^4*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^11 - 2*a*b*c^10*d + a^2*c^9*d^2
+ (b^2*c^7*d^4 - 2*a*b*c^6*d^5 + a^2*c^5*d^6)*x^8 + 4*(b^2*c^8*d^3 - 2*a*b*
c^7*d^4 + a^2*c^6*d^5)*x^6 + 6*(b^2*c^9*d^2 - 2*a*b*c^8*d^3 + a^2*c^7*d^4)*
x^4 + 4*(b^2*c^10*d - 2*a*b*c^9*d^2 + a^2*c^8*d^3)*x^2)]
```

giac [B] time = 9.31, size = 1448, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -5/128*(8*a^3*b^(3/2)*c - 7*a^4*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*
x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^5 - a*c^4*d)*s
qrt(-b^2*c^2 + a*b*c*d)) - 1/192*(120*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^3*
b^(3/2)*c*d^6 - 105*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*sqrt(b)*d^7 - 768*
(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(11/2)*c^5*d^2 + 768*(sqrt(b)*x - sqrt(b
*x^2 + a))^12*a*b^(9/2)*c^4*d^3 + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3
*b^(5/2)*c^2*d^5 - 2310*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(3/2)*c*d^6
+ 735*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^5*sqrt(b)*d^7 - 2048*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*b^(13/2)*c^6*d + 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^10*
a^2*b^(9/2)*c^4*d^3 + 8320*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b^(7/2)*c^3
*d^4 - 15600*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(5/2)*c^2*d^5 + 9800*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^5*b^(3/2)*c*d^6 - 2205*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*a^6*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(15/
2)*c^7 + 1024*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(13/2)*c^6*d - 4864*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(11/2)*c^5*d^2 + 21888*(sqrt(b)*x - sqrt(b
*x^2 + a))^8*a^3*b^(9/2)*c^4*d^3 - 38000*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^
4*b^(7/2)*c^3*d^4 + 37400*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^5*b^(5/2)*c^2*d
^5 - 18550*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(3/2)*c*d^6 + 3675*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*a^7*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*a^2*b^(13/2)*c^6*d - 9472*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(9/2)*
c^4*d^3 + 32896*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*b^(7/2)*c^3*d^4 - 35376
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(5/2)*c^2*d^5 + 18200*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*a^7*b^(3/2)*c*d^6 - 3675*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
a^8*sqrt(b)*d^7 - 768*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(11/2)*c^5*d^2
- 1536*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/2)*c^4*d^3 - 2944*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*a^6*b^(7/2)*c^3*d^4 + 12528*(sqrt(b)*x - sqrt(b*x^2
```

```
+ a))^4*a^7*b^(5/2)*c^2*d^5 - 9170*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^8*b^(3
/2)*c*d^6 + 2205*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^9*sqrt(b)*d^7 - 256*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2)*c^4*d^3 - 256*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*a^7*b^(7/2)*c^3*d^4 - 608*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^8*b^
(5/2)*c^2*d^5 + 1960*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^9*b^(3/2)*c*d^6 - 73
5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*sqrt(b)*d^7 - 16*a^8*b^(7/2)*c^3*d^4
 - 24*a^9*b^(5/2)*c^2*d^5 - 50*a^10*b^(3/2)*c*d^6 + 105*a^11*sqrt(b)*d^7)/(
(b*c^5*d^3 - a*c^4*d^4)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^4)
```

maple [B] time = 0.05, size = 28625, normalized size = 114.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^5,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{2}}}{\left(dx^2 + c\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2 + a\right)^{5/2}}{\left(d\,x^2 + c\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^5,x)
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**5,x)
[Out] Timed out
```

3.71 $\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx$

Optimal. Leaf size=30

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {402, 216, 377, 203}

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(1 + x^2),x]

[Out] -ArcSin[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx &= 2\int \frac{1}{\sqrt{1-x^2}\left(1+x^2\right)}\,dx-\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\sin^{-1}(x)+2\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)\\ &= -\sin^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 1.00

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(1 + x^2),x]
```

```
[Out] -ArcSin[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

IntegrateAlgebraic [A] time = 0.10, size = 54, normalized size = 1.80

$$2\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{1-x^2}}{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]/(1 + x^2),x]
```

```
[Out] 2*ArcTan[Sqrt[1 - x^2]/(1 + x)] - Sqrt[2]*ArcTan[(Sqrt[2]*x*Sqrt[1 - x^2])/
(-1 + x^2)]
```

fricas [A] time = 0.85, size = 42, normalized size = 1.40

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2x}\right)+2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x) + 2*arctan((sqrt(-x^2 + 1) -
1)/x)
```

giac [B] time = 0.61, size = 95, normalized size = 3.17

$$-\frac{1}{2}\pi\mathrm{sgn}(x)+\frac{1}{2}\sqrt{2}\left(\pi\mathrm{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*pi*sgn(x) + 1/2*sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x
^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - arctan(-1/2*x*((sqrt(-x^2
+ 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

maple [A] time = 0.02, size = 33, normalized size = 1.10

$$-\arcsin(x)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}\,x}{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(x^2+1),x)
```

```
[Out] -arcsin(x)-2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-x^2+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/(x^2 + 1), x)
```

mupad [B] time = 0.39, size = 83, normalized size = 2.77

$$-\operatorname{asin}(x)+\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\mathrm{i}}\right)1\mathrm{i}}{2}-\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^2}\,1\mathrm{i}}{x+1\mathrm{i}}\right)1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x^2 + 1),x)
```

```
[Out] (2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i)
/2 - asin(x) - (2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i
)/(x + 1i))*1i)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(x-1)(x+1)}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**2 + 1), x)
```

3.72 $\int \frac{\sqrt{1+x^2}}{-1+x^2} \, dx$

Optimal. Leaf size=27

$$\sinh^{-1}(x) - \sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+1}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {402, 215, 377, 207}

$$\sinh^{-1}(x) - \sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/(-1 + x^2),x]
```

```
[Out] ArcSinh[x] - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^2]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^2}}{-1+x^2} \, dx &= 2 \int \frac{1}{\left(-1+x^2\right)\sqrt{1+x^2}} \, dx + \int \frac{1}{\sqrt{1+x^2}} \, dx \\
&= \sinh^{-1}(x) + 2 \operatorname{Subst}\left(\int \frac{1}{-1+2x^2} \, dx, x, \frac{x}{\sqrt{1+x^2}}\right) \\
&= \sinh^{-1}(x) - \sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^2}}\right)
\end{aligned}$$

Mathematica [B] time = 0.03, size = 64, normalized size = 2.37

$$\frac{\log\left(\sqrt{2}\sqrt{x^2+1}-x+1\right)-\log\left(\sqrt{2}\sqrt{x^2+1}+x+1\right)+\log(1-x)-\log(x+1)}{\sqrt{2}}+\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/(-1 + x^2),x]
```

```
[Out] ArcSinh[x] + (Log[1 - x] - Log[1 + x] + Log[1 - x + Sqrt[2]*Sqrt[1 + x^2]]
- Log[1 + x + Sqrt[2]*Sqrt[1 + x^2]])/Sqrt[2]
```

IntegrateAlgebraic [B] time = 0.11, size = 57, normalized size = 2.11

$$-\log\left(\sqrt{x^2+1}-x\right)-\sqrt{2}\tanh^{-1}\left(-\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^2+1}\,x}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^2]/(-1 + x^2),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[1/Sqrt[2] - x^2/Sqrt[2] + (x*Sqrt[1 + x^2])/Sqrt[2]]) - L
og[-x + Sqrt[1 + x^2]]
```

fricas [B] time = 0.64, size = 67, normalized size = 2.48

$$\frac{1}{2}\sqrt{2}\log\left(\frac{9\,x^2-2\sqrt{2}\left(3\,x^2+1\right)-2\sqrt{x^2+1}\left(3\sqrt{2}\,x-4\,x\right)+3}{x^2-1}\right)-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((9*x^2 - 2*sqrt(2)*(3*x^2 + 1) - 2*sqrt(x^2 + 1)*(3*sqrt(2)
*x - 4*x) + 3)/(x^2 - 1)) - log(-x + sqrt(x^2 + 1))
```

giac [B] time = 0.62, size = 70, normalized size = 2.59

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|2\left(x-\sqrt{x^2+1}\right)^2-4\sqrt{2}-6\right|}{\left|2\left(x-\sqrt{x^2+1}\right)^2+4\sqrt{2}-6\right|}\right)-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(2*(x - sqrt(x^2 + 1))^2 - 4*sqrt(2) - 6)/abs(2*(x - sq
rt(x^2 + 1))^2 + 4*sqrt(2) - 6)) - log(-x + sqrt(x^2 + 1))
```

maple [B] time = 0.01, size = 84, normalized size = 3.11

$$\mathrm{arcsinh}(x)+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{(-2x+2)\sqrt{2}}{4\sqrt{-2x+(x+1)^2}}\right)}{2}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{2x+(x-1)^2}}\right)}{2}-\frac{\sqrt{-2x+(x+1)^2}}{2}+\frac{\sqrt{2x+(x-1)^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(x^2-1),x)
```

```
[Out] -1/2*((x+1)^2-2*x)^(1/2)+arcsinh(x)+1/2*2^(1/2)*arctanh(1/4*(2-2*x)*2^(1/2)
/((x+1)^2-2*x)^(1/2))+1/2*((x-1)^2+2*x)^(1/2)-1/2*2^(1/2)*arctanh(1/4*(2*x+
2)*2^(1/2)/((x-1)^2+2*x)^(1/2))
```

maxima [B] time = 2.98, size = 59, normalized size = 2.19

$$-\frac{1}{2}\sqrt{2}\,\operatorname{arsinh}\left(\frac{2x}{|2x+2|}-\frac{2}{|2x+2|}\right)-\frac{1}{2}\sqrt{2}\,\operatorname{arsinh}\left(\frac{2x}{|2x-2|}+\frac{2}{|2x-2|}\right)+\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arcsinh(2*x/abs(2*x + 2) - 2/abs(2*x + 2)) - 1/2*sqrt(2)*arcsi
nh(2*x/abs(2*x - 2) + 2/abs(2*x - 2)) + arcsinh(x)
```

mupad [B] time = 0.17, size = 59, normalized size = 2.19

$$\operatorname{asinh}(x)+\frac{\sqrt{2}\,\left(\ln(x-1)-\ln\left(x+\sqrt{2}\,\sqrt{x^2+1}\,+1\right)\right)}{2}-\frac{\sqrt{2}\,\left(\ln(x+1)-\ln\left(\sqrt{2}\,\sqrt{x^2+1}\,-x+1\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(x^2 - 1),x)
```

```
[Out] asinh(x) + (2^(1/2)*(log(x - 1) - log(x + 2^(1/2)*(x^2 + 1)^(1/2) + 1)))/2
- (2^(1/2)*(log(x + 1) - log(2^(1/2)*(x^2 + 1)^(1/2) - x + 1)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2+1}}{(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(x**2-1),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/((x - 1)*(x + 1)), x)
```

3.73 $\int \frac{\sqrt{1-x^2}}{-1+2x^2} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {402, 216, 377, 207}

$$-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
```

```
[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x^2}}{-1+2x^2} \, dx &= -\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}} \, dx\right)+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\left(-1+2x^2\right)} \, dx \\
&= -\frac{1}{2}\sin^{-1}(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2} \, dx,x,\frac{x}{\sqrt{1-x^2}}\right) \\
&= -\frac{1}{2}\sin^{-1}(x)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
```

```
[Out] -1/2*ArcSin[x] - ArcTanh[x/Sqrt[1 - x^2]]/2
```

IntegrateAlgebraic [C] time = 0.07, size = 55, normalized size = 2.20

$$-\frac{1}{2}i\log\left(\sqrt{1-x^2}-ix\right)-\frac{1}{2}i\tan^{-1}\left(-2x^2-2i\sqrt{1-x^2}\,x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
```

```
[Out] (-1/2*I)*ArcTan[1 - 2*x^2 - (2*I)*x*Sqrt[1 - x^2]] - (I/2)*Log[(-I)*x + Sqr
t[1 - x^2]]
```

fricas [B] time = 1.13, size = 74, normalized size = 2.96

$$\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}\,(x+1)-x-1}{x^2}\right)-\frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}\,(x-1)+x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="fricas")
```

```
[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(-(x^2 + sqrt(-x^2 + 1)*(x + 1) - x
 - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) + x - 1)/x^2)
```

giac [B] time = 0.61, size = 118, normalized size = 4.72

$$-\frac{1}{4}\pi\operatorname{sgn}(x)-\frac{1}{2}\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)-\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="giac")
```

```
[Out] -1/4*pi*sgn(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-
x^2 + 1) - 1)) - 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)
/x + 2)) + 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x - 2
))
```

maple [B] time = 0.04, size = 187, normalized size = 7.48

$$-\frac{\sqrt{2}\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\sqrt{2}}{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}+\frac{\sqrt{2}\,\arcsin(x)}{4}+\frac{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{4}\right)}{2}+\frac{\sqrt{2}\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\sqrt{2}}{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}-\frac{\sqrt{2}\,\arcsin(x)}{4}+\frac{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(2*x^2-1),x)
```

```
[Out] -1/2*2^(1/2)*(1/4*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)+
1/4*2^(1/2)*arcsin(x)-1/4*2^(1/2)*arctanh(((x+1/2*2^(1/2))*2^(1/2)+1)*2^(1/
2)/(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)))+1/2*2^(1/2)*(
1/4*(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)-1/4*2^(1/2)*ar
csin(x)-1/4*2^(1/2)*arctanh((-(x-1/2*2^(1/2))*2^(1/2)+1)*2^(1/2)/(-4*(x-1/2
*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)))
```

maxima [B] time = 3.07, size = 110, normalized size = 4.40

$$-\frac{1}{8}\sqrt{2}\left(2\sqrt{2}\arcsin(x)-\sqrt{2}\log\left(\frac{1}{4}\sqrt{2}+\frac{\sqrt{2}\sqrt{-x^2+1}}{|4x+2\sqrt{2}|}+\frac{1}{|4x+2\sqrt{2}|}\right)+\sqrt{2}\log\left(-\frac{1}{4}\sqrt{2}+\frac{\sqrt{2}\sqrt{-x^2+1}}{|4x-2\sqrt{2}|}+\frac{1}{|4x-2\sqrt{2}|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*(2*sqrt(2)*arcsin(x) - sqrt(2)*log(1/4*sqrt(2) + sqrt(2)*sqrt(
-x^2 + 1)/abs(4*x + 2*sqrt(2)) + 1/abs(4*x + 2*sqrt(2))) + sqrt(2)*log(-1/4
*sqrt(2) + sqrt(2)*sqrt(-x^2 + 1)/abs(4*x - 2*sqrt(2)) + 1/abs(4*x - 2*sqrt
(2))))
```

mupad [B] time = 5.35, size = 85, normalized size = 3.40

$$-\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}-1\right)1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{4}+\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}+1\right)1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{4}-\frac{\mathrm{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(2*x^2 - 1),x)
```

```
[Out] log((2^(1/2)*((2^(1/2)*x)/2 + 1)*1i + (1 - x^2)^(1/2)*1i)/(x + 2^(1/2)/2))/
4 - log((2^(1/2)*((2^(1/2)*x)/2 - 1)*1i - (1 - x^2)^(1/2)*1i)/(x - 2^(1/2)/
2))/4 - asin(x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(x-1)(x+1)}}{2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(2*x**2-1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(2*x**2 - 1), x)
```

3.74 $\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=169

$$\frac{(2bc-ad)\left(5a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}+\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-44abcd+44b^2c^2\right)}{48b^3}+\frac{5dx\sqrt{a+bx^2}}{}$$

Rubi [A] time = 0.14, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 528, 388, 217, 206}

$$\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-44abcd+44b^2c^2\right)}{48b^3}+\frac{(2bc-ad)\left(5a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}+\frac{5dx\sqrt{a+bx^2}\left(c+dx^2\right)(2bc-ad)}{24b^2}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

```
[Out] (d*(44*b^2*c^2 - 44*a*b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(48*b^3) + (5*
d*(2*b*c - a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(24*b^2) + (d*x*Sqrt[a + b*x
^2]*(c + d*x^2)^2)/(6*b) + ((2*b*c - a*d)*(8*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^
2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b} + \frac{\int \frac{\left(c+dx^2\right)\left(c(6bc-ad)+5d(2bc-ad)x^2\right)}{\sqrt{a+bx^2}}\,dx}{6b}$$

$$= \frac{5d(2bc-ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{24b^2} + \frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b} + \frac{\int \frac{c\left(24b^2c^2-14abcd+5a^2d^2\right)+d\left(44b^2c^2-\right.}{\sqrt{a+bx^2}}}{24b^2}$$

$$= \frac{d\left(44b^2c^2-44abcd+15a^2d^2\right)x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{24b^2} + \frac{dx\sqrt{a+bx^2}}{6l}$$

$$= \frac{d\left(44b^2c^2-44abcd+15a^2d^2\right)x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{24b^2} + \frac{dx\sqrt{a+bx^2}}{6l}$$

$$= \frac{d\left(44b^2c^2-44abcd+15a^2d^2\right)x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{24b^2} + \frac{dx\sqrt{a+bx^2}}{6l}$$

Mathematica [A] time = 5.09, size = 140, normalized size = 0.83

$$\frac{\sqrt{b}\,dx\sqrt{a+bx^2}\left(15a^2d^2-2abd\left(27c+5dx^2\right)+4b^2\left(18c^2+9cdx^2+2d^2x^4\right)\right)+3\left(-5a^3d^3+18a^2bcd^2-24ab^2c^2d+16b^3c^3\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{48b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[a + b*x^2]*(15*a^2*d^2 - 2*a*b*d*(27*c + 5*d*x^2) + 4*b^2
*(18*c^2 + 9*c*d*x^2 + 2*d^2*x^4)) + 3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a^
2*b*c*d^2 - 5*a^3*d^3)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(48*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.17, size = 148, normalized size = 0.88

$$\frac{\sqrt{a+bx^2}\left(15a^2d^3x-54abcd^2x-10abd^3x^3+72b^2c^2dx+36b^2cd^2x^3+8b^2d^3x^5\right)}{48b^3}+\frac{\left(5a^3d^3-18a^2bcd^2+24ab^2c^2d-16b^3c^3\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(72*b^2*c^2*d*x - 54*a*b*c*d^2*x + 15*a^2*d^3*x + 36*b^2*c
*d^2*x^3 - 10*a*b*d^3*x^3 + 8*b^2*d^3*x^5))/(48*b^3) + ((-16*b^3*c^3 + 24*a
*b^2*c^2*d - 18*a^2*b*c*d^2 + 5*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]
])/(16*b^(7/2))
```

fricas [A] time = 1.18, size = 300, normalized size = 1.78

$$\left[-\frac{3\left(16b^3c^3-24ab^2c^2d+18a^2bcd^2-5a^3d^3\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8b^3d^3x^5+2\left(18b^3cd^2-5ab^2d^3\right)x^3+3\left(24b^3c^2d-18ab^2cd^2+5a^2bd^3\right)x\right)\sqrt{bx^2+a}}{96b^4}, -\frac{3\left(16b^3c^3-24ab^2c^2d+18a^2bcd^2-5a^3d^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(8b^3d^3x^5+2\left(18b^3cd^2-5ab^2d^3\right)x^3+3\left(24b^3c^2d-18ab^2cd^2+5a^2bd^3\right)x\right)\sqrt{bx^2+a}}{48b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b
)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*b^3*d^3*x^5 + 2*(1
8*b^3*c*d^2 - 5*a*b^2*d^3)*x^3 + 3*(24*b^3*c^2*d - 18*a*b^2*c*d^2 + 5*a^2*b
*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/48*(3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a
^2*b*c*d^2 - 5*a^3*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^
```

```
3*d^3*x^5 + 2*(18*b^3*c*d^2 - 5*a*b^2*d^3)*x^3 + 3*(24*b^3*c^2*d - 18*a*b^2
*c*d^2 + 5*a^2*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

giac [A] time = 0.64, size = 150, normalized size = 0.89

$$\frac{1}{48}\left(2\left(\frac{4\,d^3x^2}{b}+\frac{18\,b^4cd^2-5\,ab^3d^3}{b^5}\right)x^2+\frac{3\left(24\,b^4c^2d-18\,ab^3cd^2+5\,a^2b^2d^3\right)}{b^5}\right)\sqrt{bx^2+a}\,x-\frac{\left(16\,b^3c^3-24\,ab^2c^2d+18\,a^2bcd^2-5\,a^3d^3\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^3*x^2/b + (18*b^4*c*d^2 - 5*a*b^3*d^3)/b^5)*x^2 + 3*(24*b^4*c^
2*d - 18*a*b^3*c*d^2 + 5*a^2*b^2*d^3)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3
*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.01, size = 228, normalized size = 1.35

$$\frac{\sqrt{b\,x^2+a}\;d^3x^5}{6b}-\frac{5\sqrt{b\,x^2+a}\;a\,d^3x^3}{24b^2}+\frac{3\sqrt{b\,x^2+a}\;c\,d^2x^3}{4b}-\frac{5a^3d^3\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{7}{2}}}+\frac{9a^2c\,d^2\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}-\frac{3a\,c^2d\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{c^3\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{5\sqrt{b\,x^2+a}\;a^2d^3x}{16b^3}-\frac{9\sqrt{b\,x^2+a}\;ac\,d^2x}{8b^2}+\frac{3\sqrt{b\,x^2+a}\;c^2dx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*d^3*x^5/b*(b*x^2+a)^(1/2)-5/24*d^3*a/b^2*x^3*(b*x^2+a)^(1/2)+5/16*d^3*a
^2/b^3*x*(b*x^2+a)^(1/2)-5/16*d^3*a^3/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
+3/4*c*d^2*x^3/b*(b*x^2+a)^(1/2)-9/8*c*d^2*a/b^2*x*(b*x^2+a)^(1/2)+9/8*c*d^
2*a^2/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+3/2*c^2*d*x/b*(b*x^2+a)^(1/2)-3
/2*c^2*d*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+c^3*ln(b^(1/2)*x+(b*x^2+a)
^(1/2))/b^(1/2)
```

maxima [A] time = 1.36, size = 199, normalized size = 1.18

$$\frac{\sqrt{bx^2+a}\,d^3x^5}{6\,b}+\frac{3\,\sqrt{bx^2+a}\,cd^2x^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}\,ad^3x^3}{24\,b^2}+\frac{3\,\sqrt{bx^2+a}\,c^2dx}{2\,b}-\frac{9\,\sqrt{bx^2+a}\,acd^2x}{8\,b^2}+\frac{5\,\sqrt{bx^2+a}\,a^2d^3x}{16\,b^3}+\frac{c^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{3\,ac^2d\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{9\,a^2cd^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{5\,a^3d^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*d^3*x^5/b + 3/4*sqrt(b*x^2 + a)*c*d^2*x^3/b - 5/24*sqrt
(b*x^2 + a)*a*d^3*x^3/b^2 + 3/2*sqrt(b*x^2 + a)*c^2*d*x/b - 9/8*sqrt(b*x^2
+ a)*a*c*d^2*x/b^2 + 5/16*sqrt(b*x^2 + a)*a^2*d^3*x/b^3 + c^3*arcsinh(b*x/s
qrt(a*b))/sqrt(b) - 3/2*a*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 9/8*a^2*c*
d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/16*a^3*d^3*arcsinh(b*x/sqrt(a*b))/b^
(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^3}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^3/(a + b*x^2)^(1/2), x)
```

sympy [A] time = 13.27, size = 400, normalized size = 2.37

$$\frac{5a^{\frac{5}{2}}d^3x}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{9a^{\frac{3}{2}}cd^2x}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}d^3x^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}\,c^2dx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{3\sqrt{a}\,cd^2x^3}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,d^3x^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^3d^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{9a^2cd^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{3ac^2d\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+c^3\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b<0\\\frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b>0\\\frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\: b>0\wedge a<0\end{cases}\right)+\frac{3cd^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{d^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*d**3*x/(16*b**3*sqrt(1 + b*x**2/a)) - 9*a**(3/2)*c*d**2*x/(8*b**
2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*d**3*x**3/(48*b**2*sqrt(1 + b*x**2/a)) +
 3*sqrt(a)*c**2*d*x*sqrt(1 + b*x**2/a)/(2*b) - 3*sqrt(a)*c*d**2*x**3/(8*b*s
qrt(1 + b*x**2/a)) - sqrt(a)*d**3*x**5/(24*b*sqrt(1 + b*x**2/a)) - 5*a**3*d
**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + 9*a**2*c*d**2*asinh(sqrt(b)*x/
sqrt(a))/(8*b**(5/2)) - 3*a*c**2*d*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) +
c**3*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)),
(sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acos
h(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + 3*c*d**2*x**5/(4*sqrt(a)*sq
rt(1 + b*x**2/a)) + d**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.75 $\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=108

$$\frac{(3a^2d^2 - 8abcd + 8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}} + \frac{3dx\sqrt{a+bx^2}\,(2bc-ad)}{8b^2} + \frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b}$$

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {416, 388, 217, 206}

$$\frac{(3a^2d^2 - 8abcd + 8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}} + \frac{3dx\sqrt{a+bx^2}\,(2bc-ad)}{8b^2} + \frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

```
[Out] (3*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b^2) + (d*x*Sqrt[a + b*x^2]*(c + d
*x^2))/(4*b) + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b} + \frac{\int \frac{c(4bc-ad)+3d(2bc-ad)x^2}{\sqrt{a+bx^2}}\,dx}{4b}$$

$$= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2} + \frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b} - \frac{(3ad(2bc-ad)-2bc(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}}}{8b^2}$$

$$= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2} + \frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b} - \frac{(3ad(2bc-ad)-2bc(4bc-ad))\,\mathrm{Subst}\left(\int\right.}{8b^2}$$

$$= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2} + \frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b} + \frac{(8b^2c^2-8abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

Mathematica [C] time = 2.49, size = 160, normalized size = 1.48

$$\frac{x\sqrt{\frac{bx^2}{a}+1}\left(-2bx^2(c+dx^2)^2\,{}_3F_2\left(\frac{3}{2},\frac{3}{2},2;1,\frac{9}{2};-\frac{bx^2}{a}\right)-4bx^2(2c^2+3cdx^2+d^2x^4)\,{}_2F_1\left(\frac{3}{2},\frac{3}{2};\frac{9}{2};-\frac{bx^2}{a}\right)+7a(15c^2+10cdx^2+3d^2x^4)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2};\frac{7}{2};-\frac{bx^2}{a}\right)\right)}{105a\sqrt{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[1 + (b*x^2)/a]*(7*a*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*Hypergeometri
c2F1[1/2, 1/2, 7/2, -((b*x^2)/a)] - 4*b*x^2*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*H
ypergeometric2F1[3/2, 3/2, 9/2, -((b*x^2)/a)] - 2*b*x^2*(c + d*x^2)^2*Hyper
geometricPFQ[{3/2, 3/2, 2}, {1, 9/2}, -((b*x^2)/a)]))/(105*a*Sqrt[a + b*x^2
])
```

IntegrateAlgebraic [A] time = 0.11, size = 95, normalized size = 0.88

$$\frac{(-3a^2d^2+8abcd-8b^2c^2)\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{8b^{5/2}}+\frac{\sqrt{a+bx^2}\,(-3ad^2x+8bcdx+2bd^2x^3)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*b*c*d*x - 3*a*d^2*x + 2*b*d^2*x^3))/(8*b^2) + ((-8*b^2*
c^2 + 8*a*b*c*d - 3*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2
))
```

fricas [A] time = 1.31, size = 192, normalized size = 1.78

$$\left[\frac{(8b^2c^2-8abcd+3a^2d^2)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2b^2d^2x^3+(8b^2cd-3abd^2)x\right)\sqrt{bx^2+a}}{16b^3}, -\frac{(8b^2c^2-8abcd+3a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(2b^2d^2x^3+(8b^2cd-3abd^2)x\right)\sqrt{bx^2+a}}{8b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) + 2*(2*b^2*d^2*x^3 + (8*b^2*c*d - 3*a*b*d^2)*x)*sqr
t(b*x^2 + a))/b^3, -1/8*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*d^2*x^3 + (8*b^2*c*d - 3*a*b*d^2)*x)
*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.62, size = 90, normalized size = 0.83

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2d^2x^2}{b}+\frac{8b^2cd-3abd^2}{b^3}\right)x-\frac{(8b^2c^2-8abcd+3a^2d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*d^2*x^2/b + (8*b^2*c*d - 3*a*b*d^2)/b^3)*x - 1/8*(8*
b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
5/2)
```

maple [A] time = 0.01, size = 131, normalized size = 1.21

$$\frac{\sqrt{b\,x^2+a}\,d^2x^3}{4b}+\frac{3a^2d^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}-\frac{acd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}+\frac{c^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{3\sqrt{b\,x^2+a}\,a\,d^2x}{8b^2}+\frac{\sqrt{b\,x^2+a}\,cdx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*d^2*x^3/b*(b*x^2+a)^(1/2)-3/8*d^2*a/b^2*x*(b*x^2+a)^(1/2)+3/8*d^2*a^2/b
^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+c*d*x/b*(b*x^2+a)^(1/2)-c*d*a/b^(3/2)*
ln(b^(1/2)*x+(b*x^2+a)^(1/2))+c^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.47, size = 109, normalized size = 1.01

$$\frac{\sqrt{bx^2+a}\,d^2x^3}{4\,b}+\frac{\sqrt{bx^2+a}\,cdx}{b}-\frac{3\,\sqrt{bx^2+a}\,ad^2x}{8\,b^2}+\frac{c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{acd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{3\,a^2d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*d^2*x^3/b + sqrt(b*x^2 + a)*c*d*x/b - 3/8*sqrt(b*x^2 +
a)*a*d^2*x/b^2 + c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - a*c*d*arcsinh(b*x/sqr
t(a*b))/b^(3/2) + 3/8*a^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^2}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^2/(a + b*x^2)^(1/2), x)
```

sympy [A] time = 6.94, size = 238, normalized size = 2.20

$$-\frac{3a^{\frac{3}{2}}d^2x}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,cdx\sqrt{1+\frac{bx^2}{a}}}{b}-\frac{\sqrt{a}\,d^2x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2d^2\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{acd\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}+c^2\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\,\mathrm{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\,\mathrm{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\,\mathrm{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)+\frac{d^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*d**2*x/(8*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*c*d*x*sqrt(1 + b*x
**2/a)/b - sqrt(a)*d**2*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*a**2*d**2*asinh(s
qrt(b)*x/sqrt(a))/(8*b**(5/2)) - a*c*d*asinh(sqrt(b)*x/sqrt(a))/b**(3/2) +
c**2*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)),
(sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acos
h(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + d**2*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a))
```

3.76 $\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 217, 206}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx &= \frac{dx\sqrt{a+bx^2}}{2b}-\frac{(-2bc+ad)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\
&= \frac{dx\sqrt{a+bx^2}}{2b}-\frac{(-2bc+ad)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\
&= \frac{dx\sqrt{a+bx^2}}{2b}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.98

$$\frac{dx\sqrt{a+bx^2}}{2b}-\frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) - ((-2*b*c + a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 59, normalized size = 1.02

$$\frac{(ad-2bc)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) + ((-2*b*c + a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

fricas [A] time = 0.67, size = 113, normalized size = 1.95

$$\left[\frac{2\,\sqrt{bx^2+a}\,bdx-(2\,bc-ad)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b^2},\ \frac{\sqrt{bx^2+a}\,bdx-(2\,bc-ad)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*d*x - (2*b*c - a*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*d*x - (2*b*c - a*d)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/b^2]
```

giac [A] time = 0.60, size = 49, normalized size = 0.84

$$\frac{\sqrt{bx^2+a}\,dx}{2\,b}-\frac{(2\,bc-ad)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*d*x/b - 1/2*(2*b*c - a*d)*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

maple [A] time = 0.01, size = 62, normalized size = 1.07

$$-\frac{ad\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{c\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\,dx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*d*x*(b*x^2+a)^(1/2)/b-1/2*d*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+c*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.33, size = 47, normalized size = 0.81

$$\frac{\sqrt{bx^2+a}\,dx}{2\,b}+\frac{c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{ad\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*d*x/b + c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*d*arcs
inh(b*x/sqrt(a*b))/b^(3/2)
```

mupad [B] time = 5.51, size = 86, normalized size = 1.48

$$\begin{cases} \frac{d\,x^3+3\,c\,x}{3\,\sqrt{a}} & \text{if } b=0 \\ \frac{c\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{a\,d\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{d\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (3*c*x + d*x^3)/(3*a^(1/2)), b ~= 0, (c*log(b^(1/2)*x + (
a + b*x^2)^(1/2)))/b^(1/2) - (a*d*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(
2*b^(3/2)) + (d*x*(a + b*x^2)^(1/2))/(2*b))
```

sympy [A] time = 2.78, size = 126, normalized size = 2.17

$$\frac{\sqrt{a}\,dx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+c\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x*sqrt(1 + b*x**2/a)/(2*b) - a*d*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) + c*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b <
0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)
*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0)))
```

3.77 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.00, size = 28, normalized size = 1.12

$$-\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a + b*x^2],x]
```

```
[Out] -(Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]/Sqrt[b])
```

fricas [A] time = 1.25, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.60, size = 23, normalized size = 0.92

$$-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x)
```

```
[Out] ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.28, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 0.12, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

sympy [A] time = 1.00, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

3.78 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {377, 208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[c]*Sqrt[b*c -
a*d])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx &= \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[c]*Sqrt[b*c -
a*d])
```

IntegrateAlgebraic [B] time = 0.13, size = 112, normalized size = 2.29

$$\frac{\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\,dx^2}{\sqrt{c}\,\sqrt{ad-bc}}-\frac{dx\sqrt{a+bx^2}}{\sqrt{c}\,\sqrt{ad-bc}}+\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{ad-bc}}\right)}{\sqrt{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

```
[Out] (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c])/Sqrt[-(b*c) + a*d] + (Sqrt[b]*
d*x^2)/(Sqrt[c]*Sqrt[-(b*c) + a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-
(b*c) + a*d])])/(Sqrt[c]*(b*c - a*d))
```

fricas [B] time = 1.45, size = 241, normalized size = 4.92

$$\left[\frac{\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2cd\right)x^2+4\left((2\,bc-ad)x^3+acx\right)\sqrt{bc^2-acd}\,\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\,\sqrt{bc^2-acd}},\ -\frac{\sqrt{-bc^2+acd}\,\arctan\left(\frac{\sqrt{-bc^2+acd}\left((2\,bc-ad)x^2+ac\right)\sqrt{bx^2+a}}{2\left(\left(b^2c^2-abcd\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)}{2\left(bc^2-acd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 -
3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x
^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2))/sqrt(b*c^2 - a*c*d), -1/2*sqrt(-b*c^2
 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*
x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x))/(b*c^2 - a*c*d)
]
```

giac [A] time = 0.60, size = 70, normalized size = 1.43

$$-\frac{\sqrt{b}\,\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(
-b^2*c^2 + a*b*c*d))/sqrt(-b^2*c^2 + a*b*c*d)
```

maple [B] time = 0.02, size = 300, normalized size = 6.12

$$-\frac{\ln\left(\frac{\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\,\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,\sqrt{\frac{ad-bc}{d}}}+\frac{\ln\left(\frac{-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\,\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,\sqrt{\frac{ad-bc}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c),x)
```

```
[Out] -1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)
*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(
1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/2/(-c
*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*
```

```
(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x
+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2 + a}\left(dx^2 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{x\sqrt{a\,d-b\,c}}{\sqrt{c}\,\sqrt{b\,x^2+a}}\right)}{\sqrt{c\,(a\,d-b\,c)}} & \text{if } 0 < a\,d-b\,c \\ \frac{\ln\left(\frac{\sqrt{c\left(b\,x^2+a\right)}+x\,\sqrt{b\,c-a\,d}}{\sqrt{c\left(b\,x^2+a\right)}-x\,\sqrt{b\,c-a\,d}}\right)}{2\,\sqrt{-c\,(a\,d-b\,c)}} & \text{if } a\,d-b\,c < 0 \\ \int \frac{1}{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)}\, dx & \text{if } a\,d-b\,c \notin \mathbb{R} \vee a\,d = b\,c \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)),x)
```

```
[Out] piecewise(0 < a*d - b*c, atan((x*(a*d - b*c)^(1/2))/(c^(1/2)*(a + b*x^2)^(1
/2)))/(c*(a*d - b*c))^(1/2), a*d - b*c < 0, log(((c*(a + b*x^2))^(1/2) + x*
(- a*d + b*c)^(1/2))/((c*(a + b*x^2))^(1/2) - x*(- a*d + b*c)^(1/2)))/(2*(-
c*(a*d - b*c))^(1/2)), ~in(a*d - b*c, 'real') | a*d == b*c, int(1/((a + b*x
^2)^(1/2)*(c + d*x^2)), x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^2}\left(c + dx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)), x)
```

3.79 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=101

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

```
[Out] -(d*x*Sqrt[a + b*x^2])/(2*c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c - a*d)*ArcTa
nh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(
3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx &= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}\\
&= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2c(bc-ad)}\\
&= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 126, normalized size = 1.25

$$\frac{x\left(\frac{\left(c+dx^2\right)(2bc-ad)\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)}{c\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}}-d\left(a+bx^2\right)\right)}{2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

```
[Out] (x*(-(d*(a + b*x^2)) + ((2*b*c - a*d)*(c + d*x^2)*ArcTanh[Sqrt[((b*c - a*d)
*x^2)/(c*(a + b*x^2))]])/(c*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))])))/(2*c
*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2))
```

IntegrateAlgebraic [A] time = 0.33, size = 132, normalized size = 1.31

$$\frac{(2bc-ad)\sqrt{ad-bc}\,\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,c+\sqrt{b}\,dx^2}{\sqrt{c}\,\sqrt{ad-bc}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx\sqrt{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c - a*d)*Sqr
t[-(b*c) + a*d]*ArcTan[(Sqrt[b]*c + Sqrt[b]*d*x^2 - d*x*Sqrt[a + b*x^2])/(S
qrt[c]*Sqrt[-(b*c) + a*d])])/(2*c^(3/2)*(b*c - a*d)^2)
```

fricas [B] time = 1.62, size = 463, normalized size = 4.58

$$\left[-\frac{4\left(bc^2d-acd^2\right)\sqrt{bx^2+a}\,x-\left(2bc^2-acd+\left(2bcd-ad^2\right)x^2\right)\sqrt{bc^2-acd}\,\log\left(\frac{\left(8b^2c^2-8abcd+a^2d^2\right)x^4+a^2c^2+2\left(4abc^2-3a^2cd\right)x^2+4\left(\left(2bc-ad\right)x^3+acx\right)\sqrt{bc^2-acd}\,\sqrt{bx^2+a}}{d^2x^4+2cdx^2+c^2}\right)}{8\left(b^2c^5-2abc^4d+a^2c^3d^2+\left(b^2c^4d-2abc^3d^2+a^2c^2d^3\right)x^2\right)},\ -\frac{2\left(bc^2d-acd^2\right)\sqrt{bx^2+a}\,x+\left(2bc^2-acd+\left(2bcd-ad^2\right)x^2\right)\sqrt{-bc^2+acd}\,\arctan\left(\frac{\sqrt{-bc^2+acd}\left(\left(2bc-ad\right)x^2+ac\right)\sqrt{bx^2+a}}{2\left(\left(b^2c^2-abcd\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)}{4\left(b^2c^5-2abc^4d+a^2c^3d^2+\left(b^2c^4d-2abc^3d^2+a^2c^2d^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a)*x - (2*b*c^2 - a*c*d + (2*b*c*
d - a*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*
x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*
x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(b^2*
c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)
*x^2), -1/4*(2*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a)*x + (2*b*c^2 - a*c*d + (
2*b*c*d - a*d^2)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*
((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c
^2 - a^2*c*d)*x)))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^4*d - 2*a*
b*c^3*d^2 + a^2*c^2*d^3)*x^2)]
```

giac [B] time = 0.63, size = 242, normalized size = 2.40

$$\frac{1}{2}\,b^{\frac{3}{2}}\left(\frac{(2bc-ad)\arctan\left(-\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 d+2bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{\left(b^2c^2-abcd\right)\sqrt{-b^2c^2+abcd}}-\frac{2\left(2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 bc-\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 ad+a^2d\right)}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4 d+4\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 bc-2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 ad+a^2d\right)\left(b^2c^2-abcd\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^(3/2)*((2*b*c - a*d)*arctan(-1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d +
 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - a*b*c*d)*sqrt(-b^2*c^2
+ a*b*c*d)) - 2*(2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - (sqrt(b)*x - sqrt(
b*x^2 + a))^2*a*d + a^2*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d
)*(b^2*c^2 - a*b*c*d)))
```

maple [B] time = 0.02, size = 809, normalized size = 8.01

$$-\frac{\sqrt{-cd}\, b \ln\left(\frac{\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{4\left(ad-bc\right)\sqrt{\frac{ad-bc}{d}}\, cd}+\frac{\sqrt{-cd}\, b \ln\left(\frac{-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{4\left(ad-bc\right)\sqrt{\frac{ad-bc}{d}}\, cd}-\frac{\ln\left(\frac{\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{4\sqrt{-cd}\,\sqrt{\frac{ad-bc}{d}}\, c}+\frac{\ln\left(\frac{-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{4\sqrt{-cd}\,\sqrt{\frac{ad-bc}{d}}\, c}+\frac{\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{4\left(ad-bc\right)\left(x-\frac{\sqrt{-cd}}{d}\right)c}+\frac{\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{4\left(ad-bc\right)\left(x+\frac{\sqrt{-cd}}{d}\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/4/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(
x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-1/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)/(
(a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+
2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2
)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/4/c/(a*d-b*c)/(x+(-c*d)^
(1/2)/d)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d
-b*c)/d)^(1/2)+1/4/c/d*(-c*d)^(1/2)*b/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((-2*
(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x
+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/
2))/(x+(-c*d)^(1/2)/d))-1/4/c/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)
^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d
)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x
-(-c*d)^(1/2)/d))+1/4/c/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2
)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/
2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*
d)^(1/2)/d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2),x)
```

```
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**2), x)
```

3.80 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=163

$$\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}}-\frac{3dx\sqrt{a+bx^2}\,(2bc-ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx\sqrt{a+bx^2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Rubi [A] time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}}-\frac{3dx\sqrt{a+bx^2}\,(2bc-ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx\sqrt{a+bx^2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

```
[Out] -(d*x*Sqrt[a + b*x^2])/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (3*d*(2*b*c - a*d)
*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)^2*(c + d*x^2)) + ((8*b^2*c^2 - 8*a*b
*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(
8*c^(5/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx = -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}+\frac{\int \frac{4bc-3ad-2bdx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\int \frac{8b^2c^2-8abcd+3a^2d^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8c^2(bc-ad)^2}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\int}{8c^2(bc-ad}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\mathrm{S}}{8c^2(}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\mathrm{ta}}{8c^{5/2}(bc-ad}$$

Mathematica [A] time = 0.67, size = 192, normalized size = 1.18

$$\frac{x\left(\frac{\left(c+dx^2\right)^2\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)}{\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}}-cd\left(a^2(-d)\left(5c+3dx^2\right)+ab\left(8c^2+cdx^2-3d^2x^4\right)+2b^2cx^2\left(4c+3dx^2\right)\right)\right)}{8c^3\sqrt{a+bx^2}\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

```
[Out] (x*(-(c*d*(2*b^2*c*x^2*(4*c + 3*d*x^2) - a^2*d*(5*c + 3*d*x^2) + a*b*(8*c^2
 + c*d*x^2 - 3*d^2*x^4))) + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*(c + d*x^2
)^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2
)/(c*(a + b*x^2))]))/(8*c^3*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)^2)
```

IntegrateAlgebraic [A] time = 0.91, size = 180, normalized size = 1.10

$$\frac{\sqrt{ad-bc}\left(3a^2d^2-8abcd+8b^2c^2\right)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}c+\sqrt{b}dx^2}{\sqrt{c}\sqrt{ad-bc}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{\sqrt{a+bx^2}\left(5acd^2x+3ad^3x^3-8bc^2dx-6bcd^2x^3\right)}{8c^2\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*b*c^2*d*x + 5*a*c*d^2*x - 6*b*c*d^2*x^3 + 3*a*d^3*x^3)
)/(8*c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (Sqrt[-(b*c) + a*d]*(8*b^2*c^2 - 8*
a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[b]*c + Sqrt[b]*d*x^2 - d*x*Sqrt[a + b*x^2
])/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(8*c^(5/2)*(b*c - a*d)^3)
```

fricas [B] time = 1.97, size = 864, normalized size = 5.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*((8*b^2*c^4 - 8*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 8*a*b*c*
d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*s
qrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2
*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a
*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(3*(2*b^2*c^3*d^2 -
 3*a*b*c^2*d^3 + a^2*c*d^4)*x^3 + (8*b^2*c^4*d - 13*a*b*c^3*d^2 + 5*a^2*c^2
*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*
c^5*d^3 + (b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x
^4 + 2*(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*x^2),
-1/16*((8*b^2*c^4 - 8*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 8*a*b*c*
d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*s
qrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*
c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(
3*(2*b^2*c^3*d^2 - 3*a*b*c^2*d^3 + a^2*c*d^4)*x^3 + (8*b^2*c^4*d - 13*a*b*c
^3*d^2 + 5*a^2*c^2*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^
2*b*c^6*d^2 - a^3*c^5*d^3 + (b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^
4 - a^3*c^3*d^5)*x^4 + 2*(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a
^3*c^4*d^4)*x^2)]
```

giac [B] time = 3.51, size = 538, normalized size = 3.30

$$-\frac{1}{8} b^{\frac{5}{2}} \left(\frac{\left(8 b^2c^2 - 8 abcd + 3 a^2d^2\right) \arctan\left(\frac{\left(\sqrt{b} x - \sqrt{bx^2+a}\right)^2 d + 2 bc - ad}{2 \sqrt{-b^2c^2+abcd}}\right)}{\left(b^4c^4 - 2 ab^3c^3d + a^2b^2c^2d^2\right) \sqrt{-b^2c^2 + abcd}} + \frac{2 \left(8 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^6 b^2c^2d - 8 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^6 abcd^2 + 3 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^6 a^2d^3 + 48 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^4 b^3c^3 - 72 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^4 ab^2c^2d + 42 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^4 a^2bcd^2 - 9 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^4 a^3d^3 + 40 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^2 a^2b^2c^2d - 40 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^2 a^3bcd^2 + 9 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^2 a^4d^3 + 6 a^4bcd^2 - 3 a^5d^3\right)}{\left(b^4c^4 - 2 ab^3c^3d + a^2b^2c^2d^2\right) \left(\left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^4 d + 4 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^2 bc - 2 \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^2 ad + a^2d\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*b^(5/2)*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*arctan(1/2*((sqrt(b)*x -
sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^4*c^4 - 2
*a*b^3*c^3*d + a^2*b^2*c^2*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) + 2*(8*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*b^2*c^2*d - 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b*c*d
^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*d^3 + 48*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*b^3*c^3 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^2*c^2*d + 42*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*a^2*b*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^3*d^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^2*c^2*d - 40*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*a^3*b*c*d^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*d
^3 + 6*a^4*b*c*d^2 - 3*a^5*d^3)/((b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2
)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*
c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2))
```

maple [B] time = 0.02, size = 1815, normalized size = 11.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x)
```

```
[Out] 1/16/(-c*d)^(1/2)/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)^2*((x-(-c*d)^(1/2)/d)^2*b+
2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-3/16/c*b/(a*d-b*c)
^2/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2
)/d)*b/d+(a*d-b*c)/d)^(1/2)-3/16/(-c*d)^(1/2)*b^2/(a*d-b*c)^2/((a*d-b*c)/d)
^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)
/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*
d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/16/(-c*d)^(1/2)/c*b/(a*d-b*c)/((a*d-
b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a
*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*
b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/16/(-c*d)^(1/2)/c/(a*d-b*c)/(
```

```
x+(-c*d)^(1/2)/d)^2*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/
d)*b/d+(a*d-b*c)/d)^(1/2)-3/16/c*b/(a*d-b*c)^2/(x+(-c*d)^(1/2)/d)*((x+(-c*d
)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+3/1
6/(-c*d)^(1/2)*b^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(
-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^
2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/
2)/d))+1/16/(-c*d)^(1/2)/c*b/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1
/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(
1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-
c*d)^(1/2)/d))+3/16/c^2/(a*d-b*c)/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(1/2)/d)^2*
b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+3/16/c^2/d*(-c*d
)^(1/2)*b/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)
/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d
)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+3/16
/c^2/(a*d-b*c)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x
-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-3/16/c^2/d*(-c*d)^(1/2)*b/(a*d-b*c)
/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/
d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1
/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-3/16/(-c*d)^(1/2)/c^2/((
a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2
*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)
/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+3/16/(-c*d)^(1/2)/c^2/((a*d
-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*(
(a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d
)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^3),x)
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
[Out] Timed out
```

3.81 $\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)\left(35a^2d^2-64abcd+24b^2c^2\right)}{24ab^3}+\frac{d\left(-35a^3d^3+120a^2bcd^2-144ab^2c^2d+64b^3c^3\right)\tanh^{-1}\left(\right.}{16b^{9/2}}$$

Rubi [A] time = 0.26, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {413, 528, 388, 217, 206}

$$-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)\left(35a^2d^2-64abcd+24b^2c^2\right)}{24ab^3}-\frac{dx\sqrt{a+bx^2}\left(290a^2bcd^2-105a^3d^3-248ab^2c^2d+48b^3c^3\right)}{48ab^4}+\frac{d\left(120a^2bcd^2-35a^3d^3-144ab^2c^2d+64b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2(6bc-7ad)}{6ab^2}+\frac{x\left(c+dx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

```
[Out] -(d*(48*b^3*c^3 - 248*a*b^2*c^2*d + 290*a^2*b*c*d^2 - 105*a^3*d^3)*x*Sqrt[a
 + b*x^2])/(48*a*b^4) - (d*(24*b^2*c^2 - 64*a*b*c*d + 35*a^2*d^2)*x*Sqrt[a
+ b*x^2]*(c + d*x^2))/(24*a*b^3) - (d*(6*b*c - 7*a*d)*x*Sqrt[a + b*x^2]*(c
+ d*x^2)^2)/(6*a*b^2) + ((b*c - a*d)*x*(c + d*x^2)^3)/(a*b*Sqrt[a + b*x^2])
 + (d*(64*b^3*c^3 - 144*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 35*a^3*d^3)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)^3}{ab\sqrt{a+bx^2}} + \frac{\int \frac{\left(c+dx^2\right)^2\left(acd-d(6bc-7ad)x^2\right)}{\sqrt{a+bx^2}}\,dx}{ab}$$

$$= -\frac{d(6bc-7ad)x\sqrt{a+bx^2}\,\left(c+dx^2\right)^2}{6ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^3}{ab\sqrt{a+bx^2}} + \frac{\int \frac{\left(c+dx^2\right)\left(acd(12bc-7ad)-d(24b\right.}{\sqrt{a+bx}}}{6ab^2}$$

$$= -\frac{d\left(24b^2c^2-64abcd+35a^2d^2\right)x\sqrt{a+bx^2}\,\left(c+dx^2\right)}{24ab^3} - \frac{d(6bc-7ad)x\sqrt{a+bx^2}\,\left(c+dx^2\right)}{6ab^2}$$

$$= -\frac{d\left(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3\right)x\sqrt{a+bx^2}}{48ab^4} - \frac{d\left(24b^2c^2-64abcd+35a\right.}{24a}$$

$$= -\frac{d\left(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3\right)x\sqrt{a+bx^2}}{48ab^4} - \frac{d\left(24b^2c^2-64abcd+35a\right.}{24a}$$

$$= -\frac{d\left(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3\right)x\sqrt{a+bx^2}}{48ab^4} - \frac{d\left(24b^2c^2-64abcd+35a\right.}{24a}$$

Mathematica [A] time = 5.20, size = 172, normalized size = 0.67

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\left(3d^2\left(19a^2d^2-56abcd+48b^2c^2\right)+2bd^3x^2(24bc-11ad)+\frac{48(bc-ad)^4}{a\left(a+bx^2\right)}+8b^2d^4x^4\right)+3d\left(-35a^3d^3+120a^2bcd^2-144ab^2c^2d+64b^3c^3\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{48b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(3*d^2*(48*b^2*c^2 - 56*a*b*c*d + 19*a^2*d^2) +
2*b*d^3*(24*b*c - 11*a*d)*x^2 + 8*b^2*d^4*x^4 + (48*(b*c - a*d)^4)/(a*(a +
b*x^2))) + 3*d*(64*b^3*c^3 - 144*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 35*a^3*d^3
)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(48*b^(9/2))
```

IntegrateAlgebraic [A] time = 0.48, size = 229, normalized size = 0.89

$$\frac{\left(35a^3d^4-120a^2bcd^3+144ab^2c^2d^2-64b^3c^3d\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{9/2}}+\frac{105a^4d^4x-360a^3bcd^3x+35a^3bd^4x^3+432a^2b^2c^2d^2x-120a^2b^2cd^3x^3-14a^2b^2d^4x^5-192ab^3c^3dx+144ab^3c^2d^2x^3+48ab^3cd^3x^5+8ab^3d^4x^7+48b^4c^4x}{48ab^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

```
[Out] (48*b^4*c^4*x - 192*a*b^3*c^3*d*x + 432*a^2*b^2*c^2*d^2*x - 360*a^3*b*c*d^3
*x + 105*a^4*d^4*x + 144*a*b^3*c^2*d^2*x^3 - 120*a^2*b^2*c*d^3*x^3 + 35*a^3
*b*d^4*x^3 + 48*a*b^3*c*d^3*x^5 - 14*a^2*b^2*d^4*x^5 + 8*a*b^3*d^4*x^7)/(48
*a*b^4*Sqrt[a + b*x^2]) + ((-64*b^3*c^3*d + 144*a*b^2*c^2*d^2 - 120*a^2*b*c
*d^3 + 35*a^3*d^4)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(9/2))
```

fricas [A] time = 1.39, size = 584, normalized size = 2.27

$$\left[-\frac{3\left(64a^2b^3c^3d-144a^3b^2c^2d^2+120a^4bcd^3-35a^5d^4+\left(64ab^4c^3d-144a^2b^3c^2d^2+120a^3b^2cd^3-35a^4bd^4\right)x^2\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8ab^4d^4x^7+2\left(24ab^4cd^3-7a^2b^3d^4\right)x^5+\left(144ab^4c^2d^2-120a^2b^3cd^3+35a^3b^2d^4\right)x^3+3\left(16b^5c^4-64ab^4c^3d+144a^2b^3c^2d^2-120a^3b^2cd^3+35a^4bd^4\right)x\right)\sqrt{bx^2+a}}{96\left(ab^6x^2+a^2b^5\right)},\ -\frac{3\left(64a^2b^3c^3d-144a^3b^2c^2d^2+120a^4bcd^3-35a^5d^4+\left(64ab^4c^3d-144a^2b^3c^2d^2+120a^3b^2cd^3-35a^4bd^4\right)x^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(8ab^4d^4x^7+2\left(24ab^4cd^3-7a^2b^3d^4\right)x^5+\left(144ab^4c^2d^2-120a^2b^3cd^3+35a^3b^2d^4\right)x^3+3\left(16b^5c^4-64ab^4c^3d+144a^2b^3c^2d^2-120a^3b^2cd^3+35a^4bd^4\right)x\right)\sqrt{bx^2+a}}{48\left(ab^6x^2+a^2b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(64*a^2*b^3*c^3*d - 144*a^3*b^2*c^2*d^2 + 120*a^4*b*c*d^3 - 35*a^
5*d^4 + (64*a*b^4*c^3*d - 144*a^2*b^3*c^2*d^2 + 120*a^3*b^2*c*d^3 - 35*a^4*
b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*
a*b^4*d^4*x^7 + 2*(24*a*b^4*c*d^3 - 7*a^2*b^3*d^4)*x^5 + (144*a*b^4*c^2*d^2
 - 120*a^2*b^3*c*d^3 + 35*a^3*b^2*d^4)*x^3 + 3*(16*b^5*c^4 - 64*a*b^4*c^3*d
 + 144*a^2*b^3*c^2*d^2 - 120*a^3*b^2*c*d^3 + 35*a^4*b*d^4)*x)*sqrt(b*x^2 +
a))/(a*b^6*x^2 + a^2*b^5), -1/48*(3*(64*a^2*b^3*c^3*d - 144*a^3*b^2*c^2*d^2
 + 120*a^4*b*c*d^3 - 35*a^5*d^4 + (64*a*b^4*c^3*d - 144*a^2*b^3*c^2*d^2 + 1
20*a^3*b^2*c*d^3 - 35*a^4*b*d^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (8*a*b^4*d^4*x^7 + 2*(24*a*b^4*c*d^3 - 7*a^2*b^3*d^4)*x^5 + (144*a
*b^4*c^2*d^2 - 120*a^2*b^3*c*d^3 + 35*a^3*b^2*d^4)*x^3 + 3*(16*b^5*c^4 - 64
*a*b^4*c^3*d + 144*a^2*b^3*c^2*d^2 - 120*a^3*b^2*c*d^3 + 35*a^4*b*d^4)*x)*s
qrt(b*x^2 + a))/(a*b^6*x^2 + a^2*b^5)]
```

giac [A] time = 0.67, size = 235, normalized size = 0.91

$$\frac{\left(\left(2\left(\frac{4\,d^4x^2}{b}+\frac{24\,ab^6cd^3-7\,a^2b^5d^4}{ab^7}\right)x^2+\frac{144\,ab^6c^2d^2-120\,a^2b^5cd^3+35\,a^3b^4d^4}{ab^7}\right)x^2+\frac{3\left(16\,b^7c^4-64\,ab^6c^3d+144\,a^2b^5c^2d^2-120\,a^3b^4cd^3+35\,a^4b^3d^4\right)}{ab^7}\right)x}{48\,\sqrt{bx^2+a}}-\frac{\left(64\,b^3c^3d-144\,ab^2c^2d^2+120\,a^2bcd^3-35\,a^3d^4\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*d^4*x^2/b + (24*a*b^6*c*d^3 - 7*a^2*b^5*d^4)/(a*b^7))*x^2 + (14
4*a*b^6*c^2*d^2 - 120*a^2*b^5*c*d^3 + 35*a^3*b^4*d^4)/(a*b^7))*x^2 + 3*(16*
b^7*c^4 - 64*a*b^6*c^3*d + 144*a^2*b^5*c^2*d^2 - 120*a^3*b^4*c*d^3 + 35*a^4
*b^3*d^4)/(a*b^7))*x/sqrt(b*x^2 + a) - 1/16*(64*b^3*c^3*d - 144*a*b^2*c^2*d
^2 + 120*a^2*b*c*d^3 - 35*a^3*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(9/2)
```

maple [A] time = 0.02, size = 340, normalized size = 1.32

$$\frac{d^4x^7}{6\sqrt{b\,x^2+a}\,b}-\frac{7a\,d^4x^5}{24\sqrt{b\,x^2+a}\,b^2}+\frac{c\,d^3x^5}{\sqrt{b\,x^2+a}\,b}+\frac{35a^2d^4x^3}{48\sqrt{b\,x^2+a}\,b^3}-\frac{5ac\,d^3x^3}{2\sqrt{b\,x^2+a}\,b^2}+\frac{3c^2d^2x^3}{\sqrt{b\,x^2+a}\,b}+\frac{35a^3d^4x}{16\sqrt{b\,x^2+a}\,b^4}-\frac{15a^2c\,d^3x}{2\sqrt{b\,x^2+a}\,b^3}+\frac{9a\,c^2d^2x}{\sqrt{b\,x^2+a}\,b^2}+\frac{c^4x}{\sqrt{b\,x^2+a}\,a}-\frac{4c^3dx}{\sqrt{b\,x^2+a}\,b}-\frac{35a^3d^4\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{9}{2}}}+\frac{15a^2c\,d^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{7}{2}}}-\frac{9a\,c^2d^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{5}{2}}}+\frac{4c^3d\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*d^4*x^7/b/(b*x^2+a)^(1/2)-7/24*d^4*a/b^2*x^5/(b*x^2+a)^(1/2)+35/48*d^4*
a^2/b^3*x^3/(b*x^2+a)^(1/2)+35/16*d^4*a^3/b^4*x/(b*x^2+a)^(1/2)-35/16*d^4*a
^3/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+c*d^3*x^5/b/(b*x^2+a)^(1/2)-5/2*c*
d^3*a/b^2*x^3/(b*x^2+a)^(1/2)-15/2*c*d^3*a^2/b^3*x/(b*x^2+a)^(1/2)+15/2*c*d
^3*a^2/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+3*c^2*d^2*x^3/b/(b*x^2+a)^(1/2
)+9*c^2*d^2*a/b^2*x/(b*x^2+a)^(1/2)-9*c^2*d^2*a/b^(5/2)*ln(b^(1/2)*x+(b*x^2
+a)^(1/2))-4*c^3*d*x/b/(b*x^2+a)^(1/2)+4*c^3*d/b^(3/2)*ln(b^(1/2)*x+(b*x^2+
a)^(1/2))+c^4*x/a/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.38, size = 311, normalized size = 1.21

$$\frac{d^4x^7}{6\,\sqrt{bx^2+a}\,b}+\frac{cd^3x^5}{\sqrt{bx^2+a}\,b}-\frac{7\,ad^4x^5}{24\,\sqrt{bx^2+a}\,b^2}+\frac{3\,c^2d^2x^3}{\sqrt{bx^2+a}\,b}-\frac{5\,acd^3x^3}{2\,\sqrt{bx^2+a}\,b^2}+\frac{35\,a^2d^4x^3}{48\,\sqrt{bx^2+a}\,b^3}+\frac{c^4x}{\sqrt{bx^2+a}\,a}-\frac{4\,c^3dx}{\sqrt{bx^2+a}\,b}+\frac{9\,ac^2d^2x}{\sqrt{bx^2+a}\,b^2}-\frac{15\,a^2cd^3x}{2\,\sqrt{bx^2+a}\,b^3}+\frac{35\,a^3d^4x}{16\,\sqrt{bx^2+a}\,b^4}+\frac{4\,c^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}-\frac{9\,ac^2d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}+\frac{15\,a^2cd^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}-\frac{35\,a^3d^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*d^4*x^7/(sqrt(b*x^2 + a)*b) + c*d^3*x^5/(sqrt(b*x^2 + a)*b) - 7/24*a*d^
4*x^5/(sqrt(b*x^2 + a)*b^2) + 3*c^2*d^2*x^3/(sqrt(b*x^2 + a)*b) - 5/2*a*c*d
^3*x^3/(sqrt(b*x^2 + a)*b^2) + 35/48*a^2*d^4*x^3/(sqrt(b*x^2 + a)*b^3) + c^
4*x/(sqrt(b*x^2 + a)*a) - 4*c^3*d*x/(sqrt(b*x^2 + a)*b) + 9*a*c^2*d^2*x/(sq
rt(b*x^2 + a)*b^2) - 15/2*a^2*c*d^3*x/(sqrt(b*x^2 + a)*b^3) + 35/16*a^3*d^4
*x/(sqrt(b*x^2 + a)*b^4) + 4*c^3*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 9*a*c^2
```

```
*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 15/2*a^2*c*d^3*arcsinh(b*x/sqrt(a*b))
/b^(7/2) - 35/16*a^3*d^4*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^2+c\right)^4}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^4/(a + b*x^2)^(3/2),x)
[Out] int((c + d*x^2)^4/(a + b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**(3/2),x)
[Out] Integral((c + d*x**2)**4/(a + b*x**2)**(3/2), x)
```

3.82 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}} \, dx$

Optimal. Leaf size=169

$$\frac{3d(5a^2d^2-12abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}-\frac{dx\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{8ab^3}-\frac{dx\sqrt{a+bx^2}(c+dx^2)(4bc-$$

Rubi [A] time = 0.20, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {413, 528, 388, 217, 206}

$$\frac{3d(5a^2d^2-12abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}-\frac{dx\sqrt{a+bx^2}(c+dx^2)(4bc-5ad)}{4ab^2}-\frac{dx\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{8ab^3}+\frac{x(c+dx^2)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

```
[Out] -(d*(2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*b^3) - (d*(4*b*
c - 5*a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(4*a*b^2) + ((b*c - a*d)*x*(c + d
*x^2)^2)/(a*b*Sqrt[a + b*x^2]) + (3*d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)^2}{ab\sqrt{a+bx^2}} + \frac{\int \frac{\left(c+dx^2\right)\left(acd-d(4bc-5ad)x^2\right)}{\sqrt{a+bx^2}}\,dx}{ab}$$

$$= -\frac{d(4bc-5ad)x\sqrt{a+bx^2}\,\left(c+dx^2\right)}{4ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^2}{ab\sqrt{a+bx^2}} + \frac{\int \frac{acd(8bc-5ad)-d(2bc-5ad)(4bc-}{\sqrt{a+bx^2}}}{4ab^2}$$

$$= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3} - \frac{d(4bc-5ad)x\sqrt{a+bx^2}\,\left(c+dx^2\right)}{4ab^2} + \frac{(bc-ad)x\,(c}{ab\sqrt{a+}}$$

$$= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3} - \frac{d(4bc-5ad)x\sqrt{a+bx^2}\,\left(c+dx^2\right)}{4ab^2} + \frac{(bc-ad)x\,(c}{ab\sqrt{a+}}$$

$$= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3} - \frac{d(4bc-5ad)x\sqrt{a+bx^2}\,\left(c+dx^2\right)}{4ab^2} + \frac{(bc-ad)x\,(c}{ab\sqrt{a+}}$$

Mathematica [A] time = 5.10, size = 122, normalized size = 0.72

$$\frac{3d\left(5a^2d^2-12abcd+8b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{7/2}} + \frac{x\sqrt{a+bx^2}\left(d^2(12bc-7ad)+\frac{8(bc-ad)^3}{a\left(a+bx^2\right)}+2bd^3x^2\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(d^2*(12*b*c - 7*a*d) + 2*b*d^3*x^2 + (8*(b*c - a*d)^3)/
(a*(a + b*x^2))))/(8*b^3) + (3*d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[b
*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.28, size = 156, normalized size = 0.92

$$\frac{-15a^3d^3x+36a^2bcd^2x-5a^2bd^3x^3-24ab^2c^2dx+12ab^2cd^2x^3+2ab^2d^3x^5+8b^3c^3x}{8ab^3\sqrt{a+bx^2}} - \frac{3\left(5a^2d^3-12abcd^2+8b^2c^2d\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

```
[Out] (8*b^3*c^3*x - 24*a*b^2*c^2*d*x + 36*a^2*b*c*d^2*x - 15*a^3*d^3*x + 12*a*b^
2*c*d^2*x^3 - 5*a^2*b*d^3*x^3 + 2*a*b^2*d^3*x^5)/(8*a*b^3*Sqrt[a + b*x^2])
- (3*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]])/(8*b^(7/2))
```

fricas [A] time = 0.89, size = 416, normalized size = 2.46

$$\left[\frac{3\left(8a^2b^2c^2d-12a^3bcd^2+5a^4d^3+\left(8ab^3c^2d-12a^2b^2cd^2+5a^3bd^3\right)x^2\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2ab^3d^3x^5+\left(12ab^3cd^2-5a^2b^2d^3\right)x^3+\left(8b^4c^3-24ab^3c^2d+36a^2b^2cd^2-15a^3bd^3\right)x\right)\sqrt{bx^2+a}}{16\left(ab^5x^2+a^2b^4\right)}, -\frac{3\left(8a^2b^2c^2d-12a^3bcd^2+5a^4d^3+\left(8ab^3c^2d-12a^2b^2cd^2+5a^3bd^3\right)x^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(2ab^3d^3x^5+\left(12ab^3cd^2-5a^2b^2d^3\right)x^3+\left(8b^4c^3-24ab^3c^2d+36a^2b^2cd^2-15a^3bd^3\right)x\right)\sqrt{bx^2+a}}{8\left(ab^5x^2+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(8*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 5*a^4*d^3 + (8*a*b^3*c^2*d - 1
2*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
```

```
)*sqrt(b)*x - a) + 2*(2*a*b^3*d^3*x^5 + (12*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^
3 + (8*b^4*c^3 - 24*a*b^3*c^2*d + 36*a^2*b^2*c*d^2 - 15*a^3*b*d^3)*x)*sqrt(
b*x^2 + a))/(a*b^5*x^2 + a^2*b^4), -1/8*(3*(8*a^2*b^2*c^2*d - 12*a^3*b*c*d^
2 + 5*a^4*d^3 + (8*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*a*b^3*d^3*x^5 + (12*a*b^3*c*d^2
 - 5*a^2*b^2*d^3)*x^3 + (8*b^4*c^3 - 24*a*b^3*c^2*d + 36*a^2*b^2*c*d^2 - 15
*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4)]
```

giac [A] time = 0.64, size = 157, normalized size = 0.93

$$\frac{\left(\left(\frac{2\,d^3x^2}{b} + \frac{12\,ab^4cd^2-5\,a^2b^3d^3}{ab^5}\right)x^2 + \frac{8\,b^5c^3-24\,ab^4c^2d+36\,a^2b^3cd^2-15\,a^3b^2d^3}{ab^5}\right)x}{8\,\sqrt{bx^2+a}} - \frac{3\left(8\,b^2c^2d - 12\,abcd^2 + 5\,a^2d^3\right)\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*d^3*x^2/b + (12*a*b^4*c*d^2 - 5*a^2*b^3*d^3)/(a*b^5))*x^2 + (8*b^5*
c^3 - 24*a*b^4*c^2*d + 36*a^2*b^3*c*d^2 - 15*a^3*b^2*d^3)/(a*b^5))*x/sqrt(b
*x^2 + a) - 3/8*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*log(abs(-sqrt(b)*x
 + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.01, size = 219, normalized size = 1.30

$$\frac{d^3x^5}{4\sqrt{b\,x^2+a}\,b} - \frac{5a\,d^3x^3}{8\sqrt{b\,x^2+a}\,b^2} + \frac{3c\,d^2x^3}{2\sqrt{b\,x^2+a}\,b} - \frac{15a^2d^3x}{8\sqrt{b\,x^2+a}\,b^3} + \frac{9ac\,d^2x}{2\sqrt{b\,x^2+a}\,b^2} + \frac{c^3x}{\sqrt{b\,x^2+a}\,a} - \frac{3c^2dx}{\sqrt{b\,x^2+a}\,b} + \frac{15a^2d^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{7}{2}}} - \frac{9ac\,d^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{5}{2}}} + \frac{3c^2d\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*d^3*x^5/b/(b*x^2+a)^(1/2)-5/8*d^3*a/b^2*x^3/(b*x^2+a)^(1/2)-15/8*d^3*a^
2/b^3*x/(b*x^2+a)^(1/2)+15/8*d^3*a^2/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+
3/2*c*d^2*x^3/b/(b*x^2+a)^(1/2)+9/2*c*d^2*a/b^2*x/(b*x^2+a)^(1/2)-9/2*c*d^2
*a/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-3*c^2*d*x/b/(b*x^2+a)^(1/2)+3*c^2*
d/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+c^3*x/a/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.40, size = 197, normalized size = 1.17

$$\frac{d^3x^5}{4\,\sqrt{bx^2+a}\,b} + \frac{3\,cd^2x^3}{2\,\sqrt{bx^2+a}\,b} - \frac{5\,ad^3x^3}{8\,\sqrt{bx^2+a}\,b^2} + \frac{c^3x}{\sqrt{bx^2+a}\,a} - \frac{3\,c^2dx}{\sqrt{bx^2+a}\,b} + \frac{9\,acd^2x}{2\,\sqrt{bx^2+a}\,b^2} - \frac{15\,a^2d^3x}{8\,\sqrt{bx^2+a}\,b^3} + \frac{3\,c^2d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{9\,acd^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{15\,a^2d^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*d^3*x^5/(sqrt(b*x^2 + a)*b) + 3/2*c*d^2*x^3/(sqrt(b*x^2 + a)*b) - 5/8*a
*d^3*x^3/(sqrt(b*x^2 + a)*b^2) + c^3*x/(sqrt(b*x^2 + a)*a) - 3*c^2*d*x/(sqr
t(b*x^2 + a)*b) + 9/2*a*c*d^2*x/(sqrt(b*x^2 + a)*b^2) - 15/8*a^2*d^3*x/(sqr
t(b*x^2 + a)*b^3) + 3*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 9/2*a*c*d^2*ar
csinh(b*x/sqrt(a*b))/b^(5/2) + 15/8*a^2*d^3*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^3}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c + d*x^2)^3/(a + b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^2\right)^3}{\left(a + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**(3/2),x)
[Out] Integral((c + d*x**2)**3/(a + b*x**2)**(3/2), x)
```

3.83 $\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}+\frac{x(bc-ad)^2}{ab^2\sqrt{a+bx^2}}+\frac{d^2x\sqrt{a+bx^2}}{2b^2}$$

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {413, 388, 217, 206}

$$-\frac{dx\sqrt{a+bx^2}\,(2bc-3ad)}{2ab^2}+\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

```
[Out] -(d*(2*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(2*a*b^2) + ((b*c - a*d)*x*(c + d*x^
2))/(a*b*Sqrt[a + b*x^2]) + (d*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(2*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{(bc-ad)x\left(c+dx^2\right)}{ab\sqrt{a+bx^2}}+\frac{\int \frac{acd-d(2bc-3ad)x^2}{\sqrt{a+bx^2}}\,dx}{ab} \\
&= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2}+\frac{(bc-ad)x\left(c+dx^2\right)}{ab\sqrt{a+bx^2}}+\frac{(d(4bc-3ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2} \\
&= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2}+\frac{(bc-ad)x\left(c+dx^2\right)}{ab\sqrt{a+bx^2}}+\frac{(d(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,-\right.}{2b^2} \\
&= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2}+\frac{(bc-ad)x\left(c+dx^2\right)}{ab\sqrt{a+bx^2}}+\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 2.53, size = 160, normalized size = 1.78

$$
\frac{x\sqrt{\frac{bx^2}{a}+1}\left(-6bx^2\left(c+dx^2\right)^2\,{}_3F_2\left(\frac{3}{2},2,\frac{5}{2};1,\frac{9}{2};-\frac{bx^2}{a}\right)-12bx^2\left(2c^2+3cdx^2+d^2x^4\right)\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{9}{2};-\frac{bx^2}{a}\right)+7a\left(15c^2+10cdx^2+3d^2x^4\right)\,{}_2F_1\left(\frac{1}{2},\frac{3}{2};\frac{7}{2};-\frac{bx^2}{a}\right)\right)}{105a^2\sqrt{a+bx^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[1 + (b*x^2)/a]*(7*a*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*Hypergeometri
c2F1[1/2, 3/2, 7/2, -((b*x^2)/a)] - 12*b*x^2*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*
Hypergeometric2F1[3/2, 5/2, 9/2, -((b*x^2)/a)] - 6*b*x^2*(c + d*x^2)^2*Hype
rgeometricPFQ[{3/2, 2, 5/2}, {1, 9/2}, -((b*x^2)/a)]))/(105*a^2*Sqrt[a + b*
x^2])
```

IntegrateAlgebraic [A] time = 0.17, size = 99, normalized size = 1.10

$$
\frac{3a^2d^2x-4abcdx+abd^2x^3+2b^2c^2x}{2ab^2\sqrt{a+bx^2}}+\frac{\left(3ad^2-4bcd\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*b^2*c^2*x - 4*a*b*c*d*x + 3*a^2*d^2*x + a*b*d^2*x^3)/(2*a*b^2*Sqrt[a + b
*x^2]) + ((-4*b*c*d + 3*a*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5
/2))
```

fricas [A] time = 0.94, size = 276, normalized size = 3.07

$$
\left[-\frac{\left(4\,a^2bcd-3\,a^3d^2+\left(4\,ab^2cd-3\,a^2bd^2\right)x^2\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(ab^2d^2x^3+\left(2\,b^3c^2-4\,ab^2cd+3\,a^2bd^2\right)x\right)\sqrt{bx^2+a}}{4\left(ab^4x^2+a^2b^3\right)},\ -\frac{\left(4\,a^2bcd-3\,a^3d^2+\left(4\,ab^2cd-3\,a^2bd^2\right)x^2\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(ab^2d^2x^3+\left(2\,b^3c^2-4\,ab^2cd+3\,a^2bd^2\right)x\right)\sqrt{bx^2+a}}{2\left(ab^4x^2+a^2b^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(b)*
log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(a*b^2*d^2*x^3 + (2*b^3
*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)
, -1/2*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (a*b^2*d^2*x^3 + (2*b^3*c^2 - 4*a*b^
2*c*d + 3*a^2*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

giac [A] time = 0.63, size = 92, normalized size = 1.02

$$\frac{\left(\frac{d^2x^2}{b} + \frac{2\,b^3c^2 - 4\,ab^2cd + 3\,a^2bd^2}{ab^3}\right)x}{2\,\sqrt{bx^2+a}} - \frac{\left(4\,bcd - 3\,ad^2\right)\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(d^2*x^2/b + (2*b^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)/(a*b^3))*x/sqrt(b*
x^2 + a) - 1/2*(4*b*c*d - 3*a*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(5/2)
```

maple [A] time = 0.01, size = 123, normalized size = 1.37

$$\frac{d^2x^3}{2\sqrt{b\,x^2+a}\;b} + \frac{3a\,d^2x}{2\sqrt{b\,x^2+a}\;b^2} + \frac{c^2x}{\sqrt{b\,x^2+a}\;a} - \frac{2cdx}{\sqrt{b\,x^2+a}\;b} - \frac{3a\,d^2\ln\left(\sqrt{b}\;x + \sqrt{b\,x^2+a}\right)}{2b^{\frac{5}{2}}} + \frac{2cd\ln\left(\sqrt{b}\;x + \sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*d^2*x^3/b/(b*x^2+a)^(1/2)+3/2*d^2*a/b^2*x/(b*x^2+a)^(1/2)-3/2*d^2*a/b^(
5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-2*c*d*x/b/(b*x^2+a)^(1/2)+2*c*d/b^(3/2)*
ln(b^(1/2)*x+(b*x^2+a)^(1/2))+c^2*x/a/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.38, size = 108, normalized size = 1.20

$$\frac{d^2x^3}{2\,\sqrt{bx^2+a}\,b} + \frac{c^2x}{\sqrt{bx^2+a}\,a} - \frac{2\,cdx}{\sqrt{bx^2+a}\,b} + \frac{3\,ad^2x}{2\,\sqrt{bx^2+a}\,b^2} + \frac{2\,cd\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{3\,ad^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*d^2*x^3/(sqrt(b*x^2 + a)*b) + c^2*x/(sqrt(b*x^2 + a)*a) - 2*c*d*x/(sqrt
(b*x^2 + a)*b) + 3/2*a*d^2*x/(sqrt(b*x^2 + a)*b^2) + 2*c*d*arcsinh(b*x/sqrt
(a*b))/b^(3/2) - 3/2*a*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^2}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c + d*x^2)^2/(a + b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^2\right)^2}{\left(a + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**2/(a + b*x**2)**(3/2), x)
```

3.84 $\int \frac{c+dx^2}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}+\frac{x(bc-ad)}{ab\sqrt{a+bx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 217, 206}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}+\frac{x(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*Sqrt[a + b*x^2]) + (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\ &= \frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\ &= \frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 1.30

$$\frac{a^{3/2}d\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b}\,x(bc-ad)}{ab^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(b*c - a*d)*x + a^(3/2)*d*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/
Sqrt[a]])/(a*b^(3/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.09, size = 58, normalized size = 1.07

$$-\frac{d\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{3/2}}-\frac{x(ad-bc)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -(((-(b*c) + a*d)*x)/(a*b*Sqrt[a + b*x^2])) - (d*Log[-(Sqrt[b]*x) + Sqrt[a
+ b*x^2]])/b^(3/2)
```

fricas [A] time = 0.89, size = 167, normalized size = 3.09

$$\left[\frac{2\left(b^2c-abd\right)\sqrt{bx^2+a}\,x+\left(abdx^2+a^2d\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\left(ab^3x^2+a^2b^2\right)},\ \frac{\left(b^2c-abd\right)\sqrt{bx^2+a}\,x-\left(abdx^2+a^2d\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{ab^3x^2+a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - a*b*d)*sqrt(b*x^2 + a)*x + (a*b*d*x^2 + a^2*d)*sqrt(b)*log
(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(a*b^3*x^2 + a^2*b^2), ((b^2*
c - a*b*d)*sqrt(b*x^2 + a)*x - (a*b*d*x^2 + a^2*d)*sqrt(-b)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2)]
```

giac [A] time = 0.64, size = 50, normalized size = 0.93

$$-\frac{d\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{(bc-ad)x}{\sqrt{bx^2+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -d*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + (b*c - a*d)*x/(sqrt(b*x
^2 + a)*a*b)
```

maple [A] time = 0.00, size = 54, normalized size = 1.00

$$\frac{cx}{\sqrt{b\,x^2+a}\;a}-\frac{dx}{\sqrt{b\,x^2+a}\;b}+\frac{d\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^(3/2),x)
```

```
[Out] -d*x/b/(b*x^2+a)^(1/2)+d/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+c*x/a/(b*x^2
+a)^(1/2)
```

maxima [A] time = 1.30, size = 46, normalized size = 0.85

$$\frac{cx}{\sqrt{bx^2+a}\,a}-\frac{dx}{\sqrt{bx^2+a}\,b}+\frac{d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*x/(sqrt(b*x^2 + a)*a) - d*x/(sqrt(b*x^2 + a)*b) + d*arcsinh(b*x/sqrt(a*b)
)/b^(3/2)
```

mupad [B] time = 5.12, size = 53, normalized size = 0.98

$$\frac{d\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}+\frac{c\,x}{a\,\sqrt{b\,x^2+a}}-\frac{d\,x}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^(3/2),x)
```

```
[Out] (d*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) + (c*x)/(a*(a + b*x^2)^(1/2)
) - (d*x)/(b*(a + b*x^2)^(1/2))
```

sympy [A] time = 5.17, size = 60, normalized size = 1.11

$$d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)+\frac{cx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(3/2),x)
```

```
[Out] d*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
c*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

3.85 $\int \frac{1}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+bx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {191}

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{x}{a\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + b*x^2])
```

fricas [A] time = 0.96, size = 23, normalized size = 1.44

$$\frac{\sqrt{bx^2+a}\,x}{abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)
```

giac [A] time = 0.60, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{bx^2 + a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] x/(sqrt(b*x^2 + a)*a)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{x}{\sqrt{b\,x^2 + a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2),x)
[Out] 1/(b*x^2+a)^(1/2)/a*x
```

maxima [A] time = 1.32, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{bx^2 + a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] x/(sqrt(b*x^2 + a)*a)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.88

$$\frac{x}{a\,\sqrt{b\,x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(3/2),x)
[Out] x/(a*(a + b*x^2)^(1/2))
```

sympy [A] time = 0.61, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2),x)
[Out] x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

3.86 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=79

$$\frac{bx}{a\sqrt{a+bx^2}\,(bc-ad)}-\frac{d\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(bc-ad)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 208}

$$\frac{bx}{a\sqrt{a+bx^2}\,(bc-ad)}-\frac{d\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}\,dx &= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}}-\frac{d\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{bc-ad}\\
&= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}}-\frac{d\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{bc-ad}\\
&= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.72, size = 309, normalized size = 3.91

$$\frac{x\left(2dx^2\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{5/2}{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)+2c\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{5/2}{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)-10dx^2\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}-15c\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}+10dx^2\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\right)+15c\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\right)\right)}{5c^2\left(a+bx^2\right)^{3/2}\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

```
[Out] (x*(-15*c*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - 10*d*x^2*Sqrt[((b*c - a
*d)*x^2)/(c*(a + b*x^2))] + 15*c*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x
^2))]] + 10*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 2*c*((
(b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*
c - a*d)*x^2)/(c*(a + b*x^2))] + 2*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2))
)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]))
/(5*c^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2)*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.24, size = 143, normalized size = 1.81

$$-\frac{bx}{a\sqrt{a+bx^2}\,(ad-bc)}-\frac{d\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\,dx^2}{\sqrt{c}\,\sqrt{ad-bc}}-\frac{dx\sqrt{a+bx^2}}{\sqrt{c}\,\sqrt{ad-bc}}+\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{ad-bc}}\right)}{\sqrt{c}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

```
[Out] -((b*x)/(a*(-(b*c) + a*d)*Sqrt[a + b*x^2])) - (d*Sqrt[-(b*c) + a*d]*ArcTan[
(Sqrt[b]*Sqrt[c])/Sqrt[-(b*c) + a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[-(b*c)
 + a*d]) - (d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*(b
*c - a*d)^2)
```

fricas [B] time = 1.22, size = 441, normalized size = 5.58

$$\left[\frac{4\left(b^2c^2-abcd\right)\sqrt{bx^2+a}\,x-\left(abdx^2+a^2d\right)\sqrt{bc^2-acd}\,\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2cd\right)x^2+4\left((2\,bc-ad)x^3+acx\right)\sqrt{bc^2-acd}\,\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\left(a^2b^2c^3-2\,a^3bc^2d+a^4cd^2+\left(ab^3c^3-2\,a^2b^2c^2d+a^3bcd^2\right)x^2\right)},\ \frac{2\left(b^2c^2-abcd\right)\sqrt{bx^2+a}\,x+\left(abdx^2+a^2d\right)\sqrt{-bc^2+acd}\,\arctan\left(\frac{\sqrt{-bc^2+acd}\left((2\,bc-ad)x^2+ac\right)\sqrt{bx^2+a}}{2\left(\left(b^2c^2-abcd\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)}{2\left(a^2b^2c^3-2\,a^3bc^2d+a^4cd^2+\left(ab^3c^3-2\,a^2b^2c^2d+a^3bcd^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(b^2*c^2 - a*b*c*d)*sqrt(b*x^2 + a)*x - (a*b*d*x^2 + a^2*d)*sqrt(b*
c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*
b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*
sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(a^2*b^2*c^3 - 2*a^3*b*c^2*d
 + a^4*c*d^2 + (a*b^3*c^3 - 2*a^2*b^2*c^2*d + a^3*b*c*d^2)*x^2), 1/2*(2*(b^
2*c^2 - a*b*c*d)*sqrt(b*x^2 + a)*x + (a*b*d*x^2 + a^2*d)*sqrt(-b*c^2 + a*c*
d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a
)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^2*b^2*c^3 - 2*a^3*
b*c^2*d + a^4*c*d^2 + (a*b^3*c^3 - 2*a^2*b^2*c^2*d + a^3*b*c*d^2)*x^2)]
```

giac [A] time = 0.62, size = 107, normalized size = 1.35

$$-\frac{\sqrt{b}\,d\arctan\left(-\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}\,(bc-ad)}+\frac{bx}{\left(abc-a^2d\right)\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -sqrt(b)*d*arctan(-1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sq
rt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*(b*c - a*d)) + b*x/((a*b*
c - a^2*d)*sqrt(b*x^2 + a))
```

maple [B]   time = 0.02, size = 618, normalized size = 7.82

$$-\frac{d\ln\left(\frac{\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,(ad-bc)\sqrt{\frac{ad-bc}{d}}}+\frac{d\ln\left(\frac{-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,(ad-bc)\sqrt{\frac{ad-bc}{d}}}-\frac{bx}{2(ad-bc)\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}\,a}-\frac{bx}{2(ad-bc)\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}\,a}+\frac{d}{2\sqrt{-cd}\,(ad-bc)\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}-\frac{d}{2\sqrt{-cd}\,(ad-bc)\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c),x)
```

```
[Out] 1/2/(-c*d)^(1/2)/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*
d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-1/2/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d)^2*b+
2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x*b-1/2/(-c*d)^(1/
2)/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/
d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2
)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/2/(-c*d)
^(1/2)/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d
)*b/d+(a*d-b*c)/d)^(1/2)-1/2/(a*d-b*c)/a/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(
1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x*b+1/2/(-c*d)^(1/2)/(a*d-b*
c)*d/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-
b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*
d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)),x)
```

```
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)), x)
```

3.87 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=143

$$-\frac{d(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}+\frac{bx(ad+2bc)}{2ac\sqrt{a+bx^2}\,(bc-ad)^2}-\frac{dx}{2c\sqrt{a+bx^2}\,\left(c+dx^2\right)(bc-ad)}$$

Rubi [A] time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$-\frac{d(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}+\frac{bx(ad+2bc)}{2ac\sqrt{a+bx^2}\,(bc-ad)^2}-\frac{dx}{2c\sqrt{a+bx^2}\,\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] (b*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]) - (d*x)/(2*c*(b*c
 - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)) - (d*(4*b*c - a*d)*ArcTanh[(Sqrt[b*c -
 a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}\,dx = -\frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,\left(c+dx^2\right)} + \frac{\int \frac{2bc-ad-2bdx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,\left(c+dx^2\right)} - \frac{\int \frac{ad(4bc-ad)}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2ac(bc-ad)^2}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,\left(c+dx^2\right)} - \frac{(d(4bc-ad))\int \frac{}{\sqrt{a+bx}}}{2c(bc-ad)}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,\left(c+dx^2\right)} - \frac{(d(4bc-ad))\,\text{Subst}\big(}{2c(}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,\left(c+dx^2\right)} - \frac{d(4bc-ad)\tanh^{-1}\big(}{2c^{3/2}(bc-ad)}$$

Mathematica [C] time = 2.67, size = 758, normalized size = 5.30

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] (x*(-2625*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - (5250*d*x^2*Sqrt[((b*c
- a*d)*x^2)/(c*(a + b*x^2))])/c - (2310*d^2*x^4*Sqrt[((b*c - a*d)*x^2)/(c*(
a + b*x^2))])/c^2 + 70*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2) + (560*d*x
^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c + (280*d^2*x^4*(((b*c - a*d
)*x^2)/(c*(a + b*x^2)))^(3/2))/c^2 + 2625*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c
*(a + b*x^2))]] + (5250*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2)
)]])/c + (2310*d^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/c^
2 - (945*(b*c - a*d)*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/
(c*(a + b*x^2)) + (2310*d*(-(b*c) + a*d)*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)
/(c*(a + b*x^2))]])/(c^2*(a + b*x^2)) + (1050*d^2*(-(b*c) + a*d)*x^6*ArcTan
h[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^3*(a + b*x^2)) + 24*(((b*c -
 a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*HypergeometricPFQ[{2, 2, 5/2}, {1, 9/2},
((b*c - a*d)*x^2)/(c*(a + b*x^2))] + (48*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b
*x^2)))^(7/2)*HypergeometricPFQ[{2, 2, 5/2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c
*(a + b*x^2))])/c + (24*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*H
ypergeometricPFQ[{2, 2, 5/2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])
/c^2))/(210*c*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2)*(a + b*x^2)^(3/2)*(
c + d*x^2))
```

IntegrateAlgebraic [A] time = 0.70, size = 173, normalized size = 1.21

$$\frac{a^2d^2x+abd^2x^3+2b^2c^2x+2b^2cdx^3}{2ac\sqrt{a+bx^2}\,\left(c+dx^2\right)(ad-bc)^2} - \frac{\sqrt{ad-bc}\,\left(4bcd-ad^2\right)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,c+\sqrt{b}\,dx^2}{\sqrt{c}\,\sqrt{ad-bc}}\right)}{2c^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] (2*b^2*c^2*x + a^2*d^2*x + 2*b^2*c*d*x^3 + a*b*d^2*x^3)/(2*a*c*(-(b*c) + a*
d)^2*Sqrt[a + b*x^2]*(c + d*x^2)) - (Sqrt[-(b*c) + a*d]*(4*b*c*d - a*d^2)*A
rcTan[(Sqrt[b]*c + Sqrt[b]*d*x^2 - d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c
) + a*d])])/(2*c^(3/2)*(b*c - a*d)^3)
```

fricas [B] time = 2.22, size = 864, normalized size = 6.04

$$\left[-\frac{\left(4a^2bc^2d - a^3cd^2 + \left(4ab^2cd^2 - a^2bd^3\right)x^4 + \left(4ab^2c^2d + 3a^2bcd^2 - a^3d^3\right)x^2\right)\sqrt{bc^2 - acd}\,\log\left(\frac{\left(8b^2c^2 - 8abcd + a^2d^2\right)x^4 + a^2c^2 + 2\left(4abc^2 - 3a^2cd\right)x^2 + 4\left((2bc - ad)x^3 + acx\right)\sqrt{bc^2 - acd}\,\sqrt{bx^2 + a}}{d^2x^4 + 2cdx^2 + c^2}\right) - 4\left(\left(2b^3c^3d - ab^2c^2d^2 - a^2bcd^3\right)x^3 + \left(2b^3c^4 - 2ab^2c^3d + a^2bc^2d^2 - a^3cd^3\right)x\right)\sqrt{bx^2 + a}}{8\left(a^2b^3c^6 - 3a^3b^2c^5d + 3a^4bc^4d^2 - a^5c^3d^3 + \left(ab^4c^5d - 3a^2b^3c^4d^2 + 3a^3b^2c^3d^3 - a^4bc^2d^4\right)x^4 + \left(ab^4c^6 - 2a^2b^3c^5d + 2a^4bc^3d^3 - a^5c^2d^4\right)x^2\right)}, \frac{\left(4a^2bc^2d - a^3cd^2 + \left(4ab^2cd^2 - a^2bd^3\right)x^4 + \left(4ab^2c^2d + 3a^2bcd^2 - a^3d^3\right)x^2\right)\sqrt{-bc^2 + acd}\,\arctan\left(\frac{\sqrt{-bc^2 + acd}\left((2bc - ad)x^2 + ac\right)\sqrt{bx^2 + a}}{2\left(\left(b^2c^2 - abcd\right)x^3 + \left(abc^2 - a^2cd\right)x\right)}\right) + 2\left(\left(2b^3c^3d - ab^2c^2d^2 - a^2bcd^3\right)x^3 + \left(2b^3c^4 - 2ab^2c^3d + a^2bc^2d^2 - a^3cd^3\right)x\right)\sqrt{bx^2 + a}}{4\left(a^2b^3c^6 - 3a^3b^2c^5d + 3a^4bc^4d^2 - a^5c^3d^3 + \left(ab^4c^5d - 3a^2b^3c^4d^2 + 3a^3b^2c^3d^3 - a^4bc^2d^4\right)x^4 + \left(ab^4c^6 - 2a^2b^3c^5d + 2a^4bc^3d^3 - a^5c^2d^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (4*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c
^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4
*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4
+ 2*c*d*x^2 + c^2)) - 4*((2*b^3*c^3*d - a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^3 +
(2*b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 - a^3*c*d^3)*x)*sqrt(b*x^2 + a))
/(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 + (a*b^4*c^
5*d - 3*a^2*b^3*c^4*d^2 + 3*a^3*b^2*c^3*d^3 - a^4*b*c^2*d^4)*x^4 + (a*b^4*c
^6 - 2*a^2*b^3*c^5*d + 2*a^4*b*c^3*d^3 - a^5*c^2*d^4)*x^2), 1/4*((4*a^2*b*c
^2*d - a^3*c*d^2 + (4*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (4*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*
d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*
b*c^2 - a^2*c*d)*x)) + 2*((2*b^3*c^3*d - a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^3 +
 (2*b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 - a^3*c*d^3)*x)*sqrt(b*x^2 + a)
)/(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 + (a*b^4*c
^5*d - 3*a^2*b^3*c^4*d^2 + 3*a^3*b^2*c^3*d^3 - a^4*b*c^2*d^4)*x^4 + (a*b^4*
c^6 - 2*a^2*b^3*c^5*d + 2*a^4*b*c^3*d^3 - a^5*c^2*d^4)*x^2)]
```

giac [B] time = 1.90, size = 318, normalized size = 2.22

$$\frac{b^2x}{\left(ab^2c^2 - 2a^2bcd + a^3d^2\right)\sqrt{bx^2 + a}} + \frac{\left(4b^{\frac{3}{2}}cd - a\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 d + 2bc - ad}{2\sqrt{-b^2c^2 + abcd}}\right)}{2\left(b^2c^3 - 2abc^2d + a^2cd^2\right)\sqrt{-b^2c^2 + abcd}} + \frac{2\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 b^{\frac{3}{2}}cd - \left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 a\sqrt{b}d^2 + a^2\sqrt{b}d^2}{\left(b^2c^3 - 2abc^2d + a^2cd^2\right)\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^4 d + 4\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 bc - 2\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 ad + a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(b*x^2 + a)) + 1/2*(4*b^(3/2
)*c*d - a*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*
c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqr
t(-b^2*c^2 + a*b*c*d)) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d - (
sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d^2 + a^2*sqrt(b)*d^2)/((b^2*c^3 -
 2*a*b*c^2*d + a^2*c*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d))
```

maple [B] time = 0.02, size = 1439, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/4/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(
x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-3/4/c*(-c*d)^(1/2)*b/(a*d-b*c)^2/(
(x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(
1/2)+3/4*b^2/(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)
^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+3/4/c*(-c*d)^(1/2)*b/(a*d-b*c)^2/((a*d-b
*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a
*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*
```

```
b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+1/4/c/(a*d-b*c)/a/((x+(-c*d)^(1
/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x*b+1/4
/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(
-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+3/4/c*(-c*d)^(1/2)*b/(a*d-b*c)^2/((x-
(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2
)+3/4*b^2/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1
/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-3/4/c*(-c*d)^(1/2)*b/(a*d-b*c)^2/((a*d-b*c)
/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b
*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+
(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/4/c/(a*d-b*c)/a/((x-(-c*d)^(1/2)/
d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x*b+1/4/c/(
-c*d)^(1/2)/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1
/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-1/4/c/(-c*d)^(1/2)/(a*d-b*c)*d/((a*d-b*c)/d)^
(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/
d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d
-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/4/c/(-c*d)^(1/2)/(a*d-b*c)*d/((x+(-c*
d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+1/
4/c/(-c*d)^(1/2)/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c
*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*
b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)
/d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{3/2}\,{\left(d\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x)
```

```
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**2), x)
```

3.88 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{3d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{7/2}}+\frac{dx\sqrt{a+bx^2}\,(4bc-ad)(3ad+2bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx(ad+4bc)}{4ac\sqrt{a+bx^2}\,\left(c+dx^2\right)(}$$

Rubi [A] time = 0.24, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$-\frac{3d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{7/2}}+\frac{dx\sqrt{a+bx^2}\,(4bc-ad)(3ad+2bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx(ad+4bc)}{4ac\sqrt{a+bx^2}\,\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\sqrt{a+bx^2}\,\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^2) + (b*(4*b*c + a*d)*x
)/(4*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)) + (d*(4*b*c - a*d)*(2*b
*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (3*d*(
8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[
a + b*x^2])])/(8*c^(5/2)*(b*c - a*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}\,dx = -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,\left(c+dx^2\right)^2} + \frac{\int \frac{4bc-3ad-4bdx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,\left(c+dx^2\right)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,\left(c+dx^2\right)} - \ldots$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,\left(c+dx^2\right)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,\left(c+dx^2\right)} + \ldots$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,\left(c+dx^2\right)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,\left(c+dx^2\right)} + \ldots$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,\left(c+dx^2\right)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,\left(c+dx^2\right)} + \ldots$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,\left(c+dx^2\right)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,\left(c+dx^2\right)} + \ldots$$

Mathematica [C] time = 5.02, size = 1392, normalized size = 6.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] (x*(-108045*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - (324135*d*x^2*Sqrt[((
b*c - a*d)*x^2)/(c*(a + b*x^2))])/c - (324135*d^2*x^4*Sqrt[((b*c - a*d)*x^2
)/(c*(a + b*x^2))])/c^2 - (103320*d^3*x^6*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*
x^2))])/c^3 + 42735*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2) + (128205*d*x
^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c + (139545*d^2*x^4*(((b*c -
a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c^2 + (46200*d^3*x^6*(((b*c - a*d)*x^2)/(
c*(a + b*x^2)))^(3/2))/c^3 - 3864*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2)
 - (4032*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2))/c - (4032*d^2*x^4
*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2))/c^2 - (1344*d^3*x^6*(((b*c - a*
d)*x^2)/(c*(a + b*x^2)))^(5/2))/c^3 + 108045*ArcTanh[Sqrt[((b*c - a*d)*x^2)
/(c*(a + b*x^2))]] + (324135*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b
*x^2))]])/c + (324135*d^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2)
)]])/c^2 + (103320*d^3*x^6*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]]
)/c^3 + (8505*(b*c - a*d)^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^
2))]])/(c^2*(a + b*x^2)^2) + (17955*d*(b*c - a*d)^2*x^6*ArcTanh[Sqrt[((b*c
- a*d)*x^2)/(c*(a + b*x^2))]])/(c^3*(a + b*x^2)^2) + (21735*d^2*(b*c - a*d)
^2*x^8*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^4*(a + b*x^2)^2
) + (7560*d^3*(b*c - a*d)^2*x^10*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x
^2))]])/(c^5*(a + b*x^2)^2) - (78750*(b*c - a*d)*x^2*ArcTanh[Sqrt[((b*c - a
*d)*x^2)/(c*(a + b*x^2))]])/(c*(a + b*x^2)) + (236250*d*(-(b*c) + a*d)*x^4*
ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^2*(a + b*x^2)) + (2475
90*d^2*(-(b*c) + a*d)*x^6*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])
```

```
/(c^3*(a + b*x^2)) + (80640*d^3*(-(b*c) + a*d)*x^8*ArcTanh[Sqrt[((b*c - a*d
)*x^2)/(c*(a + b*x^2))]])/(c^4*(a + b*x^2)) + 64*(((b*c - a*d)*x^2)/(c*(a +
 b*x^2)))^(9/2)*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d
)*x^2)/(c*(a + b*x^2))] + (192*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9
/2)*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a
 + b*x^2))])/c + (192*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9/2)*Hyp
ergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^
2))])/c^2 + (64*d^3*x^6*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9/2)*Hypergeom
etricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/
c^3))/(2520*c*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*(a + b*x^2)^(3/2)*(
c + d*x^2)^2)
```

IntegrateAlgebraic [B] time = 56.77, size = 6883, normalized size = 30.59

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 3.78, size = 1482, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*c^2*d^3 + (8*a*b^3*c^2*d
^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (16*a*b^3*c^3*d^2 - 2*a^3*b*c*d^4 +
 a^4*d^5)*x^4 + (8*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 - 7*a^3*b*c^2*d^3 + 2*a
^4*c*d^4)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x
^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x
)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*((8
*b^4*c^4*d^2 + 2*a*b^3*c^3*d^3 - 13*a^2*b^2*c^2*d^4 + 3*a^3*b*c*d^5)*x^5 +
(16*b^4*c^5*d - 4*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3 - 8*a^3*b*c^2*d^4 + 3*a
^4*c*d^5)*x^3 + (8*b^4*c^6 - 8*a*b^3*c^5*d + 12*a^2*b^2*c^4*d^2 - 17*a^3*b*
c^3*d^3 + 5*a^4*c^2*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^4*c^9 - 4*a^3*b^3*c^8*d
 + 6*a^4*b^2*c^7*d^2 - 4*a^5*b*c^6*d^3 + a^6*c^5*d^4 + (a*b^5*c^7*d^2 - 4*a
^2*b^4*c^6*d^3 + 6*a^3*b^3*c^5*d^4 - 4*a^4*b^2*c^4*d^5 + a^5*b*c^3*d^6)*x^6
 + (2*a*b^5*c^8*d - 7*a^2*b^4*c^7*d^2 + 8*a^3*b^3*c^6*d^3 - 2*a^4*b^2*c^5*d
^4 - 2*a^5*b*c^4*d^5 + a^6*c^3*d^6)*x^4 + (a*b^5*c^9 - 2*a^2*b^4*c^8*d - 2*
a^3*b^3*c^7*d^2 + 8*a^4*b^2*c^6*d^3 - 7*a^5*b*c^5*d^4 + 2*a^6*c^4*d^5)*x^2)
, 1/16*(3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*c^2*d^3 + (8*a*b^3*c^2*d
^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (16*a*b^3*c^3*d^2 - 2*a^3*b*c*d^4 +
 a^4*d^5)*x^4 + (8*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 - 7*a^3*b*c^2*d^3 + 2*a
^4*c*d^4)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c
 - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^
2*c*d)*x)) + 2*((8*b^4*c^4*d^2 + 2*a*b^3*c^3*d^3 - 13*a^2*b^2*c^2*d^4 + 3*a
^3*b*c*d^5)*x^5 + (16*b^4*c^5*d - 4*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3 - 8*a
^3*b*c^2*d^4 + 3*a^4*c*d^5)*x^3 + (8*b^4*c^6 - 8*a*b^3*c^5*d + 12*a^2*b^2*c
^4*d^2 - 17*a^3*b*c^3*d^3 + 5*a^4*c^2*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^4*c^9
 - 4*a^3*b^3*c^8*d + 6*a^4*b^2*c^7*d^2 - 4*a^5*b*c^6*d^3 + a^6*c^5*d^4 + (a
*b^5*c^7*d^2 - 4*a^2*b^4*c^6*d^3 + 6*a^3*b^3*c^5*d^4 - 4*a^4*b^2*c^4*d^5 +
a^5*b*c^3*d^6)*x^6 + (2*a*b^5*c^8*d - 7*a^2*b^4*c^7*d^2 + 8*a^3*b^3*c^6*d^3
 - 2*a^4*b^2*c^5*d^4 - 2*a^5*b*c^4*d^5 + a^6*c^3*d^6)*x^4 + (a*b^5*c^9 - 2*
a^2*b^4*c^8*d - 2*a^3*b^3*c^7*d^2 + 8*a^4*b^2*c^6*d^3 - 7*a^5*b*c^5*d^4 + 2
*a^6*c^4*d^5)*x^2)]
```

giac [B] time = 2.70, size = 643, normalized size = 2.86

$$\frac{b^{3} x}{\left(a b^{3} c^{3} - 3 a^{2} b^{2} c^{2} d + 3 a^{3} b c d^{2} - a^{4} d^{3}\right) \sqrt{b x^{2} + a}} + \frac{3 \left(8 b^{\frac{5}{2}} c^{2} d - 4 a b^{\frac{3}{2}} c d^{2} + a^{2} \sqrt{b} d^{3}\right) \arctan\left(\frac{\left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{2} d + 2 b c - a d}{2 \sqrt{-b^{2} c^{2} + a b c d}}\right)}{8 \left(b^{3} c^{5} - 3 a b^{2} c^{4} d + 3 a^{2} b c^{3} d^{2} - a^{3} c^{2} d^{3}\right) \sqrt{-b^{2} c^{2} + a b c d}} + \frac{16 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{6} b^{\frac{5}{2}} c^{2} d^{2} - 12 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{6} a b^{\frac{3}{2}} c d^{3} + 3 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{6} a^{2} \sqrt{b} d^{4} + 80 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{4} b^{\frac{7}{2}} c^{3} d - 104 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{4} a b^{\frac{5}{2}} c^{2} d^{2} + 54 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{4} a^{2} b^{\frac{3}{2}} c d^{3} - 9 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{4} a^{3} \sqrt{b} d^{4} + 64 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{2} a^{2} b^{\frac{5}{2}} c^{2} d^{2} - 52 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{2} a^{3} b^{\frac{3}{2}} c d^{3} + 9 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{2} a^{4} \sqrt{b} d^{4} + 10 a^{4} b^{\frac{3}{2}} c d^{3} - 3 a^{5} \sqrt{b} d^{4}}{4 \left(b^{3} c^{5} - 3 a b^{2} c^{4} d + 3 a^{2} b c^{3} d^{2} - a^{3} c^{2} d^{3}\right) \left(\left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{4} d + 4 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{2} b c - 2 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{2} a d + a^{2} d\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(b*x^2 +
 a)) + 3/8*(8*b^(5/2)*c^2*d - 4*a*b^(3/2)*c*d^2 + a^2*sqrt(b)*d^3)*arctan(1
/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*
d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(-b^2*c^
2 + a*b*c*d)) + 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d^2 - 1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^3 + 3*(sqrt(b)*x - sqrt(b*x
^2 + a))^6*a^2*sqrt(b)*d^4 + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3
*d - 104*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^2 + 54*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
a^3*sqrt(b)*d^4 + 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*d^2 -
52*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^3 + 9*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*a^4*sqrt(b)*d^4 + 10*a^4*b^(3/2)*c*d^3 - 3*a^5*sqrt(b)*d^4)/(
(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*((sqrt(b)*x - sqr
t(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a*d + a^2*d)^2)
```

maple [B] time = 0.03, size = 2919, normalized size = 12.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x)
```

```
[Out] 3/16/c^2/(a*d-b*c)/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2
)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+3/16/c^2/(a*d-b*c)/(x+(-c*d)^(1
/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b
*c)/d)^(1/2)-1/16/(-c*d)^(1/2)/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)^2/((x+(-c*d)^
(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-5/16/
c*b/(a*d-b*c)^2/(x+(-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(
x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-3/16/(-c*d)^(1/2)/c^2/(a*d-b*c)*d/
((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^
(1/2)-9/16/c^2*(-c*d)^(1/2)*b/(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^
(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+3/16/(-c*d)^(1/2)/c^2/(a*d-
b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b
*c)/d)^(1/2)+15/16/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((x-(-c*d)^(1/2)/d)^2*b+2
*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-15/16*b^3/(a*d-b*c)
^3/a/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c
)/d)^(1/2)*x+9/16/c^2*(-c*d)^(1/2)*b/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*
(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-15/16/(-c*d)^(1/2)*d
*b^2/(a*d-b*c)^3/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*
b/d+(a*d-b*c)/d)^(1/2)-15/16*b^3/(a*d-b*c)^3/a/((x+(-c*d)^(1/2)/d)^2*b-2*(-
c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-3/16/(-c*d)^(1/2)/c*
d*b/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b
/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/
2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+3/16/(-c*
d)^(1/2)/c*d*b/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d
)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-
2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d
))+1/16/(-c*d)^(1/2)/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)^2/((x-(-c*d)^(1/2)/d)^2
*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-5/16/c*b/(a*d-b
*c)^2/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(
1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+15/16/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((a*d-b
*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a
*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*
b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-1/4/c*b^2/(a*d-b*c)^2/a/((x+(-c
*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x
```

```
-3/16/(-c*d)^(1/2)/c*d*b/(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)
*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+9/16/c^2*(-c*d)^(1/2)*b/(a*d-b*c
)^2/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b
*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d
)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+3/16/c^2/(a*d-b*c)/a
/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)
^(1/2)*x*b+3/16/c^2/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(
-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x*b-3/16/(-c*d)^(1/2)/c^2/(a*d-b*c)*d
/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/
d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1
/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+3/16/(-c*d)^(1/2)/c^2/(a
*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*
(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x
+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-9/16/c^2*(-c*d
)^(1/2)*b/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2
)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*
d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-15/
16/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x
-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d
)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(
1/2)/d))-1/4/c*b^2/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-
(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+3/16/(-c*d)^(1/2)/c*d*b/(a*d-b*c)^
2/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x)
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
[Out] Timed out
```

3.89 $\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=255

$$\frac{x(c+dx^2)^2(bc-ad)(7ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2(35a^2d^2-80abcd+48b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}-\frac{dx\sqrt{a+bx^2}\,(c+dx^2)(-35\ldots}{12a^2b\ldots}$$

Rubi [A] time = 0.24, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {413, 526, 528, 388, 217, 206}

$$-\frac{dx\sqrt{a+bx^2}\,(c+dx^2)(-35a^2d^2+24abcd+8b^2c^2)}{12a^2b^3}-\frac{dx\sqrt{a+bx^2}\,(-170a^2bcd^2+105a^3d^3+40ab^2c^2d+16b^3c^3)}{24a^2b^4}+\frac{d^2(35a^2d^2-80abcd+48b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}+\frac{x(c+dx^2)^2(bc-ad)(7ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{x(c+dx^2)^3(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

```
[Out] -(d*(16*b^3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sqrt[a
+ b*x^2])/(24*a^2*b^4) - (d*(8*b^2*c^2 + 24*a*b*c*d - 35*a^2*d^2)*x*Sqrt[a
+ b*x^2]*(c + d*x^2))/(12*a^2*b^3) + ((b*c - a*d)*(2*b*c + 7*a*d)*x*(c + d*
x^2)^2)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b*c - a*d)*x*(c + d*x^2)^3)/(3*a*b*
(a + b*x^2)^(3/2)) + (d^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
```

```
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)^3}{3ab\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{\left(c+dx^2\right)^2\left(c(2bc+ad)-d(4bc-7ad)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}$$

$$= \frac{(bc-ad)(2bc+7ad)x\left(c+dx^2\right)^2}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x\left(c+dx^2\right)^3}{3ab\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{\left(c+dx^2\right)\left(acd(4bc-7ad)+d\left(8b^2c^2+24\right.\right.}{\sqrt{a+bx^2}}}{3a^2b^2}$$

$$= -\frac{d\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}\left(c+dx^2\right)}{12a^2b^3} + \frac{(bc-ad)(2bc+7ad)x\left(c+dx^2\right)^2}{3a^2b^2\sqrt{a+bx^2}} -$$

$$= -\frac{d\left(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3\right)x\sqrt{a+bx^2}}{24a^2b^4} - \frac{d\left(8b^2c^2+24abcd-35a^2d\right.}{12a^2b}$$

$$= -\frac{d\left(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3\right)x\sqrt{a+bx^2}}{24a^2b^4} - \frac{d\left(8b^2c^2+24abcd-35a^2d\right.}{12a^2b}$$

$$= -\frac{d\left(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3\right)x\sqrt{a+bx^2}}{24a^2b^4} - \frac{d\left(8b^2c^2+24abcd-35a^2d\right.}{12a^2b}$$

Mathematica [A] time = 5.17, size = 157, normalized size = 0.62

$$\frac{x\sqrt{a+bx^2}\left(\frac{16(bc-ad)^3(5ad+bc)}{a^2\left(a+bx^2\right)}+3d^3(16bc-11ad)+\frac{8(bc-ad)^4}{a\left(a+bx^2\right)^2}+6bd^4x^2\right)}{24b^4}+\frac{d^2\left(35a^2d^2-80abcd+48b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(3*d^3*(16*b*c - 11*a*d) + 6*b*d^4*x^2 + (8*(b*c - a*d)^
4)/(a*(a + b*x^2)^2) + (16*(b*c - a*d)^3*(b*c + 5*a*d))/(a^2*(a + b*x^2))))
/(24*b^4) + (d^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*Log[b*x + Sqrt[b]*S
qrt[a + b*x^2]])/(8*b^(9/2))
```

IntegrateAlgebraic [A] time = 0.41, size = 237, normalized size = 0.93

$$\frac{\left(-35a^2d^4+80abcd^3-48b^2c^2d^2\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{9/2}}+\frac{-105a^5d^4x+240a^4bcd^3x-140a^4bd^4x^3-144a^3b^2c^2d^2x+320a^3b^2cd^3x^3-21a^3b^2d^4x^5-192a^2b^3c^2d^2x^3+48a^2b^3cd^3x^5+6a^2b^3d^4x^7+24ab^4c^4x+32ab^4c^3dx^3+16b^5c^4x^3}{24a^2b^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

```
[Out] (24*a*b^4*c^4*x - 144*a^3*b^2*c^2*d^2*x + 240*a^4*b*c*d^3*x - 105*a^5*d^4*x
 + 16*b^5*c^4*x^3 + 32*a*b^4*c^3*d*x^3 - 192*a^2*b^3*c^2*d^2*x^3 + 320*a^3*
b^2*c*d^3*x^3 - 140*a^4*b*d^4*x^3 + 48*a^2*b^3*c*d^3*x^5 - 21*a^3*b^2*d^4*x
^5 + 6*a^2*b^3*d^4*x^7)/(24*a^2*b^4*(a + b*x^2)^(3/2)) + ((-48*b^2*c^2*d^2
+ 80*a*b*c*d^3 - 35*a^2*d^4)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(9/2
))
```

fricas [A]   time = 1.31, size = 684, normalized size = 2.68

$$\left[\frac{3\left(48a^4b^2c^2d^2 - 80a^5bcd^3 + 35a^6d^4 + \left(48a^2b^4c^2d^2 - 80a^3b^3cd^3 + 35a^4b^2d^4\right)x^4 + 2\left(48a^3b^3c^2d^2 - 80a^4b^2cd^3 + 35a^5bd^4\right)x^2\right)\sqrt{b}\log\left(-2bx^2 - 2\sqrt{bx^2+a}\sqrt{b}x - a\right) + 2\left(6a^2b^4d^4x^7 + 3\left(16a^2b^4cd^3 - 7a^3b^3d^4\right)x^5 + 4\left(4b^6c^4 + 8ab^5c^3d - 48a^2b^4c^2d^2 + 80a^3b^3cd^3 - 35a^4b^2d^4\right)x^3 + 3\left(8ab^5c^4 - 48a^3b^3c^2d^2 + 80a^4b^2cd^3 - 35a^5bd^4\right)x\right)\sqrt{bx^2+a}}{48\left(a^2b^7x^4 + 2a^3b^6x^2 + a^4b^5\right)}, -\frac{3\left(48a^4b^2c^2d^2 - 80a^5bcd^3 + 35a^6d^4 + \left(48a^2b^4c^2d^2 - 80a^3b^3cd^3 + 35a^4b^2d^4\right)x^4 + 2\left(48a^3b^3c^2d^2 - 80a^4b^2cd^3 + 35a^5bd^4\right)x^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - \left(6a^2b^4d^4x^7 + 3\left(16a^2b^4cd^3 - 7a^3b^3d^4\right)x^5 + 4\left(4b^6c^4 + 8ab^5c^3d - 48a^2b^4c^2d^2 + 80a^3b^3cd^3 - 35a^4b^2d^4\right)x^3 + 3\left(8ab^5c^4 - 48a^3b^3c^2d^2 + 80a^4b^2cd^3 - 35a^5bd^4\right)x\right)\sqrt{bx^2+a}}{24\left(a^2b^7x^4 + 2a^3b^6x^2 + a^4b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*a^6*d^4 + (48*a^2*b^4*c^
2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(48*a^3*b^3*c^2*d^2 - 80
*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) + 2*(6*a^2*b^4*d^4*x^7 + 3*(16*a^2*b^4*c*d^3 - 7*a^3*b^3*d
^4)*x^5 + 4*(4*b^6*c^4 + 8*a*b^5*c^3*d - 48*a^2*b^4*c^2*d^2 + 80*a^3*b^3*c*
d^3 - 35*a^4*b^2*d^4)*x^3 + 3*(8*a*b^5*c^4 - 48*a^3*b^3*c^2*d^2 + 80*a^4*b^
2*c*d^3 - 35*a^5*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 +
a^4*b^5), -1/24*(3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*a^6*d^4 + (48*
a^2*b^4*c^2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(48*a^3*b^3*c^
2*d^2 - 80*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (6*a^2*b^4*d^4*x^7 + 3*(16*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*
x^5 + 4*(4*b^6*c^4 + 8*a*b^5*c^3*d - 48*a^2*b^4*c^2*d^2 + 80*a^3*b^3*c*d^3
- 35*a^4*b^2*d^4)*x^3 + 3*(8*a*b^5*c^4 - 48*a^3*b^3*c^2*d^2 + 80*a^4*b^2*c*
d^3 - 35*a^5*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*
b^5)]
```

giac [A]   time = 0.68, size = 237, normalized size = 0.93

$$\frac{\left(\left(3\left(\frac{2d^4x^2}{b} + \frac{16a^2b^6cd^3 - 7a^3b^5d^4}{a^2b^7}\right)x^2 + \frac{4\left(4b^8c^4 + 8ab^7c^3d - 48a^2b^6c^2d^2 + 80a^3b^5cd^3 - 35a^4b^4d^4\right)}{a^2b^7}\right)x^2 + \frac{3\left(8ab^7c^4 - 48a^3b^5c^2d^2 + 80a^4b^4cd^3 - 35a^5b^3d^4\right)}{a^2b^7}\right)x}{24\left(bx^2+a\right)^{\frac{3}{2}}} - \frac{\left(48b^2c^2d^2 - 80abcd^3 + 35a^2d^4\right)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{8b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*d^4*x^2/b + (16*a^2*b^6*c*d^3 - 7*a^3*b^5*d^4)/(a^2*b^7))*x^2 +
 4*(4*b^8*c^4 + 8*a*b^7*c^3*d - 48*a^2*b^6*c^2*d^2 + 80*a^3*b^5*c*d^3 - 35*
a^4*b^4*d^4)/(a^2*b^7))*x^2 + 3*(8*a*b^7*c^4 - 48*a^3*b^5*c^2*d^2 + 80*a^4*
b^4*c*d^3 - 35*a^5*b^3*d^4)/(a^2*b^7))*x/(b*x^2 + a)^(3/2) - 1/8*(48*b^2*c^
2*d^2 - 80*a*b*c*d^3 + 35*a^2*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(9/2)
```

maple [A]   time = 0.02, size = 351, normalized size = 1.38

$$\frac{d^4x^7}{4\left(bx^2+a\right)^{\frac{3}{2}}b} - \frac{7ad^4x^5}{8\left(bx^2+a\right)^{\frac{3}{2}}b^2} + \frac{2cd^3x^5}{\left(bx^2+a\right)^{\frac{3}{2}}b} - \frac{35a^2d^4x^3}{24\left(bx^2+a\right)^{\frac{3}{2}}b^3} + \frac{10acd^3x^3}{3\left(bx^2+a\right)^{\frac{3}{2}}b^2} - \frac{2c^2d^2x^3}{\left(bx^2+a\right)^{\frac{3}{2}}b} + \frac{c^4x}{3\left(bx^2+a\right)^{\frac{3}{2}}a} - \frac{4c^3dx}{3\left(bx^2+a\right)^{\frac{3}{2}}b} - \frac{35a^2d^4x}{8\sqrt{bx^2+a}\,b^4} + \frac{10acd^3x}{\sqrt{bx^2+a}\,b^3} + \frac{4c^3dx}{3\sqrt{bx^2+a}\,ab} + \frac{2c^4x}{3\sqrt{bx^2+a}\,a^2} - \frac{6c^2d^2x}{\sqrt{bx^2+a}\,b^2} + \frac{35a^2d^4\ln\left(\sqrt{b}x + \sqrt{bx^2+a}\right)}{8b^{\frac{9}{2}}} - \frac{10acd^3\ln\left(\sqrt{b}x + \sqrt{bx^2+a}\right)}{b^{\frac{7}{2}}} + \frac{6c^2d^2\ln\left(\sqrt{b}x + \sqrt{bx^2+a}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/4*d^4*x^7/b/(b*x^2+a)^(3/2)-7/8*d^4*a/b^2*x^5/(b*x^2+a)^(3/2)-35/24*d^4*a
^2/b^3*x^3/(b*x^2+a)^(3/2)-35/8*d^4*a^2/b^4*x/(b*x^2+a)^(1/2)+35/8*d^4*a^2/
b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+2*c*d^3*x^5/b/(b*x^2+a)^(3/2)+10/3*c*
d^3*a/b^2*x^3/(b*x^2+a)^(3/2)+10*c*d^3*a/b^3*x/(b*x^2+a)^(1/2)-10*c*d^3*a/b
^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-2*c^2*d^2*x^3/b/(b*x^2+a)^(3/2)-6*c^2*
```

```
d^2/b^2*x/(b*x^2+a)^(1/2)+6*c^2*d^2/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-4
/3*c^3*d/b*x/(b*x^2+a)^(3/2)+4/3*c^3*d/a/b*x/(b*x^2+a)^(1/2)+1/3*c^4*x/a/(b
*x^2+a)^(3/2)+2/3*c^4/a^2*x/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.44, size = 392, normalized size = 1.54

$$\frac{d^4x^7}{4\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2\,cd^3x^5}{\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{7\,ad^4x^5}{8\left(bx^2+a\right)^{\frac{3}{2}}b^2}-2\,c^2d^2x\left(\frac{3\,x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)+\frac{10\,acd^3x\left(\frac{3\,x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)}{3\,b}-\frac{35\,a^2d^4x\left(\frac{3\,x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)}{24\,b^2}+\frac{2\,c^4x}{3\,\sqrt{bx^2+a}\,a^2}+\frac{c^4x}{3\left(bx^2+a\right)^{\frac{3}{2}}a}-\frac{4\,c^3dx}{3\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{4\,c^3dx}{3\,\sqrt{bx^2+a}\,ab}-\frac{2\,c^2d^2x}{\sqrt{bx^2+a}\,b^2}+\frac{10\,acd^3x}{3\,\sqrt{bx^2+a}\,b^3}-\frac{35\,a^2d^4x}{24\,\sqrt{bx^2+a}\,b^4}+\frac{6\,c^2d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{10\,acd^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{7}{2}}}+\frac{35\,a^2d^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*d^4*x^7/((b*x^2 + a)^(3/2)*b) + 2*c*d^3*x^5/((b*x^2 + a)^(3/2)*b) - 7/8
*a*d^4*x^5/((b*x^2 + a)^(3/2)*b^2) - 2*c^2*d^2*x*(3*x^2/((b*x^2 + a)^(3/2)*
b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 10/3*a*c*d^3*x*(3*x^2/((b*x^2 + a)^(3/2
)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b - 35/24*a^2*d^4*x*(3*x^2/((b*x^2 + a)
^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 2/3*c^4*x/(sqrt(b*x^2 + a)*a
^2) + 1/3*c^4*x/((b*x^2 + a)^(3/2)*a) - 4/3*c^3*d*x/((b*x^2 + a)^(3/2)*b) +
 4/3*c^3*d*x/(sqrt(b*x^2 + a)*a*b) - 2*c^2*d^2*x/(sqrt(b*x^2 + a)*b^2) + 10
/3*a*c*d^3*x/(sqrt(b*x^2 + a)*b^3) - 35/24*a^2*d^4*x/(sqrt(b*x^2 + a)*b^4)
+ 6*c^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 10*a*c*d^3*arcsinh(b*x/sqrt(a*
b))/b^(7/2) + 35/8*a^2*d^4*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^2+c\right)}^4}{{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^4/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c + d*x^2)^4/(a + b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{2}\right)^{4}}{\left(a + bx^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**4/(a + b*x**2)**(5/2), x)
```

3.90 $\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=172

$$\frac{x\left(c+dx^2\right)(bc-ad)(5ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}-\frac{dx\sqrt{a+bx^2}\left(-15a^2d^2+8abcd+4b^2c^2\right)}{6a^2b^3}+\frac{d^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{x\left(c\right.}{3}$$

Rubi [A] time = 0.16, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {413, 526, 388, 217, 206}

$$-\frac{dx\sqrt{a+bx^2}\left(-15a^2d^2+8abcd+4b^2c^2\right)}{6a^2b^3}+\frac{x\left(c+dx^2\right)(bc-ad)(5ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

```
[Out] -(d*(4*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(6*a^2*b^3) + (
(b*c - a*d)*(2*b*c + 5*a*d)*x*(c + d*x^2))/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((
b*c - a*d)*x*(c + d*x^2)^2)/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*(6*b*c - 5*a*d
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
```

```
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)^2}{3ab\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{\left(c+dx^2\right)\left(c(2bc+ad)-d(2bc-5ad)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}$$

$$= \frac{(bc-ad)(2bc+5ad)x\left(c+dx^2\right)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x\left(c+dx^2\right)^2}{3ab\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{acd(2bc-5ad)+d\left(4b^2c^2+8abcd-15a^2\right.}{\sqrt{a+bx^2}}}{3a^2b^2}$$

$$= -\frac{d\left(4b^2c^2+8abcd-15a^2d^2\right)x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x\left(c+dx^2\right)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x}{3ab\left(a+\right.}$$

$$= -\frac{d\left(4b^2c^2+8abcd-15a^2d^2\right)x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x\left(c+dx^2\right)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x}{3ab\left(a+\right.}$$

$$= -\frac{d\left(4b^2c^2+8abcd-15a^2d^2\right)x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x\left(c+dx^2\right)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x}{3ab\left(a+\right.}$$

Mathematica [A] time = 5.10, size = 125, normalized size = 0.73

$$\frac{x\left(3a^2d^3\left(a+bx^2\right)^2+2\left(a+bx^2\right)(bc-ad)^2(7ad+2bc)+2a(bc-ad)^3\right)}{6a^2b^3\left(a+bx^2\right)^{3/2}}+\frac{d^2(6bc-5ad)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(2*a*(b*c - a*d)^3 + 2*(b*c - a*d)^2*(2*b*c + 7*a*d)*(a + b*x^2) + 3*a^2
*d^3*(a + b*x^2)^2))/(6*a^2*b^3*(a + b*x^2)^(3/2)) + (d^2*(6*b*c - 5*a*d)*L
og[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.27, size = 162, normalized size = 0.94

$$\frac{15a^4d^3x-18a^3bcd^2x+20a^3bd^3x^3-24a^2b^2cd^2x^3+3a^2b^2d^3x^5+6ab^3c^3x+6ab^3c^2dx^3+4b^4c^3x^3}{6a^2b^3\left(a+bx^2\right)^{3/2}}+\frac{\left(5ad^3-6bcd^2\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

```
[Out] (6*a*b^3*c^3*x - 18*a^3*b*c*d^2*x + 15*a^4*d^3*x + 4*b^4*c^3*x^3 + 6*a*b^3*
c^2*d*x^3 - 24*a^2*b^2*c*d^2*x^3 + 20*a^3*b*d^3*x^3 + 3*a^2*b^2*d^3*x^5)/(6
*a^2*b^3*(a + b*x^2)^(3/2)) + ((-6*b*c*d^2 + 5*a*d^3)*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/(2*b^(7/2))
```

fricas [A] time = 1.21, size = 486, normalized size = 2.83

$$\left[-\frac{3\left(6a^4bcd^2-5a^5d^3+\left(6a^2b^3cd^2-5a^3b^2d^3\right)x^4+2\left(6a^3b^2cd^2-5a^4bd^3\right)x^2\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(3a^2b^3d^3x^5+2\left(2b^5c^3+3ab^4c^2d-12a^2b^3cd^2+10a^3b^2d^3\right)x^3+3\left(2ab^4c^3-6a^3b^2cd^2+5a^4bd^3\right)x\right)\sqrt{bx^2+a}}{12\left(a^2b^6x^4+2a^3b^5x^2+a^4b^4\right)}, -\frac{3\left(6a^4bcd^2-5a^5d^3+\left(6a^2b^3cd^2-5a^3b^2d^3\right)x^4+2\left(6a^3b^2cd^2-5a^4bd^3\right)x^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(3a^2b^3d^3x^5+2\left(2b^5c^3+3ab^4c^2d-12a^2b^3cd^2+10a^3b^2d^3\right)x^3+3\left(2ab^4c^3-6a^3b^2cd^2+5a^4bd^3\right)x\right)\sqrt{bx^2+a}}{6\left(a^2b^6x^4+2a^3b^5x^2+a^4b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(6*a^4*b*c*d^2 - 5*a^5*d^3 + (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^
4 + 2*(6*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) - 2*(3*a^2*b^3*d^3*x^5 + 2*(2*b^5*c^3 + 3*a*b^4*c^2
*d - 12*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^3 + 3*(2*a*b^4*c^3 - 6*a^3*b^2*c*
d^2 + 5*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b
^4), -1/6*(3*(6*a^4*b*c*d^2 - 5*a^5*d^3 + (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)
*x^4 + 2*(6*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (3*a^2*b^3*d^3*x^5 + 2*(2*b^5*c^3 + 3*a*b^4*c^2*d - 12*a^2
*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^3 + 3*(2*a*b^4*c^3 - 6*a^3*b^2*c*d^2 + 5*a^4
*b*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4)]
```

giac [A] time = 0.68, size = 158, normalized size = 0.92

$$\frac{\left(\left(\frac{3\,d^3x^2}{b}+\frac{2\left(2\,b^6c^3+3\,ab^5c^2d-12\,a^2b^4cd^2+10\,a^3b^3d^3\right)}{a^2b^5}\right)x^2+\frac{3\left(2\,ab^5c^3-6\,a^3b^3cd^2+5\,a^4b^2d^3\right)}{a^2b^5}\right)x}{6\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{\left(6\,bcd^2-5\,ad^3\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*d^3*x^2/b + 2*(2*b^6*c^3 + 3*a*b^5*c^2*d - 12*a^2*b^4*c*d^2 + 10*a^
3*b^3*d^3)/(a^2*b^5))*x^2 + 3*(2*a*b^5*c^3 - 6*a^3*b^3*c*d^2 + 5*a^4*b^2*d^
3)/(a^2*b^5))*x/(b*x^2 + a)^(3/2) - 1/2*(6*b*c*d^2 - 5*a*d^3)*log(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.01, size = 228, normalized size = 1.33

$$\frac{d^3x^5}{2\left(b\,x^2+a\right)^{\frac{3}{2}}b}+\frac{5a\,d^3x^3}{6\left(b\,x^2+a\right)^{\frac{3}{2}}b^2}-\frac{c\,d^2x^3}{\left(b\,x^2+a\right)^{\frac{3}{2}}b}+\frac{c^3x}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a}-\frac{c^2dx}{\left(b\,x^2+a\right)^{\frac{3}{2}}b}+\frac{5a\,d^3x}{2\sqrt{b\,x^2+a}\,b^3}+\frac{c^2dx}{\sqrt{b\,x^2+a}\,ab}+\frac{2c^3x}{3\sqrt{b\,x^2+a}\,a^2}-\frac{3c\,d^2x}{\sqrt{b\,x^2+a}\,b^2}-\frac{5a\,d^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{7}{2}}}+\frac{3c\,d^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/2*d^3*x^5/b/(b*x^2+a)^(3/2)+5/6*d^3*a/b^2*x^3/(b*x^2+a)^(3/2)+5/2*d^3*a/b
^3*x/(b*x^2+a)^(1/2)-5/2*d^3*a/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-c*d^2*
x^3/b/(b*x^2+a)^(3/2)-3*c*d^2/b^2*x/(b*x^2+a)^(1/2)+3*c*d^2/b^(5/2)*ln(b^(1
/2)*x+(b*x^2+a)^(1/2))-c^2*d/b*x/(b*x^2+a)^(3/2)+c^2*d/a/b*x/(b*x^2+a)^(1/2
)+1/3*c^3*x/a/(b*x^2+a)^(3/2)+2/3*c^3/a^2*x/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.49, size = 254, normalized size = 1.48

$$\frac{d^3x^5}{2\left(bx^2+a\right)^{\frac{3}{2}}b}-cd^2x\left(\frac{3\,x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)+\frac{5\,ad^3x\left(\frac{3\,x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)}{6\,b}+\frac{2\,c^3x}{3\,\sqrt{bx^2+a}\,a^2}+\frac{c^3x}{3\left(bx^2+a\right)^{\frac{3}{2}}a}-\frac{c^2dx}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{c^2dx}{\sqrt{bx^2+a}\,ab}-\frac{cd^2x}{\sqrt{bx^2+a}\,b^2}+\frac{5\,ad^3x}{6\,\sqrt{bx^2+a}\,b^3}+\frac{3\,cd^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{5\,ad^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*x^5/((b*x^2 + a)^(3/2)*b) - c*d^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) +
2*a/((b*x^2 + a)^(3/2)*b^2)) + 5/6*a*d^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2
*a/((b*x^2 + a)^(3/2)*b^2))/b + 2/3*c^3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^3*x
/((b*x^2 + a)^(3/2)*a) - c^2*d*x/((b*x^2 + a)^(3/2)*b) + c^2*d*x/(sqrt(b*x^
2 + a)*a*b) - c*d^2*x/(sqrt(b*x^2 + a)*b^2) + 5/6*a*d^3*x/(sqrt(b*x^2 + a)*
b^3) + 3*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/2*a*d^3*arcsinh(b*x/sqrt(
a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^3}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c + d*x^2)^3/(a + b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**3/(a + b*x**2)**(5/2), x)
```

3.91 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=105

$$\frac{x(bc-ad)(3ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {413, 385, 217, 206}

$$\frac{x(bc-ad)(3ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)*(2*b*c + 3*a*d)*x)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b*c - a*d)*
x*(c + d*x^2))/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/b^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)}{3ab\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{c(2bc+ad)+3ad^2x^2}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}$$

$$= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x\left(c+dx^2\right)}{3ab\left(a+bx^2\right)^{3/2}} + \frac{d^2\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}$$

$$= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x\left(c+dx^2\right)}{3ab\left(a+bx^2\right)^{3/2}} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b^2}$$

$$= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x\left(c+dx^2\right)}{3ab\left(a+bx^2\right)^{3/2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Mathematica [C] time = 4.15, size = 214, normalized size = 2.04

$$\frac{\sqrt{\frac{bx^2}{a}+1}\left(-16b^3x^6\left(c+dx^2\right)^2\,{}_3F_2\left(\frac{3}{2},2,\frac{7}{2};1,\frac{9}{2};-\frac{bx^2}{a}\right)+\frac{7a^2\left(15c^2+10cdx^2+3d^2x^4\right)\left(\sqrt{-\frac{bx^2\left(a+bx^2\right)}{a^2}}\left(2bx^2-3a\right)+3a\sin^{-1}\left(\sqrt{-\frac{bx^2}{a}}\right)\right)}{\sqrt{-\frac{bx^2}{a}}}-32b^3x^6\left(2c^2+3cdx^2+d^2x^4\right)\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{bx^2}{a}\right)\right)}{168a^3b^2x^3\sqrt{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 + (b*x^2)/a]*((7*a^2*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*(Sqrt[-((b*x
^2*(a + b*x^2))/a^2)]*(-3*a + 2*b*x^2) + 3*a*ArcSin[Sqrt[-((b*x^2)/a)]]))/S
qrt[-((b*x^2)/a)] - 32*b^3*x^6*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*Hypergeometric
2F1[3/2, 7/2, 9/2, -((b*x^2)/a)] - 16*b^3*x^6*(c + d*x^2)^2*HypergeometricP
FQ[{3/2, 2, 7/2}, {1, 9/2}, -((b*x^2)/a)]))/(168*a^3*b^2*x^3*Sqrt[a + b*x^2
])
```

IntegrateAlgebraic [A] time = 0.21, size = 107, normalized size = 1.02

$$\frac{-3a^3d^2x-4a^2bd^2x^3+3ab^2c^2x+2ab^2cdx^3+2b^3c^2x^3}{3a^2b^2\left(a+bx^2\right)^{3/2}}-\frac{d^2\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

```
[Out] (3*a*b^2*c^2*x - 3*a^3*d^2*x + 2*b^3*c^2*x^3 + 2*a*b^2*c*d*x^3 - 4*a^2*b*d^
2*x^3)/(3*a^2*b^2*(a + b*x^2)^(3/2)) - (d^2*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/b^(5/2)
```

fricas [A] time = 1.13, size = 318, normalized size = 3.03

$$\left[\frac{3\left(a^2b^2d^2x^4+2a^3bd^2x^2+a^4d^2\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2\left(b^4c^2+ab^3cd-2a^2b^2d^2\right)x^3+3\left(ab^3c^2-a^3bd^2\right)x\right)\sqrt{bx^2+a}}{6\left(a^2b^5x^4+2a^3b^4x^2+a^4b^3\right)}, -\frac{3\left(a^2b^2d^2x^4+2a^3bd^2x^2+a^4d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(2\left(b^4c^2+ab^3cd-2a^2b^2d^2\right)x^3+3\left(ab^3c^2-a^3bd^2\right)x\right)\sqrt{bx^2+a}}{3\left(a^2b^5x^4+2a^3b^4x^2+a^4b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*
d^2)*x^3 + 3*(a*b^3*c^2 - a^3*b*d^2)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a
```

```
^3*b^4*x^2 + a^4*b^3), -1/3*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2
)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*(b^4*c^2 + a*b^3*c*d - 2
*a^2*b^2*d^2)*x^3 + 3*(a*b^3*c^2 - a^3*b*d^2)*x)*sqrt(b*x^2 + a))/(a^2*b^5*
x^4 + 2*a^3*b^4*x^2 + a^4*b^3)]
```

giac [A] time = 0.63, size = 103, normalized size = 0.98

$$\frac{x\left(\frac{2\left(b^4c^2+ab^3cd-2a^2b^2d^2\right)x^2}{a^2b^3}+\frac{3\left(ab^3c^2-a^3bd^2\right)}{a^2b^3}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{d^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*d^2)*x^2/(a^2*b^3) + 3*(a*b^3*c^2
 - a^3*b*d^2)/(a^2*b^3))/(b*x^2 + a)^(3/2) - d^2*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 136, normalized size = 1.30

$$-\frac{d^2x^3}{3\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{c^2x}{3\left(bx^2+a\right)^{\frac{3}{2}}a}-\frac{2cdx}{3\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2cdx}{3\sqrt{bx^2+a}\,ab}+\frac{2c^2x}{3\sqrt{bx^2+a}\,a^2}-\frac{d^2x}{\sqrt{bx^2+a}\,b^2}+\frac{d^2\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*d^2*x^3/b/(b*x^2+a)^(3/2)-d^2/b^2*x/(b*x^2+a)^(1/2)+d^2/b^(5/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))-2/3*c*d/b*x/(b*x^2+a)^(3/2)+2/3*c*d/a/b*x/(b*x^2+a)
^(1/2)+1/3*c^2*x/a/(b*x^2+a)^(3/2)+2/3*c^2/a^2*x/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.39, size = 147, normalized size = 1.40

$$-\frac{1}{3}d^2x\left(\frac{3x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)+\frac{2c^2x}{3\sqrt{bx^2+a}\,a^2}+\frac{c^2x}{3\left(bx^2+a\right)^{\frac{3}{2}}a}-\frac{2cdx}{3\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2cdx}{3\sqrt{bx^2+a}\,ab}-\frac{d^2x}{3\sqrt{bx^2+a}\,b^2}+\frac{d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*d^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 2/
3*c^2*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^2*x/((b*x^2 + a)^(3/2)*a) - 2/3*c*d*x
/((b*x^2 + a)^(3/2)*b) + 2/3*c*d*x/(sqrt(b*x^2 + a)*a*b) - 1/3*d^2*x/(sqrt(
b*x^2 + a)*b^2) + d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^2}{\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c + d*x^2)^2/(a + b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**2/(a + b*x**2)**(5/2), x)
```

3.92 $\int \frac{c+dx^2}{\left(a+bx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=47

$$\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x\left(c+dx^2\right)}{3a\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {378, 191}

$$\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x\left(c+dx^2\right)}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*c*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(c + d*x^2))/(3*a*(a + b*x^2)^(3/2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{\left(a+bx^2\right)^{5/2}}\,dx&=\frac{x\left(c+dx^2\right)}{3a\left(a+bx^2\right)^{3/2}}+\frac{(2c)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\&=\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x\left(c+dx^2\right)}{3a\left(a+bx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.79

$$\frac{x\left(3ac+adx^2+2bcx^2\right)}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(3*a*c + 2*b*c*x^2 + a*d*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 37, normalized size = 0.79

$$\frac{3acx + adx^3 + 2bcx^3}{3a^2\left(a + bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (3*a*c*x + 2*b*c*x^3 + a*d*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.95, size = 54, normalized size = 1.15

$$\frac{\left((2\,bc + ad)x^3 + 3\,acx\right)\sqrt{bx^2 + a}}{3\left(a^2b^2x^4 + 2\,a^3bx^2 + a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b*c + a*d)*x^3 + 3*a*c*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^
2 + a^4)
```

giac [A] time = 0.62, size = 40, normalized size = 0.85

$$\frac{x\left(\frac{3\,c}{a} + \frac{\left(2\,b^2c + abd\right)x^2}{a^2b}\right)}{3\left(bx^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(3*c/a + (2*b^2*c + a*b*d)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.72

$$\frac{\left(ad\,x^2 + 2bc\,x^2 + 3ac\right)x}{3\left(b\,x^2 + a\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(a*d*x^2+2*b*c*x^2+3*a*c)/(b*x^2+a)^(3/2)/a^2
```

maxima [A] time = 1.37, size = 68, normalized size = 1.45

$$\frac{2\,cx}{3\sqrt{bx^2 + a}\,a^2} + \frac{cx}{3\left(bx^2 + a\right)^{\frac{3}{2}}a} - \frac{dx}{3\left(bx^2 + a\right)^{\frac{3}{2}}b} + \frac{dx}{3\sqrt{bx^2 + a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*c*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c*x/((b*x^2 + a)^(3/2)*a) - 1/3*d*x/((b
*x^2 + a)^(3/2)*b) + 1/3*d*x/(sqrt(b*x^2 + a)*a*b)
```

mupad [B] time = 4.78, size = 33, normalized size = 0.70

$$\frac{3\,a\,c\,x+a\,d\,x^3+2\,b\,c\,x^3}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^(5/2),x)
```

```
[Out] (3*a*c*x + a*d*x^3 + 2*b*c*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

sympy [B] time = 11.03, size = 144, normalized size = 3.06

$$c\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)+\frac{dx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(5/2),x)
```

```
[Out] c*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + d*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sq
rt(1 + b*x**2/a))
```

3.93 $\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{x}{3a\left(a+bx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\ &= \frac{x}{3a\left(a+bx^2\right)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.74

$$\frac{x\left(3a+2bx^2\right)}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/2),x]
```

```
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.00, size = 29, normalized size = 0.74

$$\frac{x\left(3a+2bx^2\right)}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(-5/2),x]
```

```
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.58, size = 47, normalized size = 1.21

$$\frac{\left(2\,bx^{3}+3\,ax\right)\sqrt{bx^{2}+a}}{3\,\left(a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

giac [A] time = 0.61, size = 27, normalized size = 0.69

$$\frac{x\left(\frac{2\,bx^{2}}{a^{2}}+\frac{3}{a}\right)}{3\,\left(bx^{2}+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

maple [A] time = 0.00, size = 26, normalized size = 0.67

$$\frac{\left(2b\,x^{2}+3a\right)x}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(2*b*x^2+3*a)/(b*x^2+a)^(3/2)/a^2*x
```

maxima [A] time = 1.33, size = 31, normalized size = 0.79

$$\frac{2\,x}{3\,\sqrt{bx^{2}+a}\,a^{2}}+\frac{x}{3\,\left(bx^{2}+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

mupad [B] time = 4.75, size = 28, normalized size = 0.72

$$\frac{2\,x\,\left(b\,x^2+a\right)+a\,x}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(5/2),x)
```

```
[Out] (2*x*(a + b*x^2) + a*x)/(3*a^2*(a + b*x^2)^(3/2))
```

sympy [B] time = 0.82, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)
) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*
x**2/a))
```

3.94 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=122

$$\frac{bx(2bc-5ad)}{3a^2\sqrt{a+bx^2}\,(bc-ad)^2}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(bc-ad)^{5/2}}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Rubi [A] time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$\frac{bx(2bc-5ad)}{3a^2\sqrt{a+bx^2}\,(bc-ad)^2}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(bc-ad)^{5/2}}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (b*(2*b*c - 5*a*d)*x)/(3*a^2*(b
*c - a*d)^2*Sqrt[a + b*x^2]) + (d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sq
rt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}\,dx &= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{-2bc+3ad-2bdx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}\,dx}{3a(bc-ad)}\\
&= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}}+\frac{\int \frac{3a^2d^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{3a^2(bc-ad)^2}\\
&= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}}+\frac{d^2\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{(bc-ad)^2}\\
&= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,\right.}{(bc-ad)^2}\\
&= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(bc-ad)^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 2.75, size = 775, normalized size = 6.35

$$
\frac{x\left(12c^2\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{7/2}{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)+12d^2x^4\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{7/2}{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)+24cdx^2\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{7/2}{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)+48c^2\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{7/2}{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)-105c^2\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{3/2}-315c^2\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}+315c^2\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\right)+36d^2x^4\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{7/2}{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)-56d^2x^4\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{3/2}-168d^2x^4\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}+168d^2x^4\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\right)+84cdx^2\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{7/2}{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)-140cdx^2\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{3/2}-420cdx^2\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}+420cdx^2\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\right)\right)}{63c^3\left(a+bx^2\right)^{5/2}\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{5/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

```
[Out] (x*(-315*c^2*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - 420*c*d*x^2*Sqrt[((b
*c - a*d)*x^2)/(c*(a + b*x^2))] - 168*d^2*x^4*Sqrt[((b*c - a*d)*x^2)/(c*(a
+ b*x^2))] - 105*c^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2) - 140*c*d*x^
2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2) - 56*d^2*x^4*(((b*c - a*d)*x^2)
/(c*(a + b*x^2)))^(3/2) + 315*c^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*
x^2))]] + 420*c*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 16
8*d^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 48*c^2*(((b*c
- a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, ((b*c - a
*d)*x^2)/(c*(a + b*x^2))] + 84*c*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^
(7/2)*Hypergeometric2F1[2, 7/2, 9/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 3
6*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*Hypergeometric2F1[2, 7/
2, 9/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 12*c^2*(((b*c - a*d)*x^2)/(c*(
a + b*x^2)))^(7/2)*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, ((b*c - a*d)*x^
2)/(c*(a + b*x^2))] + 24*c*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*
HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]
 + 12*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*HypergeometricPFQ[{
2, 2, 7/2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(63*c^3*(((b*c -
 a*d)*x^2)/(c*(a + b*x^2)))^(5/2)*(a + b*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.41, size = 180, normalized size = 1.48

$$
\frac{-6a^2bdx+3ab^2cx-5ab^2dx^3+2b^3cx^3}{3a^2\left(a+bx^2\right)^{3/2}(ad-bc)^2}+\frac{d^2\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\,dx^2}{\sqrt{c}\,\sqrt{ad-bc}}-\frac{dx\sqrt{a+bx^2}}{\sqrt{c}\,\sqrt{ad-bc}}+\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{ad-bc}}\right)}{\sqrt{c}\,(bc-ad)^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

```
[Out] (3*a*b^2*c*x - 6*a^2*b*d*x + 2*b^3*c*x^3 - 5*a*b^2*d*x^3)/(3*a^2*(-(b*c) +
a*d)^2*(a + b*x^2)^(3/2)) + (d^2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c]
)/Sqrt[-(b*c) + a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[-(b*c) + a*d]) - (d*x*
Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*(b*c - a*d)^3)
```

fricas [B] time = 1.74, size = 764, normalized size = 6.26

$$\left[\frac{3\left(a^2b^2d^2x^4+2a^3bd^2x^2+a^4d^2\right)\sqrt{bc^2-acd}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2+4((2bc-ad)x^3+acx)\sqrt{bc^2-acd}\sqrt{bx^2+a}}{d^2x^4+2cdx^2+c^2}\right)+4\left((2b^4c^3-7ab^3c^2d+5a^2b^2cd^2)x^3+3(ab^3c^3-3a^2b^2c^2d+2a^3bcd^2)x\right)\sqrt{bx^2+a}}{12\left(a^4b^3c^4-3a^5b^2c^3d+3a^6bc^2d^2-a^7cd^3+(a^2b^5c^4-3a^3b^4c^3d+3a^4b^3c^2d^2-a^5b^2cd^3)x^4+2(a^3b^4c^4-3a^4b^3c^3d+3a^5b^2c^2d^2-a^6bcd^3)x^2\right)},\ -\frac{3\left(a^2b^2d^2x^4+2a^3bd^2x^2+a^4d^2\right)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\left((2bc-ad)x^2+ac\right)\sqrt{bx^2+a}}{2\left((b^2c^2-abcd)x^3+(abc^2-a^2cd)x\right)}\right)-2\left((2b^4c^3-7ab^3c^2d+5a^2b^2cd^2)x^3+3(ab^3c^3-3a^2b^2c^2d+2a^3bcd^2)x\right)\sqrt{bx^2+a}}{6\left(a^4b^3c^4-3a^5b^2c^3d+3a^6bc^2d^2-a^7cd^3+(a^2b^5c^4-3a^3b^4c^3d+3a^4b^3c^2d^2-a^5b^2cd^3)x^4+2(a^3b^4c^4-3a^4b^3c^3d+3a^5b^2c^2d^2-a^6bcd^3)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(b*c^2 - a*c*d)*
log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2
*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 +
a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((2*b^4*c^3 - 7*a*b^3*c^2*d + 5*a^2*b^
2*c*d^2)*x^3 + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 2*a^3*b*c*d^2)*x)*sqrt(b*x^
2 + a))/(a^4*b^3*c^4 - 3*a^5*b^2*c^3*d + 3*a^6*b*c^2*d^2 - a^7*c*d^3 + (a^2
*b^5*c^4 - 3*a^3*b^4*c^3*d + 3*a^4*b^3*c^2*d^2 - a^5*b^2*c*d^3)*x^4 + 2*(a^
3*b^4*c^4 - 3*a^4*b^3*c^3*d + 3*a^5*b^2*c^2*d^2 - a^6*b*c*d^3)*x^2), -1/6*(
3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(-b*c^2 + a*c*d)*arctan
(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c
^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((2*b^4*c^3 - 7*a*b^3*c^2*d
 + 5*a^2*b^2*c*d^2)*x^3 + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 2*a^3*b*c*d^2)*x
)*sqrt(b*x^2 + a))/(a^4*b^3*c^4 - 3*a^5*b^2*c^3*d + 3*a^6*b*c^2*d^2 - a^7*c
*d^3 + (a^2*b^5*c^4 - 3*a^3*b^4*c^3*d + 3*a^4*b^3*c^2*d^2 - a^5*b^2*c*d^3)*
x^4 + 2*(a^3*b^4*c^4 - 3*a^4*b^3*c^3*d + 3*a^5*b^2*c^2*d^2 - a^6*b*c*d^3)*x
^2)]
```

giac [B] time = 0.64, size = 320, normalized size = 2.62

$$-\frac{\sqrt{b}\,d^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c^2+abcd}}+\frac{\left(\frac{\left(2\,b^6c^3-9\,ab^5c^2d+12\,a^2b^4cd^2-5\,a^3b^3d^3\right)x^2}{a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4}+\frac{3\left(ab^5c^3-4\,a^2b^4c^2d+5\,a^3b^3cd^2-2\,a^4b^2d^3\right)}{a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4}\right)x}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -sqrt(b)*d^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/s
qrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c^2 + a
*b*c*d)) + 1/3*((2*b^6*c^3 - 9*a*b^5*c^2*d + 12*a^2*b^4*c*d^2 - 5*a^3*b^3*d
^3)*x^2/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^
3 + a^6*b*d^4) + 3*(a*b^5*c^3 - 4*a^2*b^4*c^2*d + 5*a^3*b^3*c*d^2 - 2*a^4*b
^2*d^3)/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^
3 + a^6*b*d^4))*x/(b*x^2 + a)^(3/2)
```

maple [B] time = 0.02, size = 1070, normalized size = 8.77

$$-\frac{d^2\ln\left(\frac{\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,(ad-bc)^2\sqrt{\frac{ad-bc}{d}}}+\frac{d^2\ln\left(\frac{-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{2ad-2bc}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,(ad-bc)^2\sqrt{\frac{ad-bc}{d}}}-\frac{bdx}{2(ad-bc)^2\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}\,a}-\frac{bdx}{2(ad-bc)^2\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}\,a}+\frac{d^2}{2\sqrt{-cd}\,(ad-bc)^2\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}-\frac{d^2}{2\sqrt{-cd}\,(ad-bc)^2\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}}-\frac{bx}{6(ad-bc)\left(\left(x-\frac{\sqrt{-cd}}{d}\right)^2b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}\right)^{\frac{3}{2}}a}-\frac{bx}{6(ad-bc)\left(\left(x+\frac{\sqrt{-cd}}{d}\right)^2b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}\right)^{\frac{3}{2}}a}+\frac{d}{6\sqrt{-cd}\,(ad-bc)\left(\left(x-\frac{\sqrt{-cd}}{d}\right)^2b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}\right)^{\frac{3}{2}}}-\frac{d}{6\sqrt{-cd}\,(ad-bc)\left(\left(x+\frac{\sqrt{-cd}}{d}\right)^2b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}\right)^{\frac{3}{2}}}-\frac{bx}{3(ad-bc)\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2b+\frac{2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}\,a^2}-\frac{bx}{3(ad-bc)\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2b-\frac{2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)b}{d}+\frac{ad-bc}{d}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c),x)
```

```
[Out] 1/6/(-c*d)^(1/2)/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*
d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-1/6*b/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d)^2*
```

```
b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*x-1/3*b/(a*d-b*c
)/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*
c)/d)^(1/2)*x+1/2/(-c*d)^(1/2)*d^2/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*(-
c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-1/2*d/(a*d-b*c)^2/a/((
x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1
/2)*x*b-1/2/(-c*d)^(1/2)*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(
1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^
(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(
-c*d)^(1/2)/d))-1/6/(-c*d)^(1/2)/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*
d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-1/6*b/(a*d-b*c)/a/((x+(-
c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*
x-1/3*b/(a*d-b*c)/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2
)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-1/2/(-c*d)^(1/2)*d^2/(a*d-b*c)^2/((x+(-c*d)^(
1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-1/2*d/
(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d
+(a*d-b*c)/d)^(1/2)*x*b+1/2/(-c*d)^(1/2)*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2
)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^
(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*
c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^2+a\right)^{5/2}\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)),x)
```

```
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{5}{2}}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)), x)
```

3.95 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=202

$$\frac{bx\left(-3a^2d^2-16abcd+4b^2c^2\right)}{6a^2c\sqrt{a+bx^2}\,(bc-ad)^3}+\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}}-\frac{dx}{2c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(bc-ad)}+\frac{bx(3\ldots}{6ac\left(a+bx\ldots\right.}$$

Rubi [A] time = 0.23, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$\frac{bx\left(-3a^2d^2-16abcd+4b^2c^2\right)}{6a^2c\sqrt{a+bx^2}\,(bc-ad)^3}+\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}}-\frac{dx}{2c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(bc-ad)}+\frac{bx(3ad+2bc)}{6ac\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (b*(2*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (b*(4*b^2*c
^2 - 16*a*b*c*d - 3*a^2*d^2)*x)/(6*a^2*c*(b*c - a*d)^3*Sqrt[a + b*x^2]) - (
d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)) + (d^2*(6*b*c - a*d)*A
rcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*
d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}\,dx = -\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{\int \frac{2bc-ad-4bdx^2}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} - \frac{\int \frac{-4b^2c^2+12abc\cdots}{(a+\cdots}}{6\cdots}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{dx\cdots}{2c(bc-ad)\left(a+b\cdots\right.}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{dx\cdots}{2c(bc-ad)\left(a+b\cdots\right.}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{dx\cdots}{2c(bc-ad)\left(a+b\cdots\right.}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{dx\cdots}{2c(bc-ad)\left(a+b\cdots\right.}$$

Mathematica [A] time = 5.50, size = 170, normalized size = 0.84

$$\frac{1}{6}\left(x\sqrt{a+bx^2}\left(\frac{4b^2(4ad-bc)}{a^2\left(a+bx^2\right)(ad-bc)^3} + \frac{2b^2}{a\left(a+bx^2\right)^2(bc-ad)^2} - \frac{3d^3}{c\left(c+dx^2\right)(bc-ad)^3}\right) + \frac{3d^2(ad-6bc)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}(ad-bc)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((2*b^2)/(a*(b*c - a*d)^2*(a + b*x^2)^2) + (4*b^2*(-(b*c
) + 4*a*d))/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - (3*d^3)/(c*(b*c - a*d)^3*(
c + d*x^2))) + (3*d^2*(-6*b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]
*Sqrt[a + b*x^2])])/(c^(3/2)*(-(b*c) + a*d)^(7/2)))/6
```

IntegrateAlgebraic [A] time = 1.59, size = 261, normalized size = 1.29

$$\frac{3a^4d^3x+6a^3bd^3x^3+18a^2b^2c^2dx+18a^2b^2cd^2x^3+3a^2b^2d^3x^5-6ab^3c^3x+10ab^3c^2dx^3+16ab^3cd^2x^5-4b^4c^3x^3-4b^4c^2dx^5}{6a^2c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(ad-bc)^3} + \frac{\sqrt{ad-bc}\left(6bcd^2-ad^3\right)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}c+\sqrt{b}dx^2}{\sqrt{c}\sqrt{ad-bc}}\right)}{2c^{3/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (-6*a*b^3*c^3*x + 18*a^2*b^2*c^2*d*x + 3*a^4*d^3*x - 4*b^4*c^3*x^3 + 10*a*b
^3*c^2*d*x^3 + 18*a^2*b^2*c*d^2*x^3 + 6*a^3*b*d^3*x^3 - 4*b^4*c^2*d*x^5 + 1
6*a*b^3*c*d^2*x^5 + 3*a^2*b^2*d^3*x^5)/(6*a^2*c*(-(b*c) + a*d)^3*(a + b*x^2
)^(3/2)*(c + d*x^2)) + (Sqrt[-(b*c) + a*d]*(6*b*c*d^2 - a*d^3)*ArcTan[(Sqrt
[b]*c + Sqrt[b]*d*x^2 - d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a*d])])
/(2*c^(3/2)*(b*c - a*d)^4)
```

fricas [B] time = 3.59, size = 1440, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(6*a^4*b*c^2*d^2 - a^5*c*d^3 + (6*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6
 + (6*a^2*b^3*c^2*d^2 + 11*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^4 + (12*a^3*b^2*c
^2*d^2 + 4*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2
- 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((
2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2
*c*d*x^2 + c^2)) + 4*((4*b^5*c^4*d - 20*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3
+ 3*a^3*b^2*c*d^4)*x^5 + 2*(2*b^5*c^5 - 7*a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 +
 6*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4)*x^3 + 3*(2*a*b^4*c^5 - 8*a^2*b^3*c^4*d
+ 6*a^3*b^2*c^3*d^2 - a^4*b*c^2*d^3 + a^5*c*d^4)*x)*sqrt(b*x^2 + a))/(a^4*b
^4*c^7 - 4*a^5*b^3*c^6*d + 6*a^6*b^2*c^5*d^2 - 4*a^7*b*c^4*d^3 + a^8*c^3*d^
4 + (a^2*b^6*c^6*d - 4*a^3*b^5*c^5*d^2 + 6*a^4*b^4*c^4*d^3 - 4*a^5*b^3*c^3*
d^4 + a^6*b^2*c^2*d^5)*x^6 + (a^2*b^6*c^7 - 2*a^3*b^5*c^6*d - 2*a^4*b^4*c^5
*d^2 + 8*a^5*b^3*c^4*d^3 - 7*a^6*b^2*c^3*d^4 + 2*a^7*b*c^2*d^5)*x^4 + (2*a^
3*b^5*c^7 - 7*a^4*b^4*c^6*d + 8*a^5*b^3*c^5*d^2 - 2*a^6*b^2*c^4*d^3 - 2*a^7
*b*c^3*d^4 + a^8*c^2*d^5)*x^2), -1/12*(3*(6*a^4*b*c^2*d^2 - a^5*c*d^3 + (6*
a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (6*a^2*b^3*c^2*d^2 + 11*a^3*b^2*c*d^3 -
2*a^4*b*d^4)*x^4 + (12*a^3*b^2*c^2*d^2 + 4*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt
(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*
sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((4*
b^5*c^4*d - 20*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 + 3*a^3*b^2*c*d^4)*x^5 +
2*(2*b^5*c^5 - 7*a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 + 6*a^3*b^2*c^2*d^3 + 3*a^
4*b*c*d^4)*x^3 + 3*(2*a*b^4*c^5 - 8*a^2*b^3*c^4*d + 6*a^3*b^2*c^3*d^2 - a^4
*b*c^2*d^3 + a^5*c*d^4)*x)*sqrt(b*x^2 + a))/(a^4*b^4*c^7 - 4*a^5*b^3*c^6*d
+ 6*a^6*b^2*c^5*d^2 - 4*a^7*b*c^4*d^3 + a^8*c^3*d^4 + (a^2*b^6*c^6*d - 4*a^
3*b^5*c^5*d^2 + 6*a^4*b^4*c^4*d^3 - 4*a^5*b^3*c^3*d^4 + a^6*b^2*c^2*d^5)*x^
6 + (a^2*b^6*c^7 - 2*a^3*b^5*c^6*d - 2*a^4*b^4*c^5*d^2 + 8*a^5*b^3*c^4*d^3
- 7*a^6*b^2*c^3*d^4 + 2*a^7*b*c^2*d^5)*x^4 + (2*a^3*b^5*c^7 - 7*a^4*b^4*c^6
*d + 8*a^5*b^3*c^5*d^2 - 2*a^6*b^2*c^4*d^3 - 2*a^7*b*c^3*d^4 + a^8*c^2*d^5)
*x^2)]
```

giac [B] time = 2.07, size = 620, normalized size = 3.07

$$\frac{\left(\frac{2\left(b^{8}c^{4}-7\,ab^{7}c^{3}d+15\,a^{2}b^{6}c^{2}d^{2}-13\,a^{3}b^{5}cd^{3}+4\,a^{4}b^{4}d^{4}\right)x^{2}}{a^{2}b^{7}c^{6}-6\,a^{3}b^{6}c^{5}d+15\,a^{4}b^{5}c^{4}d^{2}-20\,a^{5}b^{4}c^{3}d^{3}+15\,a^{6}b^{3}c^{2}d^{4}-6\,a^{7}b^{2}cd^{5}+a^{8}bd^{6}}+\frac{3\left(ab^{7}c^{4}-6\,a^{2}b^{6}c^{3}d+12\,a^{3}b^{5}c^{2}d^{2}-10\,a^{4}b^{4}cd^{3}+3\,a^{5}b^{3}d^{4}\right)}{a^{2}b^{7}c^{6}-6\,a^{3}b^{6}c^{5}d+15\,a^{4}b^{5}c^{4}d^{2}-20\,a^{5}b^{4}c^{3}d^{3}+15\,a^{6}b^{3}c^{2}d^{4}-6\,a^{7}b^{2}cd^{5}+a^{8}bd^{6}}\right)x}{3\left(bx^{2}+a\right)^{\frac{3}{2}}}+\frac{\left(6\,b^{\frac{3}{2}}cd^{2}-a\sqrt{b}\,d^{3}\right)\arctan\left(-\frac{\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}d+2\,bc-ad}{2\,\sqrt{-b^{2}c^{2}+abcd}}\right)}{2\left(b^{3}c^{4}-3\,ab^{2}c^{3}d+3\,a^{2}bc^{2}d^{2}-a^{3}cd^{3}\right)\sqrt{-b^{2}c^{2}+abcd}}-\frac{2\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}b^{\frac{3}{2}}cd^{2}-\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}a\sqrt{b}\,d^{3}+a^{2}\sqrt{b}\,d^{3}}{\left(b^{3}c^{4}-3\,ab^{2}c^{3}d+3\,a^{2}bc^{2}d^{2}-a^{3}cd^{3}\right)\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{4}d+4\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}bc-2\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}ad+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^8*c^4 - 7*a*b^7*c^3*d + 15*a^2*b^6*c^2*d^2 - 13*a^3*b^5*c*d^3 + 4
*a^4*b^4*d^4)*x^2/(a^2*b^7*c^6 - 6*a^3*b^6*c^5*d + 15*a^4*b^5*c^4*d^2 - 20*
a^5*b^4*c^3*d^3 + 15*a^6*b^3*c^2*d^4 - 6*a^7*b^2*c*d^5 + a^8*b*d^6) + 3*(a*
b^7*c^4 - 6*a^2*b^6*c^3*d + 12*a^3*b^5*c^2*d^2 - 10*a^4*b^4*c*d^3 + 3*a^5*b
^3*d^4)/(a^2*b^7*c^6 - 6*a^3*b^6*c^5*d + 15*a^4*b^5*c^4*d^2 - 20*a^5*b^4*c^
3*d^3 + 15*a^6*b^3*c^2*d^4 - 6*a^7*b^2*c*d^5 + a^8*b*d^6))*x/(b*x^2 + a)^(3
/2) + 1/2*(6*b^(3/2)*c*d^2 - a*sqrt(b)*d^3)*arctan(-1/2*((sqrt(b)*x - sqrt(
b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^3*c^4 - 3*a*b^
2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(-b^2*c^2 + a*b*c*d)) - (2*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*
a*sqrt(b)*d^3 + a^2*sqrt(b)*d^3)/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^
2 - a^3*c*d^3)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d))
```

maple [B] time = 0.03, size = 2371, normalized size = 11.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/4/(-c*d)^(1/2)/c*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*
(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)
/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)
^(1/2)/d))+1/4/c*b/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-
c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*x+1/2/c*b/(a*d-b*c)/a^2/((x-(-c*d)^(1/
2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+5/4/c*
d*(-c*d)^(1/2)*b/(a*d-b*c)^3/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*
d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+5/4*d*b^2/(a*d-b*c)^3/a/((x-(-c*d)^(1/2)
/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+1/4/c/(a
*d-b*c)/(x+(-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)
^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)+1/4/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)/((x-(-c
*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-1
/4/(-c*d)^(1/2)/c*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x
-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d
)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(
1/2)/d))-5/4/c*d*(-c*d)^(1/2)*b/(a*d-b*c)^3/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d
)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+5/4*d*b^2/(a*d-b*c)^3/a/(
(x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(
1/2)*x+1/4/c*b/(a*d-b*c)/a/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)
^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*x-5/4/c*d*(-c*d)^(1/2)*b/(a*d-b*c)^3/((a*d
-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((
a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)
*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/4/c*d/(a*d-b*c)^2/a/((x-(-c*
d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x*
b-1/4/c*d/(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1
/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x*b+5/4/c*d*(-c*d)^(1/2)*b/(a*d-b*c)^3/((a*d-
b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((
a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)
*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-5/12/c*(-c*d)^(1/2)*b/(a*d-b*c
)^2/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)
/d)^(3/2)+5/12*b^2/(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+
(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*x+5/12/c*(-c*d)^(1/2)*b/(a*d-b*c)^2/
((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^
(3/2)+1/12/(-c*d)^(1/2)/c/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2
)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)+1/4/(-c*d)^(1/2)/c*d^2/(a*d-b*c
)^2/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)
/d)^(1/2)-1/12/(-c*d)^(1/2)/c/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^
(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-1/4/(-c*d)^(1/2)/c*d^2/(a*d
-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-
b*c)/d)^(1/2)+5/6*b^2/(a*d-b*c)^2/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2
)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+5/12*b^2/(a*d-b*c)^2/a/((x-(-
c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*
x+5/6*b^2/(a*d-b*c)^2/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^
(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+1/2/c*b/(a*d-b*c)/a^2/((x+(-c*d)^(1/2)/d)
^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{5/2}\,\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x)
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{5}{2}}\left(c+dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**2), x)
```

3.96 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=313

$$\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{12a^2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)^3}+\frac{d^2\left(3a^2d^2-16abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{9/2}}+\frac{dx\sqrt{a+bx^2}\left(9a^3d^3-\right.}{24a^2c^2\left(\right.}$$

Rubi [A] time = 0.40, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$\frac{dx\sqrt{a+bx^2}\left(-42a^2bcd^2+9a^3d^3-88ab^2c^2d+16b^3c^3\right)}{24a^2c^2\left(c+dx^2\right)(bc-ad)^4}+\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{12a^2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)^3}+\frac{d^2\left(3a^2d^2-16abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{9/2}}-\frac{dx}{4c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2(bc-ad)}+\frac{bx(3ad+4bc)}{12ac\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^2) + (b*(4*b*c + 3*a*
d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^2)) + (b*(8*b^2*c^2
- 40*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*Sqrt[a + b*x^2]*(c + d
*x^2)) + (d*(16*b^3*c^3 - 88*a*b^2*c^2*d - 42*a^2*b*c*d^2 + 9*a^3*d^3)*x*Sq
rt[a + b*x^2])/(24*a^2*c^2*(b*c - a*d)^4*(c + d*x^2)) + (d^2*(48*b^2*c^2 -
16*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2
])])/(8*c^(5/2)*(b*c - a*d)^(9/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3}\,dx = -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{\int \frac{4bc-3ad-6bdx^2}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} - \int$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{}{12}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{}{12}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{}{12}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{}{12}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{}{12}$$

Mathematica [A] time = 5.66, size = 221, normalized size = 0.71

$$\frac{1}{24}\left(x\sqrt{a+bx^2}\left(\frac{8b^3(2bc-11ad)}{a^2\left(a+bx^2\right)(bc-ad)^4} - \frac{8b^3}{a\left(a+bx^2\right)^2(ad-bc)^3} + \frac{3d^3(3ad-14bc)}{c^2\left(c+dx^2\right)(bc-ad)^4} - \frac{6d^3}{c\left(c+dx^2\right)^2(bc-ad)^3}\right) + \frac{3d^2\left(3a^2d^2-16abcd+48b^2c^2\right)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{5/2}(ad-bc)^{9/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((-8*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)^2) + (8*b^3*(2
*b*c - 11*a*d))/(a^2*(b*c - a*d)^4*(a + b*x^2)) - (6*d^3)/(c*(b*c - a*d)^3*
(c + d*x^2)^2) + (3*d^3*(-14*b*c + 3*a*d))/(c^2*(b*c - a*d)^4*(c + d*x^2)))
 + (3*d^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[-(b*c) + a*d]*
x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(c^(5/2)*(-(b*c) + a*d)^(9/2)))/24
```

IntegrateAlgebraic [A] time = 3.66, size = 516, normalized size = 1.65

$$-\frac{3\left(a^2d^4-8abcd^3+16b^2c^2d^2\right)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}c+\sqrt{b}dx^2}{\sqrt{c}\sqrt{ad-bc}}\right)}{8c^{5/2}(bc-ad)^4\sqrt{ad-bc}} + \frac{15a^5cd^4x+9a^5d^5x^3-48a^4bc^2d^3x-12a^4bcd^4x^3+18a^4bd^5x^5-96a^3b^2c^2d^3x^3-69a^3b^2cd^4x^5+9a^3b^2d^5x^7-96a^2b^3c^4dx-192a^2b^3c^3d^2x^3-144a^2b^3c^2d^3x^5-42a^2b^3cd^4x^7+24ab^4c^5x-40ab^4c^4dx^3-152ab^4c^3d^2x^5-88ab^4c^2d^3x^7+16b^5c^5x^3+32b^5c^4dx^5+16b^5c^3d^2x^7}{24a^2c^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2(ad-bc)^4} + \frac{abd^3\tanh^{-1}\left(\frac{\sqrt{b}dx^2}{\sqrt{c}\sqrt{bc-ad}} - \frac{dx\sqrt{a+bx^2}}{\sqrt{c}\sqrt{bc-ad}} + \frac{\sqrt{b}\sqrt{c}}{\sqrt{bc-ad}}\right)}{c^{3/2}(bc-ad)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] (24*a*b^4*c^5*x - 96*a^2*b^3*c^4*d*x - 48*a^4*b*c^2*d^3*x + 15*a^5*c*d^4*x
+ 16*b^5*c^5*x^3 - 40*a*b^4*c^4*d*x^3 - 192*a^2*b^3*c^3*d^2*x^3 - 96*a^3*b^
2*c^2*d^3*x^3 - 12*a^4*b*c*d^4*x^3 + 9*a^5*d^5*x^3 + 32*b^5*c^4*d*x^5 - 152
*a*b^4*c^3*d^2*x^5 - 144*a^2*b^3*c^2*d^3*x^5 - 69*a^3*b^2*c*d^4*x^5 + 18*a^
4*b*d^5*x^5 + 16*b^5*c^3*d^2*x^7 - 88*a*b^4*c^2*d^3*x^7 - 42*a^2*b^3*c*d^4*
x^7 + 9*a^3*b^2*d^5*x^7)/(24*a^2*c^2*(-(b*c) + a*d)^4*(a + b*x^2)^(3/2)*(c
+ d*x^2)^2) - (3*(16*b^2*c^2*d^2 - 8*a*b*c*d^3 + a^2*d^4)*ArcTan[(Sqrt[b]*c
 + Sqrt[b]*d*x^2 - d*x*Sqrt[a + b*x^2])/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(8*c
^(5/2)*(b*c - a*d)^4*Sqrt[-(b*c) + a*d]) + (a*b*d^3*ArcTanh[(Sqrt[b]*Sqrt[c
])/Sqrt[b*c - a*d] + (Sqrt[b]*d*x^2)/(Sqrt[c]*Sqrt[b*c - a*d]) - (d*x*Sqrt[
a + b*x^2])/(Sqrt[c]*Sqrt[b*c - a*d])])/(c^(3/2)*(b*c - a*d)^(9/2))
```

fricas [B] time = 12.49, size = 2250, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(48*a^4*b^2*c^4*d^2 - 16*a^5*b*c^3*d^3 + 3*a^6*c^2*d^4 + (48*a^2*b
^4*c^2*d^4 - 16*a^3*b^3*c*d^5 + 3*a^4*b^2*d^6)*x^8 + 2*(48*a^2*b^4*c^3*d^3
+ 32*a^3*b^3*c^2*d^4 - 13*a^4*b^2*c*d^5 + 3*a^5*b*d^6)*x^6 + (48*a^2*b^4*c^
4*d^2 + 176*a^3*b^3*c^3*d^3 - 13*a^4*b^2*c^2*d^4 - 4*a^5*b*c*d^5 + 3*a^6*d^
6)*x^4 + 2*(48*a^3*b^3*c^4*d^2 + 32*a^4*b^2*c^3*d^3 - 13*a^5*b*c^2*d^4 + 3*
a^6*c*d^5)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*
x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*
x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((
16*b^6*c^5*d^2 - 104*a*b^5*c^4*d^3 + 46*a^2*b^4*c^3*d^4 + 51*a^3*b^3*c^2*d^
5 - 9*a^4*b^2*c*d^6)*x^7 + (32*b^6*c^6*d - 184*a*b^5*c^5*d^2 + 8*a^2*b^4*c^
4*d^3 + 75*a^3*b^3*c^3*d^4 + 87*a^4*b^2*c^2*d^5 - 18*a^5*b*c*d^6)*x^5 + (16
*b^6*c^7 - 56*a*b^5*c^6*d - 152*a^2*b^4*c^5*d^2 + 96*a^3*b^3*c^4*d^3 + 84*a
^4*b^2*c^3*d^4 + 21*a^5*b*c^2*d^5 - 9*a^6*c*d^6)*x^3 + 3*(8*a*b^5*c^7 - 40*
a^2*b^4*c^6*d + 32*a^3*b^3*c^5*d^2 - 16*a^4*b^2*c^4*d^3 + 21*a^5*b*c^3*d^4
- 5*a^6*c^2*d^5)*x)*sqrt(b*x^2 + a))/(a^4*b^5*c^10 - 5*a^5*b^4*c^9*d + 10*a
^6*b^3*c^8*d^2 - 10*a^7*b^2*c^7*d^3 + 5*a^8*b*c^6*d^4 - a^9*c^5*d^5 + (a^2*
b^7*c^8*d^2 - 5*a^3*b^6*c^7*d^3 + 10*a^4*b^5*c^6*d^4 - 10*a^5*b^4*c^5*d^5 +
 5*a^6*b^3*c^4*d^6 - a^7*b^2*c^3*d^7)*x^8 + 2*(a^2*b^7*c^9*d - 4*a^3*b^6*c^
8*d^2 + 5*a^4*b^5*c^7*d^3 - 5*a^6*b^3*c^5*d^5 + 4*a^7*b^2*c^4*d^6 - a^8*b*c
^3*d^7)*x^6 + (a^2*b^7*c^10 - a^3*b^6*c^9*d - 9*a^4*b^5*c^8*d^2 + 25*a^5*b^
4*c^7*d^3 - 25*a^6*b^3*c^6*d^4 + 9*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6 - a^9*c^
3*d^7)*x^4 + 2*(a^3*b^6*c^10 - 4*a^4*b^5*c^9*d + 5*a^5*b^4*c^8*d^2 - 5*a^7*
b^2*c^6*d^4 + 4*a^8*b*c^5*d^5 - a^9*c^4*d^6)*x^2), -1/48*(3*(48*a^4*b^2*c^4
*d^2 - 16*a^5*b*c^3*d^3 + 3*a^6*c^2*d^4 + (48*a^2*b^4*c^2*d^4 - 16*a^3*b^3*
c*d^5 + 3*a^4*b^2*d^6)*x^8 + 2*(48*a^2*b^4*c^3*d^3 + 32*a^3*b^3*c^2*d^4 - 1
3*a^4*b^2*c*d^5 + 3*a^5*b*d^6)*x^6 + (48*a^2*b^4*c^4*d^2 + 176*a^3*b^3*c^3*
d^3 - 13*a^4*b^2*c^2*d^4 - 4*a^5*b*c*d^5 + 3*a^6*d^6)*x^4 + 2*(48*a^3*b^3*c
^4*d^2 + 32*a^4*b^2*c^3*d^3 - 13*a^5*b*c^2*d^4 + 3*a^6*c*d^5)*x^2)*sqrt(-b*
c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt
(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*b^6
*c^5*d^2 - 104*a*b^5*c^4*d^3 + 46*a^2*b^4*c^3*d^4 + 51*a^3*b^3*c^2*d^5 - 9*
a^4*b^2*c*d^6)*x^7 + (32*b^6*c^6*d - 184*a*b^5*c^5*d^2 + 8*a^2*b^4*c^4*d^3
+ 75*a^3*b^3*c^3*d^4 + 87*a^4*b^2*c^2*d^5 - 18*a^5*b*c*d^6)*x^5 + (16*b^6*c
^7 - 56*a*b^5*c^6*d - 152*a^2*b^4*c^5*d^2 + 96*a^3*b^3*c^4*d^3 + 84*a^4*b^2
*c^3*d^4 + 21*a^5*b*c^2*d^5 - 9*a^6*c*d^6)*x^3 + 3*(8*a*b^5*c^7 - 40*a^2*b^
4*c^6*d + 32*a^3*b^3*c^5*d^2 - 16*a^4*b^2*c^4*d^3 + 21*a^5*b*c^3*d^4 - 5*a^
6*c^2*d^5)*x)*sqrt(b*x^2 + a))/(a^4*b^5*c^10 - 5*a^5*b^4*c^9*d + 10*a^6*b^3
*c^8*d^2 - 10*a^7*b^2*c^7*d^3 + 5*a^8*b*c^6*d^4 - a^9*c^5*d^5 + (a^2*b^7*c^
8*d^2 - 5*a^3*b^6*c^7*d^3 + 10*a^4*b^5*c^6*d^4 - 10*a^5*b^4*c^5*d^5 + 5*a^6
*b^3*c^4*d^6 - a^7*b^2*c^3*d^7)*x^8 + 2*(a^2*b^7*c^9*d - 4*a^3*b^6*c^8*d^2
+ 5*a^4*b^5*c^7*d^3 - 5*a^6*b^3*c^5*d^5 + 4*a^7*b^2*c^4*d^6 - a^8*b*c^3*d^7
```

)*x^6 + (a^2*b^7*c^10 - a^3*b^6*c^9*d - 9*a^4*b^5*c^8*d^2 + 25*a^5*b^4*c^7*
d^3 - 25*a^6*b^3*c^6*d^4 + 9*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6 - a^9*c^3*d^7)
x^4 + 2(a^3*b^6*c^10 - 4*a^4*b^5*c^9*d + 5*a^5*b^4*c^8*d^2 - 5*a^7*b^2*c^
6*d^4 + 4*a^8*b*c^5*d^5 - a^9*c^4*d^6)*x^2)]

giac [B] time = 3.76, size = 1010, normalized size = 3.23

$$\frac{\left(\frac{\left(2 b^{10} c^{5}-19 a b^{9} c^{4} d+56 a^{2} b^{8} c^{3} d^{2}-74 a^{3} b^{7} c^{2} d^{3}+46 a^{4} b^{6} c d^{4}-11 a^{5} b^{5} d^{5}\right) x^{2}}{a^{2} b^{9} c^{8}-8 a^{3} b^{8} c^{7} d+28 a^{4} b^{7} c^{6} d^{2}-56 a^{5} b^{6} c^{5} d^{3}+70 a^{6} b^{5} c^{4} d^{4}-56 a^{7} b^{4} c^{3} d^{5}+28 a^{8} b^{3} c^{2} d^{6}-8 a^{9} b^{2} c d^{7}+a^{10} b d^{8}}+\frac{3\left(a b^{9} c^{5}-8 a^{2} b^{8} c^{4} d+22 a^{3} b^{7} c^{3} d^{2}-28 a^{4} b^{6} c^{2} d^{3}+17 a^{5} b^{5} c d^{4}-4 a^{6} b^{4} d^{5}\right)}{a^{2} b^{9} c^{8}-8 a^{3} b^{8} c^{7} d+28 a^{4} b^{7} c^{6} d^{2}-56 a^{5} b^{6} c^{5} d^{3}+70 a^{6} b^{5} c^{4} d^{4}-56 a^{7} b^{4} c^{3} d^{5}+28 a^{8} b^{3} c^{2} d^{6}-8 a^{9} b^{2} c d^{7}+a^{10} b d^{8}}\right) x}{3\left(b x^{2}+a\right)^{\frac{3}{2}}}-\frac{\left(48 b^{\frac{5}{2}} c^{2} d^{2}-16 a b^{\frac{3}{2}} c d^{3}+3 a^{2} \sqrt{b} d^{4}\right) \arctan\left(\frac{\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{2} d+2 b c-a d}{2 \sqrt{-b^{2} c^{2}+a b c d}}\right)}{8\left(b^{4} c^{6}-4 a b^{3} c^{5} d+6 a^{2} b^{2} c^{4} d^{2}-4 a^{3} b c^{3} d^{3}+a^{4} c^{2} d^{4}\right) \sqrt{-b^{2} c^{2}+a b c d}}-\frac{24\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{6} b^{\frac{5}{2}} c^{2} d^{3}-16\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{6} a b^{\frac{3}{2}} c d^{4}+3\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{6} a^{2} \sqrt{b} d^{5}+112\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{4} b^{\frac{7}{2}} c^{3} d^{2}-136\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{4} a b^{\frac{5}{2}} c^{2} d^{3}+66\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{4} a^{2} b^{\frac{3}{2}} c d^{4}-9\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{4} a^{3} \sqrt{b} d^{5}+88\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{2} a^{2} b^{\frac{5}{2}} c^{2} d^{3}-64\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{2} a^{3} b^{\frac{3}{2}} c d^{4}+9\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{2} a^{4} \sqrt{b} d^{5}+14 a^{4} b^{\frac{3}{2}} c d^{4}-3 a^{5} \sqrt{b} d^{5}}{4\left(b^{4} c^{6}-4 a b^{3} c^{5} d+6 a^{2} b^{2} c^{4} d^{2}-4 a^{3} b c^{3} d^{3}+a^{4} c^{2} d^{4}\right)\left(\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{4} d+4\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{2} b c-2\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{2} a d+a^{2} d\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/3*((2*b^10*c^5 - 19*a*b^9*c^4*d + 56*a^2*b^8*c^3*d^2 - 74*a^3*b^7*c^2*d^3
 + 46*a^4*b^6*c*d^4 - 11*a^5*b^5*d^5)*x^2/(a^2*b^9*c^8 - 8*a^3*b^8*c^7*d +
28*a^4*b^7*c^6*d^2 - 56*a^5*b^6*c^5*d^3 + 70*a^6*b^5*c^4*d^4 - 56*a^7*b^4*c
^3*d^5 + 28*a^8*b^3*c^2*d^6 - 8*a^9*b^2*c*d^7 + a^10*b*d^8) + 3*(a*b^9*c^5
- 8*a^2*b^8*c^4*d + 22*a^3*b^7*c^3*d^2 - 28*a^4*b^6*c^2*d^3 + 17*a^5*b^5*c*
d^4 - 4*a^6*b^4*d^5)/(a^2*b^9*c^8 - 8*a^3*b^8*c^7*d + 28*a^4*b^7*c^6*d^2 -
56*a^5*b^6*c^5*d^3 + 70*a^6*b^5*c^4*d^4 - 56*a^7*b^4*c^3*d^5 + 28*a^8*b^3*c
^2*d^6 - 8*a^9*b^2*c*d^7 + a^10*b*d^8))*x/(b*x^2 + a)^(3/2) - 1/8*(48*b^(5/
2)*c^2*d^2 - 16*a*b^(3/2)*c*d^3 + 3*a^2*sqrt(b)*d^4)*arctan(1/2*((sqrt(b)*x
 - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^4*c^6
- 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-
b^2*c^2 + a*b*c*d)) - 1/4*(24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d
^3 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^4 + 3*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*a^2*sqrt(b)*d^5 + 112*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7
/2)*c^3*d^2 - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^3 + 66*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^4 - 9*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^3*sqrt(b)*d^5 + 88*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c
^2*d^3 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^4 + 9*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d^5 + 14*a^4*b^(3/2)*c*d^4 - 3*a^5*sqrt(
b)*d^5)/((b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a
^4*c^2*d^4)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2)
```

maple [B] time = 0.03, size = 4495, normalized size = 14.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x)
```

```
[Out] 3/16/c^2/(a*d-b*c)/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2
)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)+3/16/c^2/(a*d-b*c)/(x+(-c*d)^(1
/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b
*c)/d)^(3/2)+1/16/(-c*d)^(1/2)/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)^2/((x-(-c*d)^
(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-7/16/
c*b/(a*d-b*c)^2/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(
x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-15/16/c^2*d*(-c*d)^(1/2)*b/(a*d-b*
c)^3/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b
*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d
)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+5/48/(-c*d)^(1/2)/c*
d*b/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b
/d+(a*d-b*c)/d)^(3/2)+5/16/(-c*d)^(1/2)/c*d^2*b/(a*d-b*c)^3/((x-(-c*d)^(1/2
)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+3/16/c^2*
b/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d
+(a*d-b*c)/d)^(3/2)*x+3/8/c^2*b/(a*d-b*c)/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*(-c
*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-3/16/(-c*d)^(1/2)/c^2
*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*
```

```
b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1
/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-7/16/c*b
/(a*d-b*c)^2/(x+(-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(
-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-35/48/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/
((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^
(3/2)-35/48*b^3/(a*d-b*c)^3/a/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c
*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*x-35/24*b^3/(a*d-b*c)^3/a^2/((x+(-c*d)^
(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-35/
16/(-c*d)^(1/2)*d^2*b^2/(a*d-b*c)^4/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*
(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+5/16/c^2*(-c*d)^(1/2)*b/(a*d-b*c)
^2/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/
d)^(3/2)+3/16/(-c*d)^(1/2)/c^2*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((-2*(
-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+
(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2
))/(x+(-c*d)^(1/2)/d))+15/16/c^2*d*(-c*d)^(1/2)*b/(a*d-b*c)^3/((x-(-c*d)^(1
/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)-15/16/c
^2*d*(-c*d)^(1/2)*b/(a*d-b*c)^3/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(
-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+3/16/c^2*b/(a*d-b*c)/a/((x+(-c*d)^(1/
2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*x+3/8/c^
2*b/(a*d-b*c)/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)
*b/d+(a*d-b*c)/d)^(1/2)*x-35/16*d*b^3/(a*d-b*c)^4/a/((x+(-c*d)^(1/2)/d)^2*b
-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+35/16/(-c*d)^(1
/2)*d^2*b^2/(a*d-b*c)^4/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(
1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(
-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-
3/8/c*b^2/(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1
/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*x-3/4/c*b^2/(a*d-b*c)^2/a^2/((x+(-c*d)^(1/2)/
d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-5/48/(-c*
d)^(1/2)/c*d*b/(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)
^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-5/16/(-c*d)^(1/2)/c*d^2*b/(a*d-b*c)^3/((x+
(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2
)-35/16*d*b^3/(a*d-b*c)^4/a/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d
)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-35/16/(-c*d)^(1/2)*d^2*b^2/(a*d-b*c)^4/
((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d
+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/
2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-3/8/c*b^2/(a*d-b*c)^2/a/(
(x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(
3/2)*x-3/4/c*b^2/(a*d-b*c)^2/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-
(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+15/16/c^2*d*(-c*d)^(1/2)*b/(a*d-b*
c)^3/((a*d-b*c)/d)^(1/2)*ln((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-
b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*
d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+5/8/c*d*b^2/(a*d-b*
c)^3/a/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b
*c)/d)^(1/2)*x+5/16/(-c*d)^(1/2)/c*d^2*b/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln
((-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2
)*((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d
)^(1/2))/(x+(-c*d)^(1/2)/d))+5/8/c*d*b^2/(a*d-b*c)^3/a/((x-(-c*d)^(1/2)/d)^
2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x-5/16/(-c*d)^
(1/2)/c*d^2*b/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln((2*(-c*d)^(1/2)*(x-(-c*d)^
(1/2)/d)*b/d+2*(a*d-b*c)/d+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*
(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))
-3/16/c^2*d/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^
(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x*b-3/16/c^2*d/(a*d-b*c)^2/a/((x+(-c*d)^(1/
2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)*x*b+35/4
8/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-
(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-35/48*b^3/(a*d-b*c)^3/a/((x-(-c*d)^(
1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)*x-35/2
4*b^3/(a*d-b*c)^3/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2
)/d)*b/d+(a*d-b*c)/d)^(1/2)*x+35/16/(-c*d)^(1/2)*d^2*b^2/(a*d-b*c)^4/((x-(-
```

```
c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(1/2)+
1/16/(-c*d)^(1/2)/c^2/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x
-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)+3/16/(-c*d)^(1/2)/c^2*d^2/(a*d-b*c)
^2/((x-(-c*d)^(1/2)/d)^2*b+2*(-c*d)^(1/2)*(x-(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/
d)^(1/2)-1/16/(-c*d)^(1/2)/c^2/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)
^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-3/16/(-c*d)^(1/2)/c^2*d^2/
(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(
a*d-b*c)/d)^(1/2)-5/16/c^2*(-c*d)^(1/2)*b/(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2
*b-2*(-c*d)^(1/2)*(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)-1/16/(-c*d)^(1/
2)/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)^2/((x+(-c*d)^(1/2)/d)^2*b-2*(-c*d)^(1/2)*
(x+(-c*d)^(1/2)/d)*b/d+(a*d-b*c)/d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{5/2}\,\left(d\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x)
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
[Out] Timed out
```

3.97 $\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^{11/2}} \, dx$

Optimal. Leaf size=224

$$\frac{16a^3x(9bc-8ad)}{315c^5\sqrt{c+dx^2}\,(bc-ad)}+\frac{8a^2x\left(a+bx^2\right)(9bc-8ad)}{315c^4\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{2ax\left(a+bx^2\right)^2(9bc-8ad)}{105c^3\left(c+dx^2\right)^{5/2}(bc-ad)}+\frac{x\left(a+bx^2\right)^3(9bc-8ad)}{63c^2\left(c+dx^2\right)^{7/2}(bc-a}$$

Rubi [A] time = 0.10, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 378, 191}

$$\frac{8a^2x\left(a+bx^2\right)(9bc-8ad)}{315c^4\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{16a^3x(9bc-8ad)}{315c^5\sqrt{c+dx^2}\,(bc-ad)}+\frac{x\left(a+bx^2\right)^3(9bc-8ad)}{63c^2\left(c+dx^2\right)^{7/2}(bc-ad)}+\frac{2ax\left(a+bx^2\right)^2(9bc-8ad)}{105c^3\left(c+dx^2\right)^{5/2}(bc-ad)}-\frac{dx\left(a+bx^2\right)^4}{9c\left(c+dx^2\right)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

```
[Out] -(d*x*(a + b*x^2)^4)/(9*c*(b*c - a*d)*(c + d*x^2)^(9/2)) + ((9*b*c - 8*a*d)
*x*(a + b*x^2)^3)/(63*c^2*(b*c - a*d)*(c + d*x^2)^(7/2)) + (2*a*(9*b*c - 8*
a*d)*x*(a + b*x^2)^2)/(105*c^3*(b*c - a*d)*(c + d*x^2)^(5/2)) + (8*a^2*(9*b
*c - 8*a*d)*x*(a + b*x^2))/(315*c^4*(b*c - a*d)*(c + d*x^2)^(3/2)) + (16*a^
3*(9*b*c - 8*a*d)*x)/(315*c^5*(b*c - a*d)*Sqrt[c + d*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^{11/2}}\,dx = -\frac{dx\left(a+bx^2\right)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}} + \frac{(9bc-8ad)\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^{9/2}}\,dx}{9c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}} + \frac{(9bc-8ad)x\left(a+bx^2\right)^3}{63c^2(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{(2a(9bc-8ad))\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{7/2}}\,dx}{21c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}} + \frac{(9bc-8ad)x\left(a+bx^2\right)^3}{63c^2(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{2a(9bc-8ad)x\left(a+bx^2\right)^2}{105c^3(bc-ad)\left(c+dx^2\right)^{5/2}} + \frac{(8a}{}$$

$$= -\frac{dx\left(a+bx^2\right)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}} + \frac{(9bc-8ad)x\left(a+bx^2\right)^3}{63c^2(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{2a(9bc-8ad)x\left(a+bx^2\right)^2}{105c^3(bc-ad)\left(c+dx^2\right)^{5/2}} + \frac{8a^2}{315}$$

$$= -\frac{dx\left(a+bx^2\right)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}} + \frac{(9bc-8ad)x\left(a+bx^2\right)^3}{63c^2(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{2a(9bc-8ad)x\left(a+bx^2\right)^2}{105c^3(bc-ad)\left(c+dx^2\right)^{5/2}} + \frac{8a^2}{315}$$

Mathematica [A]   time = 0.10, size = 163, normalized size = 0.73

$$\frac{a^3\left(315c^4x+840c^3dx^3+1008c^2d^2x^5+576cd^3x^7+128d^4x^9\right)+3a^2bcx^3\left(105c^3+126c^2dx^2+72cd^2x^4+16d^3x^6\right)+3ab^2c^2x^5\left(63c^2+36cdx^2+8d^2x^4\right)+5b^3c^3x^7\left(9c+2dx^2\right)}{315c^5\left(c+dx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

```
[Out] (5*b^3*c^3*x^7*(9*c + 2*d*x^2) + 3*a*b^2*c^2*x^5*(63*c^2 + 36*c*d*x^2 + 8*d
^2*x^4) + 3*a^2*b*c*x^3*(105*c^3 + 126*c^2*d*x^2 + 72*c*d^2*x^4 + 16*d^3*x^
6) + a^3*(315*c^4*x + 840*c^3*d*x^3 + 1008*c^2*d^2*x^5 + 576*c*d^3*x^7 + 12
8*d^4*x^9))/(315*c^5*(c + d*x^2)^(9/2))
```

IntegrateAlgebraic [A]   time = 0.42, size = 193, normalized size = 0.86

$$\frac{315a^3c^4x+840a^3c^3dx^3+1008a^3c^2d^2x^5+576a^3cd^3x^7+128a^3d^4x^9+315a^2bc^4x^3+378a^2bc^3dx^5+216a^2bc^2d^2x^7+48a^2bcd^3x^9+189ab^2c^4x^5+108ab^2c^3dx^7+24ab^2c^2d^2x^9+45b^3c^4x^7+10b^3c^3dx^9}{315c^5\left(c+dx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

```
[Out] (315*a^3*c^4*x + 315*a^2*b*c^4*x^3 + 840*a^3*c^3*d*x^3 + 189*a*b^2*c^4*x^5
+ 378*a^2*b*c^3*d*x^5 + 1008*a^3*c^2*d^2*x^5 + 45*b^3*c^4*x^7 + 108*a*b^2*c
^3*d*x^7 + 216*a^2*b*c^2*d^2*x^7 + 576*a^3*c*d^3*x^7 + 10*b^3*c^3*d*x^9 + 2
4*a*b^2*c^2*d^2*x^9 + 48*a^2*b*c*d^3*x^9 + 128*a^3*d^4*x^9)/(315*c^5*(c + d
*x^2)^(9/2))
```

fricas [A]   time = 1.62, size = 229, normalized size = 1.02

$$\frac{\left(2\left(5b^3c^3d+12ab^2c^2d^2+24a^2bcd^3+64a^3d^4\right)x^9+315a^3c^4x+9\left(5b^3c^4+12ab^2c^3d+24a^2bc^2d^2+64a^3cd^3\right)x^7+63\left(3ab^2c^4+6a^2bc^3d+16a^3c^2d^2\right)x^5+105\left(3a^2bc^4+8a^3c^3d\right)x^3\right)\sqrt{dx^2+c}}{315\left(c^5d^5x^{10}+5c^6d^4x^8+10c^7d^3x^6+10c^8d^2x^4+5c^9dx^2+c^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(2*(5*b^3*c^3*d + 12*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 64*a^3*d^4)*x^9
 + 315*a^3*c^4*x + 9*(5*b^3*c^4 + 12*a*b^2*c^3*d + 24*a^2*b*c^2*d^2 + 64*a^
```

```
3*c*d^3)*x^7 + 63*(3*a*b^2*c^4 + 6*a^2*b*c^3*d + 16*a^3*c^2*d^2)*x^5 + 105*
(3*a^2*b*c^4 + 8*a^3*c^3*d)*x^3)*sqrt(d*x^2 + c)/(c^5*d^5*x^10 + 5*c^6*d^4*
x^8 + 10*c^7*d^3*x^6 + 10*c^8*d^2*x^4 + 5*c^9*d*x^2 + c^10)
```

giac [A]   time = 0.68, size = 218, normalized size = 0.97

$$\frac{\left(\left(\left(x^2\left(\frac{2\left(5b^3c^3d^5+12ab^2c^2d^6+24a^2bcd^7+64a^3d^8\right)x^2}{c^5d^4}+\frac{9\left(5b^3c^4d^4+12ab^2c^3d^5+24a^2bc^2d^6+64a^3cd^7\right)}{c^5d^4}\right)+\frac{63\left(3ab^2c^4d^4+6a^2bc^3d^5+16a^3c^2d^6\right)}{c^5d^4}\right)x^2+\frac{105\left(3a^2bc^4d^4+8a^3c^3d^5\right)}{c^5d^4}\right)x^2+\frac{315a^3}{c}\right)x}{315\left(dx^2+c\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((x^2*(2*(5*b^3*c^3*d^5 + 12*a*b^2*c^2*d^6 + 24*a^2*b*c*d^7 + 64*a^3
*d^8)*x^2/(c^5*d^4) + 9*(5*b^3*c^4*d^4 + 12*a*b^2*c^3*d^5 + 24*a^2*b*c^2*d^
6 + 64*a^3*c*d^7)/(c^5*d^4)) + 63*(3*a*b^2*c^4*d^4 + 6*a^2*b*c^3*d^5 + 16*a
^3*c^2*d^6)/(c^5*d^4))*x^2 + 105*(3*a^2*b*c^4*d^4 + 8*a^3*c^3*d^5)/(c^5*d^4
))*x^2 + 315*a^3/c)*x/(d*x^2 + c)^(9/2)
```

maple [A]   time = 0.01, size = 190, normalized size = 0.85

$$\frac{\left(128a^3d^4x^8+48a^2bc\,d^3x^8+24a\,b^2c^2d^2x^8+10b^3c^3d\,x^8+576a^3c\,d^3x^6+216a^2b\,c^2d^2x^6+108a\,b^2c^3d\,x^6+45b^3c^4x^6+1008a^3c^2d^2x^4+378a^2b\,c^3d\,x^4+189a\,b^2c^4x^4+840a^3c^3d\,x^2+315a^2b\,c^4x^2+315a^3c^4\right)x}{315\left(d\,x^2+c\right)^{\frac{9}{2}}c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c)^(11/2),x)
```

```
[Out] 1/315*x*(128*a^3*d^4*x^8+48*a^2*b*c*d^3*x^8+24*a*b^2*c^2*d^2*x^8+10*b^3*c^3
*d*x^8+576*a^3*c*d^3*x^6+216*a^2*b*c^2*d^2*x^6+108*a*b^2*c^3*d*x^6+45*b^3*c
^4*x^6+1008*a^3*c^2*d^2*x^4+378*a^2*b*c^3*d*x^4+189*a*b^2*c^4*x^4+840*a^3*c
^3*d*x^2+315*a^2*b*c^4*x^2+315*a^3*c^4)/(d*x^2+c)^(9/2)/c^5
```

maxima [B]   time = 1.51, size = 465, normalized size = 2.08

$$-\frac{b^3x^5}{4\left(dx^2+c\right)^{\frac{9}{2}}d}-\frac{5b^3cx^3}{24\left(dx^2+c\right)^{\frac{9}{2}}d^2}-\frac{ab^2x^3}{2\left(dx^2+c\right)^{\frac{9}{2}}d}+\frac{128a^3x}{315\sqrt{dx^2+c}c^5}+\frac{64a^3x}{315\left(dx^2+c\right)^{\frac{3}{2}}c^4}+\frac{16a^3x}{105\left(dx^2+c\right)^{\frac{5}{2}}c^3}+\frac{8a^3x}{63\left(dx^2+c\right)^{\frac{7}{2}}c^2}+\frac{a^3x}{9\left(dx^2+c\right)^{\frac{9}{2}}c}+\frac{b^3x}{84\left(dx^2+c\right)^{\frac{5}{2}}d^3}+\frac{2b^3x}{63\sqrt{dx^2+c}c^2d^3}+\frac{b^3x}{63\left(dx^2+c\right)^{\frac{3}{2}}cd^3}+\frac{5b^3cx}{504\left(dx^2+c\right)^{\frac{7}{2}}d^3}-\frac{5b^3c^2x}{72\left(dx^2+c\right)^{\frac{9}{2}}d^3}+\frac{ab^2x}{42\left(dx^2+c\right)^{\frac{7}{2}}d^2}+\frac{8ab^2x}{105\sqrt{dx^2+c}c^3d^2}+\frac{4ab^2x}{105\left(dx^2+c\right)^{\frac{3}{2}}c^2d^2}+\frac{ab^2x}{35\left(dx^2+c\right)^{\frac{5}{2}}cd^2}-\frac{ab^2cx}{6\left(dx^2+c\right)^{\frac{9}{2}}d^2}-\frac{a^2bx}{3\left(dx^2+c\right)^{\frac{9}{2}}d}+\frac{16a^2bx}{105\sqrt{dx^2+c}c^4d}+\frac{8a^2bx}{105\left(dx^2+c\right)^{\frac{3}{2}}c^3d}+\frac{2a^2bx}{35\left(dx^2+c\right)^{\frac{5}{2}}c^2d}+\frac{a^2bx}{21\left(dx^2+c\right)^{\frac{7}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^3*x^5/((d*x^2 + c)^(9/2)*d) - 5/24*b^3*c*x^3/((d*x^2 + c)^(9/2)*d^2)
 - 1/2*a*b^2*x^3/((d*x^2 + c)^(9/2)*d) + 128/315*a^3*x/(sqrt(d*x^2 + c)*c^5
) + 64/315*a^3*x/((d*x^2 + c)^(3/2)*c^4) + 16/105*a^3*x/((d*x^2 + c)^(5/2)*
c^3) + 8/63*a^3*x/((d*x^2 + c)^(7/2)*c^2) + 1/9*a^3*x/((d*x^2 + c)^(9/2)*c)
 + 1/84*b^3*x/((d*x^2 + c)^(5/2)*d^3) + 2/63*b^3*x/(sqrt(d*x^2 + c)*c^2*d^3
) + 1/63*b^3*x/((d*x^2 + c)^(3/2)*c*d^3) + 5/504*b^3*c*x/((d*x^2 + c)^(7/2)
*d^3) - 5/72*b^3*c^2*x/((d*x^2 + c)^(9/2)*d^3) + 1/42*a*b^2*x/((d*x^2 + c)^
(7/2)*d^2) + 8/105*a*b^2*x/(sqrt(d*x^2 + c)*c^3*d^2) + 4/105*a*b^2*x/((d*x^
2 + c)^(3/2)*c^2*d^2) + 1/35*a*b^2*x/((d*x^2 + c)^(5/2)*c*d^2) - 1/6*a*b^2*
c*x/((d*x^2 + c)^(9/2)*d^2) - 1/3*a^2*b*x/((d*x^2 + c)^(9/2)*d) + 16/105*a^
2*b*x/(sqrt(d*x^2 + c)*c^4*d) + 8/105*a^2*b*x/((d*x^2 + c)^(3/2)*c^3*d) + 2
/35*a^2*b*x/((d*x^2 + c)^(5/2)*c^2*d) + 1/21*a^2*b*x/((d*x^2 + c)^(7/2)*c*d
)
```

mupad [B]   time = 5.15, size = 326, normalized size = 1.46

$$\frac{x\left(\frac{a^3}{9c}-\frac{c\left(\frac{c\left(\frac{b^3}{9d}-\frac{ab^2}{3c}\right)}{d}+\frac{a^2b}{3c}\right)}{d}\right)}{\left(d\,x^2+c\right)^{9/2}}-\frac{x\left(\frac{b^3}{5\,d^3}-\frac{16\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-4\,b^3\,c^3}{105\,c^3\,d^3}\right)}{\left(d\,x^2+c\right)^{5/2}}+\frac{x\left(\frac{c\left(\frac{b^3}{7\,d^2}-\frac{b^2\,(3\,a\,d-b\,c)}{7\,c\,d^2}\right)}{d}+\frac{8\,a^3\,d^3+3\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+b^3\,c^3}{63\,c^2\,d^3}\right)}{\left(d\,x^2+c\right)^{7/2}}+\frac{x\left(64\,a^3\,d^3+24\,a^2\,b\,c\,d^2+12\,a\,b^2\,c^2\,d+5\,b^3\,c^3\right)}{315\,c^4\,d^3\,\left(d\,x^2+c\right)^{3/2}}+\frac{x\left(128\,a^3\,d^3+48\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+10\,b^3\,c^3\right)}{315\,c^5\,d^3\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/(c + d*x^2)^(11/2),x)
```

```
[Out] (x*(a^3/(9*c) - (c*((c*(b^3/(9*d) - (a*b^2)/(3*c)))/d + (a^2*b)/(3*c)))/d))
/(c + d*x^2)^(9/2) - (x*(b^3/(5*d^3) - (16*a^3*d^3 - 4*b^3*c^3 + 3*a*b^2*c^
2*d + 6*a^2*b*c*d^2)/(105*c^3*d^3)))/(c + d*x^2)^(5/2) + (x*((c*(b^3/(7*d^2
) - (b^2*(3*a*d - b*c))/(7*c*d^2)))/d + (8*a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2)/(63*c^2*d^3)))/(c + d*x^2)^(7/2) + (x*(64*a^3*d^3 + 5*b^
3*c^3 + 12*a*b^2*c^2*d + 24*a^2*b*c*d^2))/(315*c^4*d^3*(c + d*x^2)^(3/2)) +
 (x*(128*a^3*d^3 + 10*b^3*c^3 + 24*a*b^2*c^2*d + 48*a^2*b*c*d^2))/(315*c^5*
d^3*(c + d*x^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**(11/2),x)
```

```
[Out] Timed out
```

3.98 $\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=174

$$\frac{8a^2x(7bc-6ad)}{105c^4\sqrt{c+dx^2}\,(bc-ad)}+\frac{4ax\left(a+bx^2\right)(7bc-6ad)}{105c^3\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{x\left(a+bx^2\right)^2(7bc-6ad)}{35c^2\left(c+dx^2\right)^{5/2}(bc-ad)}-\frac{dx\left(a+bx^2\right)^3}{7c\left(c+dx^2\right)^{7/2}(bc-ad)}$$

Rubi [A] time = 0.07, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 378, 191}

$$\frac{8a^2x(7bc-6ad)}{105c^4\sqrt{c+dx^2}\,(bc-ad)}+\frac{x\left(a+bx^2\right)^2(7bc-6ad)}{35c^2\left(c+dx^2\right)^{5/2}(bc-ad)}+\frac{4ax\left(a+bx^2\right)(7bc-6ad)}{105c^3\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{dx\left(a+bx^2\right)^3}{7c\left(c+dx^2\right)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```

```
[Out] -(d*x*(a + b*x^2)^3)/(7*c*(b*c - a*d)*(c + d*x^2)^(7/2)) + ((7*b*c - 6*a*d)
*x*(a + b*x^2)^2)/(35*c^2*(b*c - a*d)*(c + d*x^2)^(5/2)) + (4*a*(7*b*c - 6*
a*d)*x*(a + b*x^2))/(105*c^3*(b*c - a*d)*(c + d*x^2)^(3/2)) + (8*a^2*(7*b*c
 - 6*a*d)*x)/(105*c^4*(b*c - a*d)*Sqrt[c + d*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = -\frac{dx(a+bx^2)^3}{7c(bc-ad)(c+dx^2)^{7/2}} + \frac{(7bc-6ad)\int \frac{(a+bx^2)^2}{(c+dx^2)^{7/2}}\,dx}{7c(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^3}{7c(bc-ad)(c+dx^2)^{7/2}} + \frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)(c+dx^2)^{5/2}} + \frac{(4a(7bc-6ad))\int \frac{a+bx^2}{(c+dx^2)^{5/2}}\,dx}{35c^2(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^3}{7c(bc-ad)(c+dx^2)^{7/2}} + \frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)(c+dx^2)^{5/2}} + \frac{4a(7bc-6ad)x(a+bx^2)}{105c^3(bc-ad)(c+dx^2)^{3/2}} + \frac{(8a^2}{}$$

$$= -\frac{dx(a+bx^2)^3}{7c(bc-ad)(c+dx^2)^{7/2}} + \frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)(c+dx^2)^{5/2}} + \frac{4a(7bc-6ad)x(a+bx^2)}{105c^3(bc-ad)(c+dx^2)^{3/2}} + \frac{8}{105c}$$

Mathematica [A] time = 0.07, size = 107, normalized size = 0.61

$$\frac{3a^2(35c^3x+70c^2dx^3+56cd^2x^5+16d^3x^7)+2abcx^3(35c^2+28cdx^2+8d^2x^4)+3b^2c^2x^5(7c+2dx^2)}{105c^4(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```

```
[Out] (3*b^2*c^2*x^5*(7*c + 2*d*x^2) + 2*a*b*c*x^3*(35*c^2 + 28*c*d*x^2 + 8*d^2*x
^4) + 3*a^2*(35*c^3*x + 70*c^2*d*x^3 + 56*c*d^2*x^5 + 16*d^3*x^7))/(105*c^4
*(c + d*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.23, size = 118, normalized size = 0.68

$$\frac{105a^2c^3x+210a^2c^2dx^3+168a^2cd^2x^5+48a^2d^3x^7+70abc^3x^3+56abc^2dx^5+16abcd^2x^7+21b^2c^3x^5+6b^2c^2dx^7}{105c^4(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```

```
[Out] (105*a^2*c^3*x + 70*a*b*c^3*x^3 + 210*a^2*c^2*d*x^3 + 21*b^2*c^3*x^5 + 56*a
*b*c^2*d*x^5 + 168*a^2*c*d^2*x^5 + 6*b^2*c^2*d*x^7 + 16*a*b*c*d^2*x^7 + 48*
a^2*d^3*x^7)/(105*c^4*(c + d*x^2)^(7/2))
```

fricas [A] time = 0.97, size = 151, normalized size = 0.87

$$\frac{\left(2\left(3b^2c^2d+8abcd^2+24a^2d^3\right)x^7+105a^2c^3x+7\left(3b^2c^3+8abc^2d+24a^2cd^2\right)x^5+70\left(abc^3+3a^2c^2d\right)x^3\right)\sqrt{dx^2+c}}{105\left(c^4d^4x^8+4c^5d^3x^6+6c^6d^2x^4+4c^7dx^2+c^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*b^2*c^2*d + 8*a*b*c*d^2 + 24*a^2*d^3)*x^7 + 105*a^2*c^3*x + 7*(
3*b^2*c^3 + 8*a*b*c^2*d + 24*a^2*c*d^2)*x^5 + 70*(a*b*c^3 + 3*a^2*c^2*d)*x^
3)*sqrt(d*x^2 + c)/(c^4*d^4*x^8 + 4*c^5*d^3*x^6 + 6*c^6*d^2*x^4 + 4*c^7*d*x
^2 + c^8)
```

giac [A] time = 0.64, size = 138, normalized size = 0.79

$$\frac{\left(\left(x^{2}\left(\frac{2\left(3b^{2}c^{2}d^{4}+8abcd^{5}+24a^{2}d^{6}\right)x^{2}}{c^{4}d^{3}}+\frac{7\left(3b^{2}c^{3}d^{3}+8abc^{2}d^{4}+24a^{2}cd^{5}\right)}{c^{4}d^{3}}\right)+\frac{70\left(abc^{3}d^{3}+3a^{2}c^{2}d^{4}\right)}{c^{4}d^{3}}\right)x^{2}+\frac{105a^{2}}{c}\right)x}{105\left(dx^{2}+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*(2*(3*b^2*c^2*d^4 + 8*a*b*c*d^5 + 24*a^2*d^6)*x^2/(c^4*d^3) + 7
*(3*b^2*c^3*d^3 + 8*a*b*c^2*d^4 + 24*a^2*c*d^5)/(c^4*d^3)) + 70*(a*b*c^3*d^
3 + 3*a^2*c^2*d^4)/(c^4*d^3))*x^2 + 105*a^2/c)*x/(d*x^2 + c)^(7/2)
```

maple [A] time = 0.01, size = 115, normalized size = 0.66

$$\frac{\left(48a^{2}d^{3}x^{6}+16abc\,d^{2}x^{6}+6b^{2}c^{2}d\,x^{6}+168a^{2}c\,d^{2}x^{4}+56ab\,c^{2}d\,x^{4}+21b^{2}c^{3}x^{4}+210a^{2}c^{2}d\,x^{2}+70ab\,c^{3}x^{2}+105a^{2}c^{3}\right)x}{105\left(d\,x^{2}+c\right)^{\frac{7}{2}}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(9/2),x)
```

```
[Out] 1/105*x*(48*a^2*d^3*x^6+16*a*b*c*d^2*x^6+6*b^2*c^2*d*x^6+168*a^2*c*d^2*x^4+
56*a*b*c^2*d*x^4+21*b^2*c^3*x^4+210*a^2*c^2*d*x^2+70*a*b*c^3*x^2+105*a^2*c^
3)/(d*x^2+c)^(7/2)/c^4
```

maxima [A] time = 1.50, size = 249, normalized size = 1.43

$$-\frac{b^{2}x^{3}}{4\left(dx^{2}+c\right)^{\frac{7}{2}}d}+\frac{16a^{2}x}{35\sqrt{dx^{2}+c}\,c^{4}}+\frac{8a^{2}x}{35\left(dx^{2}+c\right)^{\frac{3}{2}}c^{3}}+\frac{6a^{2}x}{35\left(dx^{2}+c\right)^{\frac{5}{2}}c^{2}}+\frac{a^{2}x}{7\left(dx^{2}+c\right)^{\frac{7}{2}}c}+\frac{3b^{2}x}{140\left(dx^{2}+c\right)^{\frac{5}{2}}d^{2}}+\frac{2b^{2}x}{35\sqrt{dx^{2}+c}\,c^{2}d^{2}}+\frac{b^{2}x}{35\left(dx^{2}+c\right)^{\frac{3}{2}}cd^{2}}-\frac{3b^{2}cx}{28\left(dx^{2}+c\right)^{\frac{7}{2}}d^{2}}-\frac{2abx}{7\left(dx^{2}+c\right)^{\frac{7}{2}}d}+\frac{16abx}{105\sqrt{dx^{2}+c}\,c^{3}d}+\frac{8abx}{105\left(dx^{2}+c\right)^{\frac{3}{2}}c^{2}d}+\frac{2abx}{35\left(dx^{2}+c\right)^{\frac{5}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*x^3/((d*x^2 + c)^(7/2)*d) + 16/35*a^2*x/(sqrt(d*x^2 + c)*c^4) + 8/
35*a^2*x/((d*x^2 + c)^(3/2)*c^3) + 6/35*a^2*x/((d*x^2 + c)^(5/2)*c^2) + 1/7
*a^2*x/((d*x^2 + c)^(7/2)*c) + 3/140*b^2*x/((d*x^2 + c)^(5/2)*d^2) + 2/35*b
^2*x/(sqrt(d*x^2 + c)*c^2*d^2) + 1/35*b^2*x/((d*x^2 + c)^(3/2)*c*d^2) - 3/2
8*b^2*c*x/((d*x^2 + c)^(7/2)*d^2) - 2/7*a*b*x/((d*x^2 + c)^(7/2)*d) + 16/10
5*a*b*x/(sqrt(d*x^2 + c)*c^3*d) + 8/105*a*b*x/((d*x^2 + c)^(3/2)*c^2*d) + 2
/35*a*b*x/((d*x^2 + c)^(5/2)*c*d)
```

mupad [B] time = 4.99, size = 176, normalized size = 1.01

$$\frac{x\left(\frac{a^{2}}{7c}+\frac{c\left(\frac{b^{2}}{7d}-\frac{2ab}{7c}\right)}{d}\right)}{\left(d\,x^{2}+c\right)^{7/2}}-\frac{x\left(\frac{b^{2}}{5d^{2}}-\frac{6a^{2}d^{2}+2abcd-b^{2}c^{2}}{35c^{2}d^{2}}\right)}{\left(d\,x^{2}+c\right)^{5/2}}+\frac{x\left(24a^{2}d^{2}+8abcd+3b^{2}c^{2}\right)}{105c^{3}d^{2}\left(d\,x^{2}+c\right)^{3/2}}+\frac{x\left(48a^{2}d^{2}+16abcd+6b^{2}c^{2}\right)}{105c^{4}d^{2}\sqrt{d\,x^{2}+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^(9/2),x)
```

```
[Out] (x*(a^2/(7*c) + (c*(b^2/(7*d) - (2*a*b)/(7*c)))/d))/(c + d*x^2)^(7/2) - (x*
(b^2/(5*d^2) - (6*a^2*d^2 - b^2*c^2 + 2*a*b*c*d)/(35*c^2*d^2)))/(c + d*x^2)
^(5/2) + (x*(24*a^2*d^2 + 3*b^2*c^2 + 8*a*b*c*d))/(105*c^3*d^2*(c + d*x^2)^
(3/2)) + (x*(48*a^2*d^2 + 6*b^2*c^2 + 16*a*b*c*d))/(105*c^4*d^2*(c + d*x^2)
^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(9/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(9/2), x)
```

3.99 $\int \frac{a+bx^2}{\left(c+dx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2x(4ad+bc)}{15c^3d\sqrt{c+dx^2}}+\frac{x(4ad+bc)}{15c^2d\left(c+dx^2\right)^{3/2}}-\frac{x(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 192, 191}

$$\frac{2x(4ad+bc)}{15c^3d\sqrt{c+dx^2}}+\frac{x(4ad+bc)}{15c^2d\left(c+dx^2\right)^{3/2}}-\frac{x(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```

```
[Out] -((b*c - a*d)*x)/(5*c*d*(c + d*x^2)^(5/2)) + ((b*c + 4*a*d)*x)/(15*c^2*d*(c
 + d*x^2)^(3/2)) + (2*(b*c + 4*a*d)*x)/(15*c^3*d*Sqrt[c + d*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{\left(c+dx^2\right)^{7/2}}\,dx &= -\frac{(bc-ad)x}{5cd\left(c+dx^2\right)^{5/2}}+\frac{(bc+4ad)\int \frac{1}{\left(c+dx^2\right)^{5/2}}\,dx}{5cd}\\
&= -\frac{(bc-ad)x}{5cd\left(c+dx^2\right)^{5/2}}+\frac{(bc+4ad)x}{15c^2d\left(c+dx^2\right)^{3/2}}+\frac{(2(bc+4ad))\int \frac{1}{\left(c+dx^2\right)^{3/2}}\,dx}{15c^2d}\\
&= -\frac{(bc-ad)x}{5cd\left(c+dx^2\right)^{5/2}}+\frac{(bc+4ad)x}{15c^2d\left(c+dx^2\right)^{3/2}}+\frac{2(bc+4ad)x}{15c^3d\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.65

$$\frac{a\left(15c^2x+20cdx^3+8d^2x^5\right)+bcx^3\left(5c+2dx^2\right)}{15c^3\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```

```
[Out] (b*c*x^3*(5*c + 2*d*x^2) + a*(15*c^2*x + 20*c*d*x^3 + 8*d^2*x^5))/(15*c^3*(
c + d*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 60, normalized size = 0.66

$$\frac{15ac^2x+20acdx^3+8ad^2x^5+5bc^2x^3+2bcdx^5}{15c^3\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```

```
[Out] (15*a*c^2*x + 5*b*c^2*x^3 + 20*a*c*d*x^3 + 2*b*c*d*x^5 + 8*a*d^2*x^5)/(15*c
^3*(c + d*x^2)^(5/2))
```

fricas [A] time = 0.75, size = 87, normalized size = 0.96

$$\frac{\left(2\left(bcd+4\,ad^2\right)x^5+15\,ac^2x+5\left(bc^2+4\,acd\right)x^3\right)\sqrt{dx^2+c}}{15\left(c^3d^3x^6+3\,c^4d^2x^4+3\,c^5dx^2+c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(b*c*d + 4*a*d^2)*x^5 + 15*a*c^2*x + 5*(b*c^2 + 4*a*c*d)*x^3)*sqrt(
d*x^2 + c)/(c^3*d^3*x^6 + 3*c^4*d^2*x^4 + 3*c^5*d*x^2 + c^6)
```

giac [A] time = 0.62, size = 72, normalized size = 0.79

$$\frac{\left(x^2\left(\frac{2\left(bcd^3+4\,ad^4\right)x^2}{c^3d^2}+\frac{5\left(bc^2d^2+4\,acd^3\right)}{c^3d^2}\right)+\frac{15\,a}{c}\right)x}{15\left(dx^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^2*(2*(b*c*d^3 + 4*a*d^4)*x^2/(c^3*d^2) + 5*(b*c^2*d^2 + 4*a*c*d^3)/
(c^3*d^2)) + 15*a/c)*x/(d*x^2 + c)^(5/2)
```

maple [A] time = 0.00, size = 57, normalized size = 0.63

$$\frac{\left(8a\,d^2x^4+2bcd\,x^4+20acd\,x^2+5b\,c^2x^2+15c^2a\right)x}{15\left(d\,x^2+c\right)^{\frac{5}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(7/2),x)
```

```
[Out] 1/15*x*(8*a*d^2*x^4+2*b*c*d*x^4+20*a*c*d*x^2+5*b*c^2*x^2+15*a*c^2)/(d*x^2+c
)^(5/2)/c^3
```

maxima [A] time = 1.45, size = 103, normalized size = 1.13

$$\frac{8\,ax}{15\,\sqrt{dx^2+c}\,c^3}+\frac{4\,ax}{15\,\left(dx^2+c\right)^{\frac{3}{2}}c^2}+\frac{ax}{5\,\left(dx^2+c\right)^{\frac{5}{2}}c}-\frac{bx}{5\,\left(dx^2+c\right)^{\frac{5}{2}}d}+\frac{2\,bx}{15\,\sqrt{dx^2+c}\,c^2d}+\frac{bx}{15\,\left(dx^2+c\right)^{\frac{3}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*a*x/(sqrt(d*x^2 + c)*c^3) + 4/15*a*x/((d*x^2 + c)^(3/2)*c^2) + 1/5*a*x
/((d*x^2 + c)^(5/2)*c) - 1/5*b*x/((d*x^2 + c)^(5/2)*d) + 2/15*b*x/(sqrt(d*x
^2 + c)*c^2*d) + 1/15*b*x/((d*x^2 + c)^(3/2)*c*d)
```

mupad [B] time = 4.85, size = 87, normalized size = 0.96

$$\frac{8\,a\,d\,x\,\left(d\,x^2+c\right)^2-3\,b\,c^3\,x+2\,b\,c\,x\,\left(d\,x^2+c\right)^2+b\,c^2\,x\,\left(d\,x^2+c\right)+3\,a\,c^2\,d\,x+4\,a\,c\,d\,x\,\left(d\,x^2+c\right)}{15\,c^3\,d\,\left(d\,x^2+c\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(c + d*x^2)^(7/2),x)
```

```
[Out] (8*a*d*x*(c + d*x^2)^2 - 3*b*c^3*x + 2*b*c*x*(c + d*x^2)^2 + b*c^2*x*(c + d
*x^2) + 3*a*c^2*d*x + 4*a*c*d*x*(c + d*x^2))/(15*c^3*d*(c + d*x^2)^(5/2))
```

sympy [B] time = 27.99, size = 566, normalized size = 6.22

$$a\left(\frac{15c^5x}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}+\frac{35c^4dx^3}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}+\frac{28c^3d^2x^5}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}+\frac{8c^2d^3x^7}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}\right)+b\left(\frac{5cx^3}{15c^{\frac{9}{2}}\sqrt{1+\frac{dx^2}{c}}+30c^{\frac{7}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{5}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}}+\frac{2dx^5}{15c^{\frac{9}{2}}\sqrt{1+\frac{dx^2}{c}}+30c^{\frac{7}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{5}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] a*(15*c**5*x/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1
+ d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d**3
*x**6*sqrt(1 + d*x**2/c)) + 35*c**4*d*x**3/(15*c**(17/2)*sqrt(1 + d*x**2/c)
 + 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 +
 d*x**2/c) + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**2/c)) + 28*c**3*d**2*x**5
/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c)
+ 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d**3*x**6*sqrt(1
 + d*x**2/c)) + 8*c**2*d**3*x**7/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(
15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c)
 + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**2/c))) + b*(5*c*x**3/(15*c**(9/2)*s
qrt(1 + d*x**2/c) + 30*c**(7/2)*d*x**2*sqrt(1 + d*x**2/c) + 15*c**(5/2)*d**
2*x**4*sqrt(1 + d*x**2/c)) + 2*d*x**5/(15*c**(9/2)*sqrt(1 + d*x**2/c) + 30*
c**(7/2)*d*x**2*sqrt(1 + d*x**2/c) + 15*c**(5/2)*d**2*x**4*sqrt(1 + d*x**2/
c)))
```

3.100 $\int \frac{1}{\left(c+dx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=39

$$\frac{2x}{3c^2\sqrt{c+dx^2}}+\frac{x}{3c\left(c+dx^2\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{2x}{3c^2\sqrt{c+dx^2}}+\frac{x}{3c\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(-5/2),x]
```

```
[Out] x/(3*c*(c + d*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + d*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+dx^2\right)^{5/2}} \, dx &= \frac{x}{3c\left(c+dx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(c+dx^2\right)^{3/2}} \, dx}{3c}\\ &= \frac{x}{3c\left(c+dx^2\right)^{3/2}}+\frac{2x}{3c^2\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.74

$$\frac{x\left(3c+2dx^2\right)}{3c^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(-5/2),x]
```

```
[Out] (x*(3*c + 2*d*x^2))/(3*c^2*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 29, normalized size = 0.74

$$\frac{x\left(3c+2dx^2\right)}{3c^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(-5/2),x]
```

```
[Out] (x*(3*c + 2*d*x^2))/(3*c^2*(c + d*x^2)^(3/2))
```

fricas [A] time = 0.87, size = 47, normalized size = 1.21

$$\frac{\left(2\,dx^3 + 3\,cx\right)\sqrt{dx^2 + c}}{3\,\left(c^2d^2x^4 + 2\,c^3dx^2 + c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*d*x^3 + 3*c*x)*sqrt(d*x^2 + c)/(c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4)
```

giac [A] time = 0.60, size = 27, normalized size = 0.69

$$\frac{x\left(\frac{2\,dx^2}{c^2} + \frac{3}{c}\right)}{3\,\left(dx^2 + c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*d*x^2/c^2 + 3/c)/(d*x^2 + c)^(3/2)
```

maple [A] time = 0.00, size = 26, normalized size = 0.67

$$\frac{\left(2d\,x^2 + 3c\right)x}{3\,\left(d\,x^2 + c\right)^{\frac{3}{2}}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*x*(2*d*x^2+3*c)/(d*x^2+c)^(3/2)/c^2
```

maxima [A] time = 1.34, size = 31, normalized size = 0.79

$$\frac{2\,x}{3\,\sqrt{dx^2 + c}\,c^2} + \frac{x}{3\,\left(dx^2 + c\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(d*x^2 + c)*c^2) + 1/3*x/((d*x^2 + c)^(3/2)*c)
```

mupad [B] time = 4.79, size = 28, normalized size = 0.72

$$\frac{2\,x\,\left(d\,x^2 + c\right) + c\,x}{3\,c^2\,\left(d\,x^2 + c\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x^2)^(5/2),x)
```

[Out] (2*x*(c + d*x^2) + c*x)/(3*c^2*(c + d*x^2)^(3/2))

sympy [B] time = 0.83, size = 95, normalized size = 2.44

$$\frac{3cx}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^2}{c}}+3c^{\frac{5}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}}+\frac{2dx^3}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^2}{c}}+3c^{\frac{5}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x**2+c)**(5/2),x)

[Out] 3*c*x/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2)*d*x**2*sqrt(1 + d*x**2/c)
) + 2*d*x**3/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2)*d*x**2*sqrt(1 + d*
x**2/c))

3.101 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}\,(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 205}

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{bc-ad}\\ &= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{bc-ad}\\ &= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [C] time = 2.72, size = 236, normalized size = 2.99

$$-\frac{\frac{15c\left(3c+2dx^2\right)\left(c\left(a+bx^2\right)\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}-a\left(c+dx^2\right)\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)\right)}{\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}}+\frac{4x^4\left(c+dx^2\right)(bc-ad)^2\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)}{a+bx^2}}{15c^3x\left(a+bx^2\right)\sqrt{c+dx^2}\,(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -1/15*((15*c*(3*c + 2*d*x^2)*(c*(a + b*x^2)*Sqrt[(a*(b*c - a*d)*x^2*(c + d*
x^2))/(c^2*(a + b*x^2)^2)] - a*(c + d*x^2)*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c
*(a + b*x^2))]]))/Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2)]
 + (4*(b*c - a*d)^2*x^4*(c + d*x^2)*Hypergeometric2F1[2, 2, 7/2, ((b*c - a*
d)*x^2)/(c*(a + b*x^2))])/(a + b*x^2))/(c^3*(-(b*c) + a*d)*x*(a + b*x^2)*Sq
rt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.26, size = 133, normalized size = 1.68

$$-\frac{dx}{c\sqrt{c+dx^2}\,(bc-ad)}-\frac{b\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) - (b*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt
[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x
^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

fricas [B] time = 1.41, size = 442, normalized size = 5.59

$$\left[-\frac{4\left(abcd-a^2d^2\right)\sqrt{dx^2+c}\,x-\left(bcdx^2+bc^2\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(bc-2\,ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\left(ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+\left(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3\right)x^2\right)},\ -\frac{2\left(abcd-a^2d^2\right)\sqrt{dx^2+c}\,x-\left(bcdx^2+bc^2\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2\,ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)}{2\left(ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+\left(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(-
a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*
a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*
d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^2*c^4 - 2*a^2*b*c^3*
d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2), -1/2*(2
*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(a*b*c - a
^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a*b^2*c^4 - 2*a^2*
b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2)]
```

giac [A] time = 0.62, size = 107, normalized size = 1.35

$$\frac{b\sqrt{d}\,\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,(bc-ad)}-\frac{dx}{\left(bc^2-acd\right)\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqr
t(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) - d*x/((b*c^2 -
 a*c*d)*sqrt(d*x^2 + c))
```

maple [B] time = 0.04, size = 628, normalized size = 7.95

$$\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}}-\frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}}+\frac{b}{2\sqrt{-ab}\,(ad-bc)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}-\frac{b}{2\sqrt{-ab}\,(ad-bc)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}+\frac{dx}{2(ad-bc)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,c}+\frac{dx}{2(ad-bc)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2/(-a*b)^(1/2)/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/(-
a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2)))-1/2/(-a*b)^(1/2)/(a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)/c/((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
*d+1/2/(-a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1
/b*(-a*b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)\left(dx^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^{2}+a\right)\,{\left(d\,x^{2}+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{2}\right)\left(c+dx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.102 $\int \frac{1}{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}\, dx$

Optimal. Leaf size=100

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(3/
2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}\, dx &= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)}+\frac{(bc-2ad)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\, dx}{2a(bc-ad)}\\
&= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)}+\frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\, dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2a(bc-ad)}\\
&= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.78, size = 405, normalized size = 4.05

$$\frac{x\sqrt{c+dx^2}\left(-30dx^2\sqrt{\frac{ax^2(c+dx^2)(bc-ad)}{c^2(a+bx^2)^2}}-45c\sqrt{\frac{ax^2(c+dx^2)(bc-ad)}{c^2(a+bx^2)^2}}+16dx^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{5/2}{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)+16c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{5/2}{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)+30dx^2\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\right)+45c\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\right)\right)}{30c^2\left(a+bx^2\right)^2\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{3/2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[c + d*x^2]*(-45*c*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*
x^2)^2)] - 30*d*x^2*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2
)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 30*d*x^2*ArcSin
[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 16*c*(((b*c - a*d)*x^2)/(c*(a +
 b*x^2)))^(5/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F1[2,
3, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 16*d*x^2*(((b*c - a*d)*x^2)/(c
*(a + b*x^2)))^(5/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F
1[2, 3, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(30*c^2*(((b*c - a*d)*x^2
)/(c*(a + b*x^2)))^(3/2)*(a + b*x^2)^2*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))
])
```

IntegrateAlgebraic [A] time = 0.36, size = 122, normalized size = 1.22

$$\frac{(2ad-bc)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{3/2}}-\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(b*x*Sqrt[c + d*x^2])/(a*(-(b*c) + a*d)*(a + b*x^2)) + ((-(b*c) + 2*a*
d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b
*c - a*d]])])/(2*a^(3/2)*(b*c - a*d)^(3/2))
```

fricas [B] time = 1.35, size = 459, normalized size = 4.59

$$\left[\frac{4\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}\,x-\left(abc-2a^2d+\left(b^2c-2abd\right)x^2\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2-4\left(\left(bc-2ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)}{8\left(a^3b^2c^2-2a^4bcd+a^5d^2+\left(a^2b^3c^2-2a^3b^2cd+a^4bd^2\right)x^2\right)}, \frac{2\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}\,x+\sqrt{abc-a^2d}\left(abc-2a^2d+\left(b^2c-2abd\right)x^2\right)\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)}{4\left(a^3b^2c^2-2a^4bcd+a^5d^2+\left(a^2b^3c^2-2a^3b^2cd+a^4bd^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - (a*b*c - 2*a^2*d + (b^2*c -
 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3
*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2
)*x^2), 1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + sqrt(a*b*c - a^2*d)*
(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((
b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2
 - a^2*c*d)*x)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^
3*b^2*c*d + a^4*b*d^2)*x^2)]
```

giac [B] time = 0.62, size = 225, normalized size = 2.25

$$-\frac{1}{2}d^{\frac{3}{2}}\left(\frac{(bc-2ad)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad-bc^2\right)}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)\left(abcd-a^2d^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
 b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sqrt
(d)*x - sqrt(d*x^2 + c))^2*b*c - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d - b*
c^2)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b*c*d - a^2*d^2))
)
```

maple [B] time = 0.02, size = 823, normalized size = 8.23

$$\frac{\sqrt{-ab}\,d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\;ab}-\frac{\sqrt{-ab}\,d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\;ab}-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\;a}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\;a}-\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\,(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)a}-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\,(ad-bc)\left(x-\frac{\sqrt{-ab}}{b}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4
/a/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2
*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4/a/(-a*b)^(1/2)/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(
a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/4/a/(a*d-b*c)/(x-(-a*b)^(1/2
)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2)+1/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.103 $\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=149

$$\frac{c(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{x\sqrt{c+dx^2}\,(3bc-4ad)}{8a^2\left(a+bx^2\right)(bc-ad)}+\frac{bx\left(c+dx^2\right)^{3/2}}{4a\left(a+bx^2\right)^2(bc-ad)}$$

Rubi [A] time = 0.08, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {382, 378, 377, 205}

$$\frac{x\sqrt{c+dx^2}\,(3bc-4ad)}{8a^2\left(a+bx^2\right)(bc-ad)}+\frac{c(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{bx\left(c+dx^2\right)^{3/2}}{4a\left(a+bx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

```
[Out] ((3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*a^2*(b*c - a*d)*(a + b*x^2)) + (b*x*
(c + d*x^2)^(3/2))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (c*(3*b*c - 4*a*d)*Arc
Tan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*(b*c - a*d)^
(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^3}\,dx = \frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{(3bc-4ad)\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx}{4a(bc-ad)}$$

$$= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{(c(3bc-4ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{8a^2(bc-ad)}$$

$$= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{(c(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,\right.}{8a^2(bc-ad)}$$

$$= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{c(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [**A**] time = 5.17, size = 130, normalized size = 0.87

$$\frac{\frac{\sqrt{a}\,x\sqrt{c+dx^2}\left(-4a^2d+ab\left(5c-2dx^2\right)+3b^2cx^2\right)}{\left(a+bx^2\right)^2(bc-ad)} + \frac{c(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*x*Sqrt[c + d*x^2]*(-4*a^2*d + 3*b^2*c*x^2 + a*b*(5*c - 2*d*x^2)))
/((b*c - a*d)*(a + b*x^2)^2) + (c*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2))/(8*a^(5/2))
```

IntegrateAlgebraic [**A**] time = 1.25, size = 154, normalized size = 1.03

$$\frac{\left(4acd-3bc^2\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{8a^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^2}\left(4a^2dx-5abcx+2abdx^3-3b^2cx^3\right)}{8a^2\left(a+bx^2\right)^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-5*a*b*c*x + 4*a^2*d*x - 3*b^2*c*x^3 + 2*a*b*d*x^3))/(8*a
^2*(-(b*c) + a*d)*(a + b*x^2)^2) + ((-3*b*c^2 + 4*a*c*d)*ArcTan[(a*Sqrt[d]
+ b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(5/
2)*(b*c - a*d)^(3/2))
```

fricas [**B**] time = 1.56, size = 698, normalized size = 4.68

$$\left[-\frac{\left(3a^2bc^2-4a^3cd+\left(3b^3c^2-4ab^2cd\right)x^4+2\left(3ab^2c^2-4a^2bcd\right)x^2\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2-4\left(\left(bc-2ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)-4\left(\left(3ab^3c^2-5a^2b^2cd+2a^3bd^2\right)x^3+\left(5a^2b^2c^2-9a^3bcd+4a^4d^2\right)x\right)\sqrt{dx^2+c}}{32\left(a^5b^2c^2-2a^6bcd+a^7d^2+\left(a^3b^4c^2-2a^4b^3cd+a^5b^2d^2\right)x^4+2\left(a^4b^3c^2-2a^5b^2cd+a^6bd^2\right)x^2\right)}, \frac{\left(3a^2bc^2-4a^3cd+\left(3b^3c^2-4ab^2cd\right)x^4+2\left(3ab^2c^2-4a^2bcd\right)x^2\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)+2\left(\left(3ab^3c^2-5a^2b^2cd+2a^3bd^2\right)x^3+\left(5a^2b^2c^2-9a^3bcd+4a^4d^2\right)x\right)\sqrt{dx^2+c}}{16\left(a^5b^2c^2-2a^6bcd+a^7d^2+\left(a^3b^4c^2-2a^4b^3cd+a^5b^2d^2\right)x^4+2\left(a^4b^3c^2-2a^5b^2cd+a^6bd^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*((3*a^2*b*c^2 - 4*a^3*c*d + (3*b^3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*b
^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d
)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
```

```
a^2)) - 4*((3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^3 + (5*a^2*b^2*c^
2 - 9*a^3*b*c*d + 4*a^4*d^2)*x)*sqrt(d*x^2 + c))/(a^5*b^2*c^2 - 2*a^6*b*c*d
 + a^7*d^2 + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*x^4 + 2*(a^4*b^3*c
^2 - 2*a^5*b^2*c*d + a^6*b*d^2)*x^2), 1/16*((3*a^2*b*c^2 - 4*a^3*c*d + (3*b
^3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c -
 a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*((3*a*b^3*c^2
- 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^3 + (5*a^2*b^2*c^2 - 9*a^3*b*c*d + 4*a^4*d
^2)*x)*sqrt(d*x^2 + c))/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2 + (a^3*b^4*c^2
 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*x^4 + 2*(a^4*b^3*c^2 - 2*a^5*b^2*c*d + a^6*
b*d^2)*x^2)]
```

<u>**giac**</u> [B] time = 3.13, size = 487, normalized size = 3.27

$$-\frac{\left(3\,bc^2\sqrt{d}-4\,acd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{8\left(a^2bc-a^3d\right)\sqrt{abcd-a^2d^2}}-\frac{3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b^3c^2\sqrt{d}-4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6ab^2cd^{\frac{3}{2}}-9\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^3c^3\sqrt{d}+30\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ab^2c^2d^{\frac{3}{2}}-40\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2bcd^{\frac{5}{2}}+16\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^3d^{\frac{7}{2}}+9\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^3c^4\sqrt{d}-28\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ab^2c^3d^{\frac{3}{2}}+16\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2bc^2d^{\frac{5}{2}}-3\,b^3c^5\sqrt{d}+2\,ab^2c^4d^{\frac{3}{2}}}{4\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)^2\left(a^2b^2c-a^3bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*b*c^2*sqrt(d) - 4*a*c*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c - a^3*d)*sqrt(a
*b*c*d - a^2*d^2)) - 1/4*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^3*c^2*sqrt(d)
 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b^2*c*d^(3/2) - 9*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*b^3*c^3*sqrt(d) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b^2*c^
2*d^(3/2) - 40*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*b*c*d^(5/2) + 16*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a^3*d^(7/2) + 9*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^
3*c^4*sqrt(d) - 28*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^3*d^(3/2) + 16*(
sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b*c^2*d^(5/2) - 3*b^3*c^5*sqrt(d) + 2*a*
b^2*c^4*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)^2*(a^2*b
^2*c - a^3*b*d))
```

<u>**maple**</u> [B] time = 0.03, size = 5177, normalized size = 34.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

<u>**maxima**</u> [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^3, x)
```

<u>**mupad**</u> [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^3,x)
```

```
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}}{\left(a + bx^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**3, x)
```

3.104 $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^4}\,dx$

Optimal. Leaf size=199

$$\frac{c^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{16a^{7/2}(bc-ad)^{3/2}}+\frac{cx\sqrt{c+dx^2}\,(5bc-6ad)}{16a^3\left(a+bx^2\right)(bc-ad)}+\frac{x\left(c+dx^2\right)^{3/2}(5bc-6ad)}{24a^2\left(a+bx^2\right)^2(bc-ad)}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a\left(a+bx^2\right)^3(bc-ad)}$$

Rubi [A] time = 0.11, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {382, 378, 377, 205}

$$\frac{c^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{16a^{7/2}(bc-ad)^{3/2}}+\frac{x\left(c+dx^2\right)^{3/2}(5bc-6ad)}{24a^2\left(a+bx^2\right)^2(bc-ad)}+\frac{cx\sqrt{c+dx^2}\,(5bc-6ad)}{16a^3\left(a+bx^2\right)(bc-ad)}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a\left(a+bx^2\right)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

```
[Out] (c*(5*b*c - 6*a*d)*x*Sqrt[c + d*x^2])/(16*a^3*(b*c - a*d)*(a + b*x^2)) + ((
5*b*c - 6*a*d)*x*(c + d*x^2)^(3/2))/(24*a^2*(b*c - a*d)*(a + b*x^2)^2) + (b
*x*(c + d*x^2)^(5/2))/(6*a*(b*c - a*d)*(a + b*x^2)^3) + (c^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(16*a^(7/2)*(b*c -
 a*d)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^4} dx = \frac{bx\left(c+dx^2\right)^{5/2}}{6a(bc-ad)\left(a+bx^2\right)^3} + \frac{(5bc-6ad)\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^3} dx}{6a(bc-ad)}$$

$$= \frac{(5bc-6ad)x\left(c+dx^2\right)^{3/2}}{24a^2(bc-ad)\left(a+bx^2\right)^2} + \frac{bx\left(c+dx^2\right)^{5/2}}{6a(bc-ad)\left(a+bx^2\right)^3} + \frac{(c(5bc-6ad))\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2} dx}{8a^2(bc-ad)}$$

$$= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)\left(a+bx^2\right)} + \frac{(5bc-6ad)x\left(c+dx^2\right)^{3/2}}{24a^2(bc-ad)\left(a+bx^2\right)^2} + \frac{bx\left(c+dx^2\right)^{5/2}}{6a(bc-ad)\left(a+bx^2\right)^3} + \frac{\left(c^2(5bc-\right.}{}$$

$$= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)\left(a+bx^2\right)} + \frac{(5bc-6ad)x\left(c+dx^2\right)^{3/2}}{24a^2(bc-ad)\left(a+bx^2\right)^2} + \frac{bx\left(c+dx^2\right)^{5/2}}{6a(bc-ad)\left(a+bx^2\right)^3} + \frac{\left(c^2(5bc-\right.}{}$$

$$= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)\left(a+bx^2\right)} + \frac{(5bc-6ad)x\left(c+dx^2\right)^{3/2}}{24a^2(bc-ad)\left(a+bx^2\right)^2} + \frac{bx\left(c+dx^2\right)^{5/2}}{6a(bc-ad)\left(a+bx^2\right)^3} + \frac{c^2(5bc-}{1}$$

Mathematica [A] time = 5.27, size = 179, normalized size = 0.90

$$\frac{\frac{3c^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}} - \frac{\sqrt{a}x\sqrt{c+dx^2}\left(-6a^3d\left(5c+2dx^2\right)+a^2b\left(33c^2-22cdx^2-4d^2x^4\right)+8ab^2cx^2\left(5c-dx^2\right)+15b^3c^2x^4\right)}{\left(a+bx^2\right)^3(ad-bc)}}{48a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

```
[Out] (-((Sqrt[a]*x*Sqrt[c + d*x^2]*(15*b^3*c^2*x^4 + 8*a*b^2*c*x^2*(5*c - d*x^2)
 - 6*a^3*d*(5*c + 2*d*x^2) + a^2*b*(33*c^2 - 22*c*d*x^2 - 4*d^2*x^4)))/((-(
b*c) + a*d)*(a + b*x^2)^3)) + (3*c^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2))/(48*a^(7/2))
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

```
[Out] $Aborted
```

fricas [B] time = 2.23, size = 972, normalized size = 4.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*a^3*b*c^3 - 6*a^4*c^2*d + (5*b^4*c^3 - 6*a*b^3*c^2*d)*x^6 + 3
*(5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a^2*b^2*c^3 - 6*a^3*b*c^2*d)*x^
2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^
2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b
```

```
*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*a*b^4*c^
3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^5 + 2*(20*a^2*b^3*c
^3 - 31*a^3*b^2*c^2*d + 5*a^4*b*c*d^2 + 6*a^5*d^3)*x^3 + 3*(11*a^3*b^2*c^3
- 21*a^4*b*c^2*d + 10*a^5*c*d^2)*x)*sqrt(d*x^2 + c))/(a^7*b^2*c^2 - 2*a^8*b
*c*d + a^9*d^2 + (a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x^6 + 3*(a^5*b
^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2)*x^4 + 3*(a^6*b^3*c^2 - 2*a^7*b^2*c*d
+ a^8*b*d^2)*x^2), 1/96*(3*(5*a^3*b*c^3 - 6*a^4*c^2*d + (5*b^4*c^3 - 6*a*b^
3*c^2*d)*x^6 + 3*(5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a^2*b^2*c^3 - 6
*a^3*b*c^2*d)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c
 - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
a^2*c*d)*x)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^
4*b*d^3)*x^5 + 2*(20*a^2*b^3*c^3 - 31*a^3*b^2*c^2*d + 5*a^4*b*c*d^2 + 6*a^5
*d^3)*x^3 + 3*(11*a^3*b^2*c^3 - 21*a^4*b*c^2*d + 10*a^5*c*d^2)*x)*sqrt(d*x^
2 + c))/(a^7*b^2*c^2 - 2*a^8*b*c*d + a^9*d^2 + (a^4*b^5*c^2 - 2*a^5*b^4*c*d
 + a^6*b^3*d^2)*x^6 + 3*(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2)*x^4 + 3
*(a^6*b^3*c^2 - 2*a^7*b^2*c*d + a^8*b*d^2)*x^2)]
```

giac [B] time = 2.81, size = 919, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="giac")
```

```
[Out] -1/16*(5*b*c^3*sqrt(d) - 6*a*c^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x
^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c - a^4*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/24*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^5*c^3*s
qrt(d) - 18*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b^4*c^2*d^(3/2) - 75*(sqrt(d
)*x - sqrt(d*x^2 + c))^8*b^5*c^4*sqrt(d) + 240*(sqrt(d)*x - sqrt(d*x^2 + c)
)^8*a*b^4*c^3*d^(3/2) - 180*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*b^3*c^2*d^(
5/2) - 96*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^3*b^2*c*d^(7/2) + 96*(sqrt(d)*x
 - sqrt(d*x^2 + c))^8*a^4*b*d^(9/2) + 150*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b
^5*c^5*sqrt(d) - 620*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b^4*c^4*d^(3/2) + 96
8*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*b^3*c^3*d^(5/2) - 720*(sqrt(d)*x - sq
rt(d*x^2 + c))^6*a^3*b^2*c^2*d^(7/2) + 64*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a
^4*b*c*d^(9/2) + 128*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^5*d^(11/2) - 150*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*b^5*c^6*sqrt(d) + 600*(sqrt(d)*x - sqrt(d*x^2
+ c))^4*a*b^4*c^5*d^(3/2) - 864*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*b^3*c^4
*d^(5/2) + 288*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^3*b^2*c^3*d^(7/2) + 96*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*a^4*b*c^2*d^(9/2) + 75*(sqrt(d)*x - sqrt(d*x^2
 + c))^2*b^5*c^7*sqrt(d) - 210*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^4*c^6*d^
(3/2) + 72*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^3*c^5*d^(5/2) + 48*(sqrt(d
)*x - sqrt(d*x^2 + c))^2*a^3*b^2*c^4*d^(7/2) - 15*b^5*c^8*sqrt(d) + 8*a*b^4
*c^7*d^(3/2) + 4*a^2*b^3*c^6*d^(5/2))/((a^3*b^3*c - a^4*b^2*d)*((sqrt(d)*x
- sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)^3)
```

maple [B] time = 0.05, size = 13964, normalized size = 70.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^4,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{3/2}}{\left(b\,x^2+a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^4,x)
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**4,x)
[Out] Timed out
```

3.105 $\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=20

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {21, 191}

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx &= \frac{d\int \frac{1}{\left(c+dx^2\right)^{3/2}}\,dx}{b}\\ &= \frac{dx}{bc\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.08, size = 20, normalized size = 1.00

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]
[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

fricas [A] time = 0.87, size = 27, normalized size = 1.35

$$\frac{\sqrt{dx^2 + c}\, dx}{bcdx^2 + bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] sqrt(d*x^2 + c)*d*x/(b*c*d*x^2 + b*c^2)
```

giac [A] time = 0.60, size = 18, normalized size = 0.90

$$\frac{dx}{\sqrt{dx^2 + c}\, bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] d*x/(sqrt(d*x^2 + c)*b*c)
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{dx}{\sqrt{d\,x^2 + c}\; bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x)
[Out] d*x/b/c/(d*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2 + \frac{bc}{d}\right)\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + b*c/d)*sqrt(d*x^2 + c)), x)
```

mupad [B] time = 4.77, size = 18, normalized size = 0.90

$$\frac{d\,x}{b\,c\,\sqrt{d\,x^2 + c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^2)^(1/2)*(b*x^2 + (b*c)/d)),x)
[Out] (d*x)/(b*c*(c + d*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{d \int \frac{1}{c\sqrt{c+dx^2} + dx^2\sqrt{c+dx^2}}\, dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x**2)/(d*x**2+c)**(1/2),x)
```

```
[Out] d*Integral(1/(c*sqrt(c + d*x**2) + d*x**2*sqrt(c + d*x**2)), x)/b
```

3.106 $\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx$

Optimal. Leaf size=25

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*(1 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx &= \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*(1 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

IntegrateAlgebraic [A] time = 0.08, size = 33, normalized size = 1.32

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{1-x^2}}{x^2-1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - x^2]*(1 + x^2)),x]
[Out] -(ArcTan[(Sqrt[2]*x*Sqrt[1 - x^2])/(-1 + x^2)]/Sqrt[2])
```

fricas [A] time = 0.63, size = 23, normalized size = 0.92

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x)
```

giac [B] time = 0.60, size = 51, normalized size = 2.04

$$\frac{1}{4}\sqrt{2}\left(\pi\operatorname{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) - 1)^2/x^
2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

maple [A] time = 0.01, size = 28, normalized size = 1.12

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}\,x}{x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(-x^2+1)^(1/2),x)
[Out] -1/2*2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^2+1\right)\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 1)*sqrt(-x^2 + 1)), x)
```

mupad [B] time = 0.37, size = 79, normalized size = 3.16

$$\frac{\sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\mathrm{i}}\right)1\mathrm{i}}{4}-\frac{\sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^2}\,1\mathrm{i}}{x+1\mathrm{i}}\right)1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(x^2 + 1)),x)
```

```
[Out] (2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i)
/4 - (2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i)/(x + 1i)
)*1i)/4
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 1)), x)
```

3.107 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {377, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

IntegrateAlgebraic [B] time = 0.13, size = 103, normalized size = 2.10

$$-\frac{\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{\sqrt{a}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[
b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])]/(Sqrt[a]*Sqr
t[b*c - a*d]))
```

fricas [B] time = 1.20, size = 241, normalized size = 4.92

$$\left[-\frac{\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left((bc-2\,ad)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\left(abc-a^2d\right)},\ \frac{\arctan\left(\frac{\sqrt{abc-a^2d}\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)}{2\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2
*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-
a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*b*c - a^2*d
), 1/2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))/sqrt(a*b*c - a^2*d)
]
```

giac [A] time = 0.60, size = 70, normalized size = 1.43

$$-\frac{\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(
a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

maple [B] time = 0.01, size = 306, normalized size = 6.24

$$-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/(
-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
```

```
2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{dx^2+c}}\right)}{\sqrt{-a(ad-bc)}} & \text{if } 0 < bc-ad \\ \frac{\ln\left(\frac{\sqrt{a\left(dx^2+c\right)}+x\sqrt{ad-bc}}{\sqrt{a\left(dx^2+c\right)}-x\sqrt{ad-bc}}\right)}{2\sqrt{a(ad-bc)}} & \text{if } bc-ad < 0 \\ \int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}\,dx & \text{if } bc-ad \notin \mathbb{R} \vee ad = bc \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] piecewise(0 < - a*d + b*c, atan((x*(- a*d + b*c)^(1/2))/(a^(1/2)*(c + d*x^2
)^(1/2)))/(-a*(a*d - b*c))^(1/2), - a*d + b*c < 0, log(((a*(c + d*x^2))^(1/
2) + x*(a*d - b*c)^(1/2))/((a*(c + d*x^2))^(1/2) - x*(a*d - b*c)^(1/2)))/(2
*(a*(a*d - b*c))^(1/2)), ~in(- a*d + b*c, 'real') | a*d == b*c, int(1/((a +
 b*x^2)*(c + d*x^2)^(1/2)), x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.108 $\int \frac{-1+x^2}{\left(1+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\sinh^{-1}(x)-\frac{2x}{\sqrt{x^2+1}}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {385, 215}

$$\sinh^{-1}(x)-\frac{2x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] (-2*x)/Sqrt[1 + x^2] + ArcSinh[x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{\left(1+x^2\right)^{3/2}}\,dx &= -\frac{2x}{\sqrt{1+x^2}}+\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= -\frac{2x}{\sqrt{1+x^2}}+\sinh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\sinh^{-1}(x)-\frac{2x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] (-2*x)/Sqrt[1 + x^2] + ArcSinh[x]
```

IntegrateAlgebraic [A] time = 0.05, size = 29, normalized size = 1.93

$$-\frac{2x}{\sqrt{x^2+1}}-\log\left(\sqrt{x^2+1}-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] (-2*x)/Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]
```

fricas [B] time = 1.21, size = 44, normalized size = 2.93

$$-\frac{2\,x^2+\left(x^2+1\right)\log\left(-x+\sqrt{x^2+1}\right)+2\,\sqrt{x^2+1}\,x+2}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*x^2 + (x^2 + 1)*log(-x + sqrt(x^2 + 1)) + 2*sqrt(x^2 + 1)*x + 2)/(x^2 +
 1)
```

giac [A] time = 0.57, size = 25, normalized size = 1.67

$$-\frac{2\,x}{\sqrt{x^2+1}}-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -2*x/sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$-\frac{2x}{\sqrt{x^2+1}}+\operatorname{arcsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)^(3/2),x)
```

```
[Out] arcsinh(x)-2*x/(x^2+1)^(1/2)
```

maxima [A] time = 2.78, size = 13, normalized size = 0.87

$$-\frac{2\,x}{\sqrt{x^2+1}}+\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*x/sqrt(x^2 + 1) + arcsinh(x)
```

mupad [B] time = 0.04, size = 27, normalized size = 1.80

$$\frac{\operatorname{asinh}(x)+x^2\operatorname{asinh}(x)-2\,x\,\sqrt{x^2+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^2 + 1)^(3/2),x)
```

```
[Out] (asinh(x) + x^2*asinh(x) - 2*x*(x^2 + 1)^(1/2))/(x^2 + 1)
```

sympy [B] time = 4.68, size = 31, normalized size = 2.07

$$\frac{x^2\operatorname{asinh}(x)}{x^2+1}-\frac{2x}{\sqrt{x^2+1}}+\frac{\operatorname{asinh}(x)}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)**(3/2),x)
```

```
[Out] x**2*asinh(x)/(x**2 + 1) - 2*x/sqrt(x**2 + 1) + asinh(x)/(x**2 + 1)
```

3.109 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}$$

Rubi [A] time = 0.03, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) +
 ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)
]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(
2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a
- b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}$$

Mathematica [C] time = 0.05, size = 162, normalized size = 0.79

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((a - b*x^2)^
(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)
```

```
/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + App
ellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

IntegrateAlgebraic [F] time = 4.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((a - b*x^2)^(1/3)*(3*a + b*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{3}}\left(b\,x^2+3a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a-b\,x^2\right)^{1/3}\left(b\,x^2+3\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x)
[Out] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

3.110 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=44

$$\frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}+\frac{9x}{2\sqrt[3]{a-bx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {413, 383}

$$\frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}+\frac{9x}{2\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
```

```
[Out] (9*x)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2))/(2*(a - b*x^2)^(4/3))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx &= \frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}-\frac{3\int \frac{-12a^2b+4ab^2x^2}{(a-bx^2)^{4/3}}\,dx}{8ab}\\ &= \frac{9x}{2\sqrt[3]{a-bx^2}}+\frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}\end{aligned}$$

Mathematica [A] time = 5.03, size = 24, normalized size = 0.55

$$\frac{9ax-3bx^3}{(a-bx^2)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
```

```
[Out] (9*a*x - 3*b*x^3)/(a - b*x^2)^(4/3)
```

IntegrateAlgebraic [F] time = 21.56, size = 0, normalized size = 0.00

$$\int \frac{(3a + bx^2)^2}{(a - bx^2)^{7/3}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
[Out] Defer[IntegrateAlgebraic][(3*a + b*x^2)^2/(a - b*x^2)^(7/3), x]
```

fricas [A] time = 1.37, size = 42, normalized size = 0.95

$$-\frac{3 \left(bx^3 - 3\,ax\right)\left(-bx^2 + a\right)^{\frac{2}{3}}}{b^2x^4 - 2\,abx^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="fricas")
[Out] -3*(b*x^3 - 3*a*x)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*x^2 + a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + 3\,a\right)^2}{\left(-bx^2 + a\right)^{\frac{7}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(7/3), x)
```

maple [A] time = 0.00, size = 24, normalized size = 0.55

$$\frac{3\left(-b\,x^2 + 3a\right)x}{\left(-b\,x^2 + a\right)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x)
[Out] 3/(-b*x^2+a)^(4/3)*x*(-b*x^2+3*a)
```

maxima [A] time = 1.89, size = 33, normalized size = 0.75

$$\frac{3\left(bx^3 - 3\,ax\right)}{\left(bx^2 - a\right)\left(-bx^2 + a\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="maxima")
[Out] 3*(b*x^3 - 3*a*x)/((b*x^2 - a)*(-b*x^2 + a)^(1/3))
```

mupad [B] time = 4.78, size = 27, normalized size = 0.61

$$\frac{3\,x\,\left(a - b\,x^2\right) + 6\,a\,x}{\left(a - b\,x^2\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)^2/(a - b*x^2)^(7/3),x)
```

```
[Out] (3*x*(a - b*x^2) + 6*a*x)/(a - b*x^2)^(4/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3a + bx^2\right)^2}{\left(a - bx^2\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**2/(-b*x**2+a)**(7/3),x)
```

```
[Out] Integral((3*a + b*x**2)**2/(a - b*x**2)**(7/3), x)
```

3.111 $\int \frac{1}{\left(-3a-bx^2\right)\sqrt[3]{-a+bx^2}}\,dx$

Optimal. Leaf size=252

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{bx^2-a}\right)}{\sqrt[3]{-a}\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{-a}\sqrt{b}x}{\sqrt{a}\left(\sqrt[3]{2}\sqrt[3]{bx^2-a}+\sqrt[3]{-a}\right)}\right)}{2\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {393}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{bx^2-a}\right)}{\sqrt[3]{-a}\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{-a}\sqrt{b}x}{\sqrt{a}\left(\sqrt[3]{2}\sqrt[3]{bx^2-a}+\sqrt[3]{-a}\right)}\right)}{2\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqrt[a
]*Sqrt[b]) - ArcTan[(Sqrt[3]*Sqrt[a]*((-a)^(1/3) - 2^(1/3)*(-a + b*x^2)^(1/
3)))/((-a)^(1/3)*Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqrt[a]*Sqrt[b])
 + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b]) - Ar
cTanh[((-a)^(1/3)*Sqrt[b]*x)/(Sqrt[a]*((-a)^(1/3) + 2^(1/3)*(-a + b*x^2)^(1
/3)))]/(2*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b])
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(-3a-bx^2\right)\sqrt[3]{-a+bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{-a+bx^2}\right)}{\sqrt[3]{-a}\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-$$

Mathematica [C] time = 0.14, size = 163, normalized size = 0.65

$$-\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{bx^2-a}\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
```

```
[Out] (-9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((-a + b*x^2
)^(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^
```

```
2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + A
ppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))
```

IntegrateAlgebraic [F] time = 5.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(-3a - bx^2)\sqrt[3]{-a + bx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
[Out] Defer[IntegrateAlgebraic][1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(bx^2 + 3a)(bx^2 - a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{(-b\,x^2 - 3a)\,(b\,x^2 - a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
[Out] int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(bx^2 + 3a)(bx^2 - a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{1}{(b\,x^2 - a)^{1/3}\,(b\,x^2 + 3\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((b*x^2 - a)^(1/3)*(3*a + b*x^2)),x)
```

```
[Out] -int(1/((b*x^2 - a)^(1/3)*(3*a + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-3*a)/(b*x**2-a)**(1/3),x)
```

```
[Out] -Integral(1/(3*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(1/3)), x)
```

3.112 $\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=202

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a+bx^2}+\sqrt[3]{a}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}$$

Rubi [A] time = 0.03, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {392}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a+bx^2}+\sqrt[3]{a}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[b]*
x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a + b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt
[b]) - ArcTanh[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sq
rt[b]) - ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/(S
qrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b])
```

Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(3a-bx^2\right)\sqrt[3]{a+bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{}}{}\right)}{2\,2^{2/3}}$$

Mathematica [C] time = 0.16, size = 166, normalized size = 0.82

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)}{\left(3a-bx^2\right)\sqrt[3]{a+bx^2}\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), (b*x^2)/(3*a)])/((3*a - b*x
^2)*(a + b*x^2)^(1/3)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), (b*x^2)
```

```
/(3*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), (b*x^2)/(3*a)]
- AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), (b*x^2)/(3*a)])))
```

IntegrateAlgebraic [F] time = 4.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(3a - bx^2)\sqrt[3]{a + bx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
[Out] Defer[IntegrateAlgebraic][1/((3*a - b*x^2)*(a + b*x^2)^(1/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(bx^2 + a)^{\frac{1}{3}}(bx^2 - 3a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{(-b\,x^2 + 3a)(b\,x^2 + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
[Out] int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(bx^2 + a)^{\frac{1}{3}}(bx^2 - 3a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(b\,x^2 + a)^{1/3}\,(3\,a - b\,x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/3)*(3*a - b*x^2)),x)
```

```
[Out] int(1/((a + b*x^2)^(1/3)*(3*a - b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-3a\sqrt[3]{a+bx^2}+bx^2\sqrt[3]{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+3*a)/(b*x**2+a)**(1/3),x)
```

```
[Out] -Integral(1/(-3*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3)), x)
```

3.113 $\int \frac{1}{\left(c-dx^2\right)\sqrt[3]{c+3dx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c+3dx^2}+\sqrt[3]{c}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}\,c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}$$

Rubi [A] time = 0.04, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {392}

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c+3dx^2}+\sqrt[3]{c}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}\,c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2*2^(2/3)*Sqrt[3]*c^(5/6)*Sqrt[d]) +
(Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c + 3*d*x^
2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[Sqrt[c]/(Sqrt[d]*x)]/(2*
2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*(c + 3*d*x^2
)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```

Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(c-dx^2\right)\sqrt[3]{c+3dx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}\,c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\sqrt[6]{c}\right.}{2\,2}$$

Mathematica [C] time = 0.15, size = 153, normalized size = 0.75

$$\frac{3cxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)}{\left(c-dx^2\right)\sqrt[3]{c+3dx^2}\left(2dx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)\right)+3cF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```

```
[Out] (3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*d*x^2)/c, (d*x^2)/c])/((c - d*x^2)*(c
 + 3*d*x^2)^(1/3)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (-3*d*x^2)/c, (d*x^2)/c]
```

```
+ 2*d*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (-3*d*x^2)/c, (d*x^2)/c] - AppellF1[3
/2, 4/3, 1, 5/2, (-3*d*x^2)/c, (d*x^2)/c])))
```

IntegrateAlgebraic [F] time = 4.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-dx^2\right)\sqrt[3]{c+3dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
[Out] Defer[IntegrateAlgebraic][1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(3\,dx^2+c\right)^{\frac{1}{3}}\left(dx^2-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-d\,x^2+c\right)\left(3d\,x^2+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
[Out] int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(3\,dx^2+c\right)^{\frac{1}{3}}\left(dx^2-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c-d\,x^2\right)\left(3\,d\,x^2+c\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x)
[Out] int(1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-c\sqrt[3]{c+3dx^2}+dx^2\sqrt[3]{c+3dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**2+c)/(3*d*x**2+c)**(1/3),x)
[Out] -Integral(1/(-c*(c + 3*d*x**2)**(1/3) + d*x**2*(c + 3*d*x**2)**(1/3)), x)
```

3.114 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}$$

Rubi [A] time = 0.03, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) +
 ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)
]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(
2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a
- b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{}{\sqrt[6]{a}}\right)}{2\,2}$$

Mathematica [C] time = 0.04, size = 162, normalized size = 0.79

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((a - b*x^2)^
(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)
```

```
/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + App
ellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((a - b*x^2)^(1/3)*(3*a + b*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{3}}\left(b\,x^2+3a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a-b\,x^2\right)^{1/3}\left(b\,x^2+3\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x)
```

```
[Out] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a - bx^2}\,(3a + bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
```

```
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

3.115 $\int \frac{1}{\sqrt[3]{c-3dx^2}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c-3dx^2}+\sqrt[3]{c}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}$$

Rubi [A] time = 0.03, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c-3dx^2}+\sqrt[3]{c}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

```
[Out] ArcTan[Sqrt[c]/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d]) + ArcTan[(c^(1/6)*(
c^(1/3) - 2^(1/3)*(c - 3*d*x^2)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqr
t[d]) - ArcTanh[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2*2^(2/3)*Sqrt[3]*c^(5/6)*Sqr
t[d]) + (Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c
 - 3*d*x^2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\left(c+dx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{}{\sqrt[6]{c}\left(\sqrt[3]{}\right.}\right.}{2\,2^{2/3}c^{5}}$$

Mathematica [C] time = 0.16, size = 156, normalized size = 0.76

$$\frac{3cxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)}{\sqrt[3]{c-3dx^2}\left(c+dx^2\right)\left(2dx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)\right)+3cF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

```
[Out] (3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (3*d*x^2)/c, -((d*x^2)/c)])/((c - 3*d*x^2
)^(1/3)*(c + d*x^2)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (3*d*x^2)/c, -((d*x^2)/
```

```
c)] + 2*d*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (3*d*x^2)/c, -((d*x^2)/c)] + App
ellF1[3/2, 4/3, 1, 5/2, (3*d*x^2)/c, -((d*x^2)/c)])))
```

IntegrateAlgebraic [F] time = 4.63, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^2+c\right)\left(-3\,dx^2+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3d\,x^2+c\right)^{\frac{1}{3}}\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
[Out] int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^2+c\right)\left(-3\,dx^2+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d\,x^2+c\right)\,{\left(c-3\,d\,x^2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^2)*(c - 3*d*x^2)^(1/3)),x)
```

```
[Out] int(1/((c + d*x^2)*(c - 3*d*x^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x**2+c)**(1/3)/(d*x**2+c),x)
```

```
[Out] Integral(1/((c - 3*d*x**2)**(1/3)*(c + d*x**2)), x)
```

3.116 $\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)} dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

Rubi [A] time = 0.02, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)} dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

Mathematica [C] time = 0.04, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)
*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

IntegrateAlgebraic [F] time = 3.94, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((1 - x^2)^(1/3)*(3 + x^2)), x]
```

fricas [B] time = 4.45, size = 1943, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 +
27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x)
+ 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 -
 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432
^(5/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432
^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)
*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x
)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3
)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(
3)*(x^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^
2))*(-x^2 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)
^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3)*log(2592*(
6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3
) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1
)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6
+ 9*x^4 + 27*x^2 + 27)) - 1/1296*432^(5/6)*arctan(1/36*(432^(5/6)*(x^5 - 18
*x^3 + 9*x)*(-x^2 + 1)^(1/3) + sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(-x
^2 + 1)^(2/3) - 288*sqrt(3)*(2*x^4 - 3*x^2)*(-x^2 + 1)^(1/3) + 6*432^(1/6)*
(x^6 + 141*x^4 - 153*x^2 + 27)) - 648*432^(1/6)*(3*x^3 - x)*(-x^2 + 1)^(2/3
) - 72*sqrt(3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) - 1/2
592*432^(5/6)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9
- 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) - 2*(-x^2 + 1)^(2/3)*(432^(5/6)
*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*
x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 1
3158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 140
4*x^6 + 1350*x^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 1
69776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 -
 189*x^2 + 27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*
x^3 - 3*x) + 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4
 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*43
2^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(-x^2 + 1)^(2/3) -
18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2
 + 729) + 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 -
 243*x) - (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*
x^5 + 5589*x^3 - 243*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6
 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 18783
9*x^4 - 21870*x^2 + 729)) - 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(
3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) +
 2*(-x^2 + 1)^(2/3)*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*
sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)
*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*
sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^
```

```
12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sq
rt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 -
 x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2
 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^
6 + 2808*x^4 - 243*x^2) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^
3 + 27*x))*(-x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 426
60*x^6 + 58239*x^4 - 14094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 45
18*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 -
 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)
*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 1
10727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2 + 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

maple [C] time = 16.87, size = 938, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] 1/36*RootOf(_Z^6+108)*ln(-(-4050*RootOf(_Z^6+108)*x^4+RootOf(_Z^6+108)^4*x^
6+225*RootOf(_Z^6+108)^4*x^4-72*x^5*RootOf(_Z^6+108)^4+1296*x^5*RootOf(_Z^6
+108)+72*x^3*RootOf(_Z^6+108)^4-189*RootOf(_Z^6+108)^4*x^2+3402*RootOf(_Z^6
+108)*x^2-1296*RootOf(_Z^6+108)*x^3+6*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^5
-108*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^4+144*RootOf(_Z^6+108)^5*(-x^2+1)^
(1/3)*x^3+108*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^2-36*RootOf(_Z^6+108)^2*(
-x^2+1)^(1/3)*x^5-54*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x+648*RootOf(_Z^6+10
8)^2*(-x^2+1)^(1/3)*x^4-864*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3-648*RootO
f(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x-48
6*RootOf(_Z^6+108)+27*RootOf(_Z^6+108)^4-1296*(-x^2+1)^(2/3)*x^4+9072*(-x^2
+1)^(2/3)*x^3-3888*(-x^2+1)^(2/3)*x^2-3888*(-x^2+1)^(2/3)*x-18*RootOf(_Z^6+
108)*x^6)/(x^2+3)^3)-1/432*ln((RootOf(_Z^6+108)^4*x^6+72*x^5*RootOf(_Z^6+10
8)^4+225*RootOf(_Z^6+108)^4*x^4+36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5-72
*x^3*RootOf(_Z^6+108)^4+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4-189*RootO
f(_Z^6+108)^4*x^2+864*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3+648*(-x^2+1)^(2
/3)*x^4-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2+4536*(-x^2+1)^(2/3)*x^3+2
7*RootOf(_Z^6+108)^4-324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+1944*(-x^2+1)^
(2/3)*x^2-1944*(-x^2+1)^(2/3)*x)/(x^2+3)^3)*RootOf(_Z^6+108)^4-1/72*ln((Roo
tOf(_Z^6+108)^4*x^6+72*x^5*RootOf(_Z^6+108)^4+225*RootOf(_Z^6+108)^4*x^4+36
*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5-72*x^3*RootOf(_Z^6+108)^4+648*RootOf
(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4-189*RootOf(_Z^6+108)^4*x^2+864*RootOf(_Z^6+
108)^2*(-x^2+1)^(1/3)*x^3+648*(-x^2+1)^(2/3)*x^4-648*RootOf(_Z^6+108)^2*(-x
^2+1)^(1/3)*x^2+4536*(-x^2+1)^(2/3)*x^3+27*RootOf(_Z^6+108)^4-324*RootOf(_Z
^6+108)^2*(-x^2+1)^(1/3)*x+1944*(-x^2+1)^(2/3)*x^2-1944*(-x^2+1)^(2/3)*x)/(
x^2+3)^3)*RootOf(_Z^6+108)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^2\right)^{1/3}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.117 $\int \frac{1}{\left(3-x^2\right)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {392}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(3-x^2\right)\sqrt[3]{1+x^2}}\,dx=-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Mathematica [C] time = 0.04, size = 124, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{\left(x^2-3\right)\sqrt[3]{x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*
(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

IntegrateAlgebraic [F] time = 4.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((3 - x^2)*(1 + x^2)^(1/3)), x]
```

fricas [B] time = 4.16, size = 1685, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2592*432^(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7
- 373248*x^5 - 69984*x^3 - sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 +
 6030*x^7 + 7182*x^5 + 2511*x^3 + 243*x) + 3*432^(1/6)*sqrt(3)*(x^12 + 924*
x^10 - 33363*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) + (432^(5/6)*sqr
t(3)*(x^10 - 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) + 432*sqrt(3)*2^(1/3)*(13
*x^9 - 177*x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) + 36*(96*x^10 - 4032*x^
8 - 2592*x^6 + sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 2
43*x))*(x^2 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^
2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 -
18*x^2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 +
 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8
+ 1206*x^6 + 756*x^4 + 81*x^2) + 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 207
0*x^5 - 756*x^3 - 81*x))*(x^2 + 1)^(2/3) + 18*sqrt(3)*(x^12 + 1422*x^10 + 2
1447*x^8 + 27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^
11 - 681*x^9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) + 3888*sqrt(3)*2^(1/
3)*(x^10 + 44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178
*x^10 + 46791*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/2592*432^
(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7 - 373248*x^
5 - 69984*x^3 + sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 + 6030*x^7 +
 7182*x^5 + 2511*x^3 + 243*x) - 3*432^(1/6)*sqrt(3)*(x^12 + 924*x^10 - 3336
3*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) - (432^(5/6)*sqrt(3)*(x^10
- 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) - 432*sqrt(3)*2^(1/3)*(13*x^9 - 177*
x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) - 36*(96*x^10 - 4032*x^8 - 2592*x^
6 - sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 243*x))*(x^2
 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3
)*(432^(5/6)*(x^3 + x) - 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sq
rt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6
 - 9*x^4 + 27*x^2 - 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8 + 1206*x^6
+ 756*x^4 + 81*x^2) - 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 2070*x^5 - 756
*x^3 - 81*x))*(x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 + 1422*x^10 + 21447*x^8 +
27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^11 - 681*x^
9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) - 3888*sqrt(3)*2^(1/3)*(x^10 +
44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178*x^10 + 467
91*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/5184*432^(5/6)*log(-
(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + 27) + 864*(9*x^3 + sqrt(3)*(x^4 + 9*x^2
) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) + 432*(x^2 +
1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 -
 9*x^4 + 27*x^2 - 27)) - 1/5184*432^(5/6)*log((432^(5/6)*(x^6 + 69*x^4 + 63
*x^2 + 27) - 864*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 43
2*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x
^3 + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/10
368*432^(5/6)*log(31104*(2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^2 + 1
)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^
```

```
2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 + 9*x)
)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*log(31104*(2*2^(2/3)*(x^
6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) - 24*2^(1
/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/
3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^2+3\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^2+1\right)^{1/3}\,\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
[Out] -int(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

3.118 $\int \frac{3-x}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=96

$$-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left((x+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{1-x}\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}$$

Rubi [A] time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1008}

$$-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left((x+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{1-x}\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 + x)^(2/3))/(Sqrt[3]*(1 - x)^(1/3
))])/2^(2/3)) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(1/3)*(1 - x)^(1/3) + (
1 + x)^(2/3)])/(2*2^(2/3))
```

Rule 1008

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[(Sqrt[3]*h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^
(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*a^(1/3)*f), x] + (-Simp[(
3*h*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2^(5/3)*a^
(1/3)*f), x] + Simp[(h*Log[d + f*x^2])/(2^(5/3)*a^(1/3)*f), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rubi steps

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1+x)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}-\frac{\log\left(3+x^2\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x}+(1+x)^{2/3}\right)}{2\ 2^{2/3}}$$

Mathematica [C] time = 0.15, size = 143, normalized size = 1.49

$$-\frac{1}{6}x^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -1/6*(x^2*AppellF1[1, 1/3, 1, 2, x^2, -1/3*x^2]) - (27*x*AppellF1[1/2, 1/3,
 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3,
1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] -
 AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

IntegrateAlgebraic [A] time = 0.24, size = 166, normalized size = 1.73

$$\frac{\log\left(2\sqrt[3]{1-x^2}+2^{2/3}x+2^{2/3}\right)}{2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}x^2-2\left(1-x^2\right)^{2/3}+\left(2^{2/3}x+2^{2/3}\right)\sqrt[3]{1-x^2}-2\sqrt[3]{2}x-\sqrt[3]{2}\right)}{2\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^2}}{\sqrt[3]{1-x^2}-2^{2/3}x-2^{2/3}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^2)^(1/3))/(-2^(2/3) - 2^(2/3)*x + (1 - x^2)
^(1/3))])/2^(2/3) + Log[2^(2/3) + 2^(2/3)*x + 2*(1 - x^2)^(1/3)]/2^(2/3) -
Log[-2^(1/3) - 2*2^(1/3)*x - 2^(1/3)*x^2 + (2^(2/3) + 2^(2/3)*x)*(1 - x^2)^
(1/3) - 2*(1 - x^2)^(2/3)]/(2*2^(2/3))
```

fricas [B] time = 16.35, size = 285, normalized size = 2.97

$$-\frac{1}{6} \cdot 4^{\frac{1}{6}} \sqrt{3} \arctan\left(\frac{4^{\frac{1}{6}} \sqrt{3} \left(12 \cdot 4^{\frac{2}{3}} \left(x^{4} + 3 x^{3} + 3 x^{2} + 9 x\right) \left(-x^{2} + 1\right)^{\frac{2}{3}} + 4^{\frac{1}{3}} \left(x^{6} - 18 x^{5} - 117 x^{4} - 36 x^{3} + 207 x^{2} + 54 x - 27\right) + 12 \left(x^{5} + 19 x^{4} + 42 x^{3} + 6 x^{2} - 27 x - 9\right) \left(-x^{2} + 1\right)^{\frac{1}{3}}\right)}{6 \left(x^{6} + 54 x^{5} + 171 x^{4} + 108 x^{3} - 81 x^{2} - 162 x - 27\right)}\right) - \frac{1}{24} \cdot 4^{\frac{2}{3}} \log\left(\frac{6 \cdot 4^{\frac{2}{3}} \left(x^{2} + 3 x\right) \left(-x^{2} + 1\right)^{\frac{2}{3}} + 4^{\frac{1}{3}} \left(x^{4} + 18 x^{3} + 24 x^{2} - 18 x - 9\right) - 6 \left(x^{3} + 7 x^{2} + 3 x - 3\right) \left(-x^{2} + 1\right)^{\frac{1}{3}}}{x^{4} + 6 x^{2} + 9}\right) + \frac{1}{12} \cdot 4^{\frac{2}{3}} \log\left(\frac{4^{\frac{2}{3}} \left(x^{2} + 3\right) + 6 \cdot 4^{\frac{1}{3}} \left(-x^{2} + 1\right)^{\frac{1}{3}} (x + 1) + 12 \left(-x^{2} + 1\right)^{\frac{2}{3}}}{x^{2} + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(x^4 + 3*x^3 +
3*x^2 + 9*x)*(-x^2 + 1)^(2/3) + 4^(1/3)*(x^6 - 18*x^5 - 117*x^4 - 36*x^3 +
207*x^2 + 54*x - 27) + 12*(x^5 + 19*x^4 + 42*x^3 + 6*x^2 - 27*x - 9)*(-x^2
+ 1)^(1/3))/(x^6 + 54*x^5 + 171*x^4 + 108*x^3 - 81*x^2 - 162*x - 27)) - 1/2
4*4^(2/3)*log((6*4^(2/3)*(x^2 + 3*x)*(-x^2 + 1)^(2/3) + 4^(1/3)*(x^4 + 18*x
^3 + 24*x^2 - 18*x - 9) - 6*(x^3 + 7*x^2 + 3*x - 3)*(-x^2 + 1)^(1/3))/(x^4
+ 6*x^2 + 9)) + 1/12*4^(2/3)*log((4^(2/3)*(x^2 + 3) + 6*4^(1/3)*(-x^2 + 1)^
(1/3)*(x + 1) + 12*(-x^2 + 1)^(2/3))/(x^2 + 3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x - 3}{\left(x^{2} + 3\right) \left(-x^{2} + 1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

maple [C] time = 7.83, size = 1033, normalized size = 10.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+3)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] 1/2*RootOf(_Z^3-2)*ln((12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)^2*RootOf(_Z^3-2)^2*x^2+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*RootOf(_Z^3-2)^3*x^2+36*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2)^2*RootOf(_Z^3-2)^2*x+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2)*RootOf(_Z^3-2)^3*x-18*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-12*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2
)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-9*(-x^2+1)^(1/3)*RootOf(_Z
^3-2)^2*x-12*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*RootOf(_Z^3-2)+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x
^2-9*(-x^2+1)^(1/3)*RootOf(_Z^3-2)^2+RootOf(_Z^3-2)*x^2+36*RootOf(RootOf(_Z
^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+6*RootOf(_Z^3-2)*x-6*(-x^2+1)^(2/3)-1
8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-3*RootOf(_Z^3-2))/(2*
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+x-3)
/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+
x+3))+RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln(-(4*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+6*RootOf(R
ootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2+12*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x+18*RootOf
```

```
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x-18*(-x^2+1
)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^
2-6*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*Root
Of(_Z^3-2)*x-9*(-x^2+1)^(1/3)*RootOf(_Z^3-2)^2*x-6*(-x^2+1)^(1/3)*RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+2*RootOf(RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2-9*(-x^2+1)^(1/3)*RootOf(_Z^3-2)^2+
3*RootOf(_Z^3-2)*x^2-12*(-x^2+1)^(2/3)+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+4*_Z^2)+9*RootOf(_Z^3-2))/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(
_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+x-3)/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+x+3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-3}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x-3}{\left(1-x^2\right)^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 3)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] -int((x - 3)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^2\sqrt[3]{1-x^2}+3\sqrt[3]{1-x^2}}\,dx-\int \left(-\frac{3}{x^2\sqrt[3]{1-x^2}+3\sqrt[3]{1-x^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -Integral(x/(x**2*(1 - x**2)**(1/3) + 3*(1 - x**2)**(1/3)), x) - Integral(-
3/(x**2*(1 - x**2)**(1/3) + 3*(1 - x**2)**(1/3)), x)
```

3.119 $\int \frac{3+x}{\sqrt[3]{1-x^2}\left(3+x^2\right)} dx$

Optimal. Leaf size=95

$$\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{2\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{2^{2/3}}$$

Rubi [A] time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1008}

$$\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{2\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - x)^(2/3))/(Sqrt[3]*(1 + x)^(1/3))
])/2^(2/3) + Log[3 + x^2]/(2*2^(2/3)) - (3*Log[(1 - x)^(2/3) + 2^(1/3)*(1 +
 x)^(1/3)])/(2*2^(2/3))
```

Rule 1008

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[(Sqrt[3]*h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^
(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*a^(1/3)*f), x] + (-Simp[(
3*h*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2^(5/3)*a^
(1/3)*f), x] + Simp[(h*Log[d + f*x^2])/(2^(5/3)*a^(1/3)*f), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rubi steps

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\left(3+x^2\right)} dx=\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{2^{2/3}}+\frac{\log\left(3+x^2\right)}{2\ 2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{1+x}\right)}{2\ 2^{2/3}}$$

Mathematica [C] time = 0.11, size = 143, normalized size = 1.51

$$\frac{1}{6}x^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (x^2*AppellF1[1, 1/3, 1, 2, x^2, -1/3*x^2])/6 - (27*x*AppellF1[1/2, 1/3, 1,
 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3, 1,
3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] - Ap
pellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

IntegrateAlgebraic [A] time = 0.23, size = 168, normalized size = 1.77

$$-\frac{\log\left(-2\sqrt[3]{1-x^2}+2^{2/3}x-2^{2/3}\right)}{2^{2/3}}+\frac{\log\left(\sqrt[3]{2}x^2+2\left(1-x^2\right)^{2/3}+\left(2^{2/3}x-2^{2/3}\right)\sqrt[3]{1-x^2}-2\sqrt[3]{2}x+\sqrt[3]{2}\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^2}}{\sqrt[3]{1-x^2}+2^{2/3}x-2^{2/3}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^2)^(1/3))/(-2^(2/3) + 2^(2/3)*x + (1 - x^
2)^(1/3))])/2^(2/3)) - Log[-2^(2/3) + 2^(2/3)*x - 2*(1 - x^2)^(1/3)]/2^(2/3
) + Log[2^(1/3) - 2*2^(1/3)*x + 2^(1/3)*x^2 + (-2^(2/3) + 2^(2/3)*x)*(1 - x
^2)^(1/3) + 2*(1 - x^2)^(2/3)]/(2*2^(2/3))
```

fricas [B] time = 15.63, size = 315, normalized size = 3.32

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(x^{4}-3x^{3}+3x^{2}-9x\right)\left(-x^{2}+1\right)^{\frac{2}{3}}+12\,(-1)^{\frac{1}{3}}\left(x^{5}-19x^{4}+42x^{3}-6x^{2}-27x+9\right)\left(-x^{2}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\left(x^{6}+18x^{5}-117x^{4}+36x^{3}+207x^{2}-54x-27\right)\right)}{6\left(x^{6}-54x^{5}+171x^{4}-108x^{3}-81x^{2}+162x-27\right)}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{6\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{2}-3x\right)\left(-x^{2}+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^{4}-18x^{3}+24x^{2}+18x-9\right)-6\left(x^{3}-7x^{2}+3x+3\right)\left(-x^{2}+1\right)^{\frac{1}{3}}}{x^{4}+6x^{2}+9}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}(x-1)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{2}+3\right)-12\left(-x^{2}+1\right)^{\frac{2}{3}}}{x^{2}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(-1)
^(2/3)*(x^4 - 3*x^3 + 3*x^2 - 9*x)*(-x^2 + 1)^(2/3) + 12*(-1)^(1/3)*(x^5 -
19*x^4 + 42*x^3 - 6*x^2 - 27*x + 9)*(-x^2 + 1)^(1/3) + 4^(1/3)*(x^6 + 18*x^
5 - 117*x^4 + 36*x^3 + 207*x^2 - 54*x - 27))/(x^6 - 54*x^5 + 171*x^4 - 108*
x^3 - 81*x^2 + 162*x - 27)) - 1/24*4^(2/3)*(-1)^(1/3)*log(-(6*4^(2/3)*(-1)^
(1/3)*(x^2 - 3*x)*(-x^2 + 1)^(2/3) - 4^(1/3)*(-1)^(2/3)*(x^4 - 18*x^3 + 24*
x^2 + 18*x - 9) - 6*(x^3 - 7*x^2 + 3*x + 3)*(-x^2 + 1)^(1/3))/(x^4 + 6*x^2
+ 9)) + 1/12*4^(2/3)*(-1)^(1/3)*log(-(6*4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3)
*(x - 1) + 4^(2/3)*(-1)^(1/3)*(x^2 + 3) - 12*(-x^2 + 1)^(2/3))/(x^2 + 3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+3}{\left(x^{2}+3\right)\left(-x^{2}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

maple [C] time = 7.62, size = 1553, normalized size = 16.35

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+3)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] -1/2*ln(-(8*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z
^3+2)^2*x^2-2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z
^3+2)^3*x^2-24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf
(_Z^3+2)^2*x+6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_
Z^3+2)^3*x+18*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*
_Z^2)*RootOf(_Z^3+2)^2-18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootO
f(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x-6*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2*x+18*(
-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z
^3+2)-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2+6*(-x^2+1)^
(1/3)*RootOf(_Z^3+2)^2+RootOf(_Z^3+2)*x^2-12*(-x^2+1)^(2/3)-12*RootOf(RootO
f(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+3*RootOf(_Z^3+2))/(2*RootOf(RootOf(
_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x-3)/(2*RootOf(Roo
tOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x+3))*RootOf(_
Z^3+2)-ln(-(8*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(
_Z^3+2)^2*x^2-2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(
_Z^3+2)^3*x^2-24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*Root
```

```
Of(_Z^3+2)^2*x+6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf
(_Z^3+2)^3*x+18*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+
4*_Z^2)*RootOf(_Z^3+2)^2-18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*Roo
tOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x-6*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2*x+18
*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(
_Z^3+2)-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2+6*(-x^2+1
)^(1/3)*RootOf(_Z^3+2)^2+RootOf(_Z^3+2)*x^2-12*(-x^2+1)^(2/3)-12*RootOf(Roo
tOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+3*RootOf(_Z^3+2))/(2*RootOf(RootO
f(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x-3)/(2*RootOf(R
ootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x+3))*RootOf
(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+RootOf(RootOf(_Z^3+2)^2+2*_Z*
RootOf(_Z^3+2)+4*_Z^2)*ln(-(8*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4
*_Z^2)^2*RootOf(_Z^3+2)^2*x^2+6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)
+4*_Z^2)*RootOf(_Z^3+2)^3*x^2-24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2
)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x-18*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+
2)+4*_Z^2)*RootOf(_Z^3+2)^3*x-18*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_
Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2+18*(-x^2+1)^(1/3)*RootOf(RootOf(_
Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x+3*(-x^2+1)^(1/3)*Root
Of(_Z^3+2)^2*x-18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2
)+4*_Z^2)*RootOf(_Z^3+2)-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z
^2)*x^2-3*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2-3*RootOf(_Z^3+2)*x^2+24*RootOf(Ro
otOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x+18*RootOf(_Z^3+2)*x+6*(-x^2+1)
^(2/3)+12*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+9*RootOf(_Z^3
+2))/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^
2*x-x-3)/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3
+2)^2*x-x+3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+3}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+3}{\left(1-x^2\right)^{1/3}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int((x + 3)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+3}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral((x + 3)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.120 $\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Rubi [A] time = 0.03, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {394}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) + (Sqrt[b]*ArcTan[
(a^(1/3) - (a + b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d) - (S
qrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a + b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(4*Sqrt[3]*a^(5/6)*d)
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}$$

Mathematica [C] time = 0.17, size = 169, normalized size = 1.12

$$\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{a+bx^2}\left(9a+bx^2\right)\left(27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -1/9*(b*x^2)/a])/(d*(a +
 b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a),
```

```
-1/9*(b*x^2)/a] - 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), -1/9*(b
*x^2)/a] + 3*AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), -1/9*(b*x^2)/a])))
```

IntegrateAlgebraic [F] time = 5.42, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}\left(d\,x^2+\frac{9ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)
[Out] int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^2+a\right)^{1/3}\,\left(d\,x^2+\frac{9\,a\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)),x)
```

```
[Out] int(1/((a + b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{1}{9a\sqrt[3]{a+bx^2}+bx^2\sqrt[3]{a+bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/3)/(9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(9*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3)), x)/d
```

3.121 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=153

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Rubi [A] time = 0.03, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {395}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a - b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*Sqrt[3]*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*
a^(5/6)*d) + (Sqrt[b]*ArcTanh[(a^(1/3) - (a - b*x^2)^(1/3))^2/(3*a^(1/6)*Sq
rt[b]*x)])/(12*a^(5/6)*d)
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx=-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-b}\right.}{3\sqrt[6]{a}\sqrt{b}}\right.}{12a^{5/6}d}$$

Mathematica [C] time = 0.18, size = 167, normalized size = 1.09

$$-\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\sqrt[3]{a-bx^2}\left(9a-bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)+27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/(d*(a - b*
x^2)^(1/3)*(9*a - b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2
```

```
)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, (b*x^2)/(9*a)] +
3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, (b*x^2)/(9*a)])))
```

IntegrateAlgebraic [F] time = 5.40, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{3}}\left(d\,x^2-\frac{9ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(-9*a/b*d+d*x^2),x)
[Out] int(1/(-b*x^2+a)^(1/3)/(-9*a/b*d+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-b\,x^2\right)}^{1/3}\,\left(d\,x^2-\frac{9\,a\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/3)*(d*x^2 - (9*a*d)/b)),x)
```

```
[Out] int(1/((a - b*x^2)^(1/3)*(d*x^2 - (9*a*d)/b)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{1}{-9a\sqrt[3]{a-bx^2}+bx^2\sqrt[3]{a-bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(-9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(-9*a*(a - b*x**2)**(1/3) + b*x**2*(a - b*x**2)**(1/3)), x)/d
```

3.122 $\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Rubi [A] time = 0.03, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {395}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a + b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*Sqrt[3]*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*
a^(5/6)*d) - (Sqrt[b]*ArcTanh[(a^(1/3) + (-a + b*x^2)^(1/3))^2/(3*a^(1/6)*S
qrt[b]*x)])/(12*a^(5/6)*d)
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx=\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}$$

Mathematica [C] time = 0.15, size = 168, normalized size = 1.11

$$-\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\left(9a-bx^2\right)\sqrt[3]{bx^2-a}\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)+27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/(d*(9*a -
b*x^2)*(-a + b*x^2)^(1/3)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^
```

```
2)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, (b*x^2)/(9*a)] +
 3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, (b*x^2)/(9*a)])))
```

IntegrateAlgebraic [F] time = 5.43, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^2-a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2-a)^{\frac{1}{3}}\left(d\,x^2-\frac{9ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-a)^(1/3)/(d*x^2-9*a/b*d),x)
[Out] int(1/(b*x^2-a)^(1/3)/(d*x^2-9*a/b*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^2-a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\,x^2-a)^{1/3}\left(d\,x^2-\frac{9\,a\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2 - a)^(1/3)*(d*x^2 - (9*a*d)/b)),x)
```

```
[Out] int(1/((b*x^2 - a)^(1/3)*(d*x^2 - (9*a*d)/b)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{1}{-9a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-a)**(1/3)/(-9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(-9*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(1/3)), x)/
d
```

3.123 $\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=153

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Rubi [A] time = 0.03, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {394}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) - (Sqrt[b]*ArcTan
[(a^(1/3) + (-a - b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d) +
(Sqrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a - b*x^2)^(1/3)))/(Sqrt[b]*
x)])/(4*Sqrt[3]*a^(5/6)*d)
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a}\right.}{\sqrt{b}x}\right.}{4\sqrt{3}a^{5/6}d}$$

Mathematica [C] time = 0.16, size = 172, normalized size = 1.12

$$\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{-a-bx^2}\left(9a+bx^2\right)\left(27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -1/9*(b*x^2)/a])/(d*(-a
- b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a),
```

```
-1/9*(b*x^2)/a] - 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), -1/9*(
b*x^2)/a] + 3*AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), -1/9*(b*x^2)/a])))
```

IntegrateAlgebraic [F] time = 5.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2-a\right)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2-a\right)^{\frac{1}{3}}\left(d\,x^2+\frac{9ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-a)^(1/3)/(d*x^2+9*a/b*d),x)
[Out] int(1/(-b*x^2-a)^(1/3)/(d*x^2+9*a/b*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2-a\right)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(-b\,x^2-a\right)^{1/3}\,\left(d\,x^2+\frac{9\,a\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- a - b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)),x)
```

```
[Out] int(1/((- a - b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{1}{9a\sqrt[3]{-a-bx^2}+bx^2\sqrt[3]{-a-bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-a)**(1/3)/(9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(9*a*(-a - b*x**2)**(1/3) + b*x**2*(-a - b*x**2)**(1/3)), x)/d
```

3.124 $\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx$

<u>**Optimal**</u>. Leaf size=151

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}$$

<u>**Rubi**</u> **[A]** time = 0.03, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {394}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*2^(5/6)*d) + (Sqrt[b]*ArcTan[
(2^(1/3) - (2 + b*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[b]*x)])/(12*2^(5/6)*d) - (S
qrt[b]*ArcTanh[(2^(1/6)*Sqrt[3]*(2^(1/3) - (2 + b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(4*2^(5/6)*Sqrt[3]*d)
```

<u>Rule 394</u>

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

<u>Rubi steps</u>

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx=\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

<u>**Mathematica**</u> **[C]** time = 0.15, size = 148, normalized size = 0.98

$$-\frac{27bxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)}{d\sqrt[3]{bx^2+2}\left(bx^2+18\right)\left(bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)\right)-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```

```
[Out] (-27*b*x*AppellF1[1/2, 1/3, 1, 3/2, -1/2*(b*x^2), -1/18*(b*x^2)])/(d*(2 + b
*x^2)^(1/3)*(18 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -1/2*(b*x^2), -1/1
```

```
8*(b*x^2)] + b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -1/2*(b*x^2), -1/18*(b*x^2)]
 + 3*AppellF1[3/2, 4/3, 1, 5/2, -1/2*(b*x^2), -1/18*(b*x^2)])))
```

IntegrateAlgebraic [F] time = 5.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+2\right)^{\frac{1}{3}}\left(dx^2+\frac{18\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+2\right)^{\frac{1}{3}}\left(d\,x^2+\frac{18d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2)^(1/3)/(18/b*d+d*x^2),x)
[Out] int(1/(b*x^2+2)^(1/3)/(18/b*d+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+2\right)^{\frac{1}{3}}\left(dx^2+\frac{18\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{18\,d}{b}+d\,x^2\right)\left(b\,x^2+2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((18*d)/b + d*x^2)*(b*x^2 + 2)^(1/3)),x)
```

```
[Out] int(1/(((18*d)/b + d*x^2)*(b*x^2 + 2)^(1/3)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{1}{bx^2\sqrt[3]{bx^2+2}+18\sqrt[3]{bx^2+2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2)**(1/3)/(18*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(b*x**2*(b*x**2 + 2)**(1/3) + 18*(b*x**2 + 2)**(1/3)), x)/d
```

3.125 $\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)}{\sqrt{b}x}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\,2^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\,2^{5/6}d}$$

Rubi [A] time = 0.02, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {395}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)}{\sqrt{b}x}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\,2^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\,2^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(2^(1/6)*Sqrt[3]*(2^(1/3) + (-2 + b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*2^(5/6)*Sqrt[3]*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*
2^(5/6)*d) - (Sqrt[b]*ArcTanh[(2^(1/3) + (-2 + b*x^2)^(1/3))^2/(3*2^(1/6)*S
qrt[b]*x)])/(12*2^(5/6)*d)
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx=\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)}{\sqrt{b}x}\right)}{4\,2^{5/6}\sqrt{3}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\,2^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-\right.}}{3\sqrt[6]{2}}\right.}{12\,2^{5/6}d}$$

Mathematica [C] time = 0.17, size = 148, normalized size = 1.01

$$\frac{27bxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)}{d\left(bx^2-18\right)\sqrt[3]{bx^2-2}\left(bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)\right)+27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```

```
[Out] (27*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/2, (b*x^2)/18])/(d*(-18 + b*x^2)
*(-2 + b*x^2)^(1/3)*(27*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/2, (b*x^2)/18] +
```

```
b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/2, (b*x^2)/18] + 3*AppellF1[3/2,
4/3, 1, 5/2, (b*x^2)/2, (b*x^2)/18])))
```

IntegrateAlgebraic [F] time = 5.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2-2\right)^{\frac{1}{3}}\left(dx^2-\frac{18\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2-2\right)^{\frac{1}{3}}\left(d\,x^2-\frac{18d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-2)^(1/3)/(-18/b*d+d*x^2),x)
[Out] int(1/(b*x^2-2)^(1/3)/(-18/b*d+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2-2\right)^{\frac{1}{3}}\left(dx^2-\frac{18\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{1}{\left(\frac{18\,d}{b}-d\,x^2\right)\left(b\,x^2-2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(((18*d)/b - d*x^2)*(b*x^2 - 2)^(1/3)),x)
```

```
[Out] int(-1/(((18*d)/b - d*x^2)*(b*x^2 - 2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{1}{bx^2\sqrt[3]{bx^2-2}-18\sqrt[3]{bx^2-2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-2)**(1/3)/(-18*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(b*x**2*(b*x**2 - 2)**(1/3) - 18*(b*x**2 - 2)**(1/3)), x)/d
```

3.126 $\int \frac{1}{\sqrt[3]{2+3x^2}\left(6d+dx^2\right)}\,dx$

Optimal. Leaf size=123

$$\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)}{x}\right)}{4\,2^{5/6}d}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}$$

Rubi [A] time = 0.02, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {394}

$$\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)}{x}\right)}{4\,2^{5/6}d}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTan[(2^(1/3) - (2 + 3*x^2)^(1/
3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/6)*(2^(1
/3) - (2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{2+3x^2}\left(6d+dx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}+\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)}{x}\right)}{4\,2^{5/6}d}$$

Mathematica [C] time = 0.12, size = 136, normalized size = 1.11

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\left(x^2+6\right)\sqrt[3]{3x^2+2}\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2])/(d*(6 + x^2)*(2 + 3
*x^2)^(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2] + x^2*(App
ellF1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -1/6*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2
, (-3*x^2)/2, -1/6*x^2])))
```

IntegrateAlgebraic [F] time = 4.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,\left(6d+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^2+6\,d\right)\left(3\,x^2+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="giac")
[Out] integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

maple [C] time = 58.10, size = 549, normalized size = 4.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x)
[Out] -1/24*(24*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*ln(-(192
*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^6
*x-4*RootOf(_Z^6+54)^7*x-288*(3*x^2+2)^(1/3)*RootOf(_Z^6+54)^4*RootOf(RootO
f(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x+6*(3*x^2+2)^(1/3)*RootOf(_Z^
6+54)^5*x+9*x^2*RootOf(_Z^6+54)^4-18*RootOf(_Z^6+54)^4-108*RootOf(_Z^6+54)^
2*(3*x^2+2)^(1/3)+324*(3*x^2+2)^(2/3))/(x^2+6))-RootOf(_Z^6+54)*ln(-(768*Ro
otOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*RootOf(_Z^6+54)^5*
x-16*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+5
4)^6*x+1152*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*Root
Of(_Z^6+54)^3*(3*x^2+2)^(1/3)*x-72*(3*x^2+2)^(1/3)*RootOf(_Z^6+54)^4*RootOf
(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x+(3*x^2+2)^(1/3)*RootOf
(_Z^6+54)^5*x-36*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*R
ootOf(_Z^6+54)^3*x^2+72*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*
_Z^2)*RootOf(_Z^6+54)^3-432*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+
576*_Z^2)*RootOf(_Z^6+54)*(3*x^2+2)^(1/3)+18*RootOf(_Z^6+54)^2*(3*x^2+2)^(1
/3)+54*(3*x^2+2)^(2/3))/(x^2+6)))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^2+6\,d\right)\left(3\,x^2+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(3\,x^2+2\right)^{1/3}\,\left(d\,x^2+6\,d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x^2 + 2)^(1/3)*(6*d + d*x^2)),x)
```

```
[Out] int(1/((3*x^2 + 2)^(1/3)*(6*d + d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2\sqrt[3]{3x^2+2}+6\sqrt[3]{3x^2+2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+2)**(1/3)/(d*x**2+6*d),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 + 2)**(1/3) + 6*(3*x**2 + 2)**(1/3)), x)/d
```

3.127 $\int \frac{1}{\sqrt[3]{2-3x^2}\left(-6d+dx^2\right)}\,dx$

Optimal. Leaf size=123

$$-\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\,2^{5/6}d}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}\,d}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}\,d}$$

Rubi [A] time = 0.02, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {395}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\,2^{5/6}d}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}\,d}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] -ArcTan[(2^(1/6)*(2^(1/3) - (2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d) - ArcTanh[
x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/3) - (2 - 3*x^2)^(1/3))^2/
(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{2-3x^2}\left(-6d+dx^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\,2^{5/6}d}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}\,d}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}\,d}$$

Mathematica [C] time = 0.13, size = 136, normalized size = 1.11

$$\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)}{d\sqrt[3]{2-3x^2}\left(x^2-6\right)\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] (9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(2 - 3*x^2)^(1/3)*(-6
 + x^2)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6] + x^2*(AppellF1[3/2
, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (3*x^2)/2,
x^2/6])))
```

IntegrateAlgebraic [F] time = 4.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{2-3x^2}\left(-6d+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^2-6\,d\right)\left(-3\,x^2+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

maple [C] time = 59.07, size = 1061, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x)
```

```
[Out] 1/24*(24*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*ln((-4608
*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)
^5*x+288*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z
^6-54)^6*x-4*RootOf(_Z^6-54)^7*x+6912*RootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-
54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*(-3*x^2+2)^(1/3)*x-144*RootOf(_Z^6-
54)^4*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*(-3*x^2+2)^(
1/3)*x-216*RootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)
+576*_Z^2)*x^2+9*x^2*RootOf(_Z^6-54)^4-432*RootOf(RootOf(_Z^6-54)^2-24*_Z*R
ootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3+18*RootOf(_Z^6-54)^4-2592*RootOf
(_Z^6-54)*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*(-3*x^2+
2)^(1/3)+324*(-3*x^2+2)^(2/3))/(x^2-6))-24*ln((-768*RootOf(RootOf(_Z^6-54)^
2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)^5*x+16*RootOf(RootOf(_Z
^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^6*x+1152*RootOf(_Z
^6-54)^3*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*(-3*x^2
+2)^(1/3)*x-72*RootOf(_Z^6-54)^4*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6
-54)+576*_Z^2)*(-3*x^2+2)^(1/3)*x+(-3*x^2+2)^(1/3)*RootOf(_Z^6-54)^5*x+36*R
ootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*x
^2+72*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-
54)^3+432*RootOf(_Z^6-54)*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+57
6*_Z^2)*(-3*x^2+2)^(1/3)-18*(-3*x^2+2)^(1/3)*RootOf(_Z^6-54)^2+54*(-3*x^2+2
)^(2/3))/(x^2-6))*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)+
ln((-768*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(
```

```
_Z^6-54)^5*x+16*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*Ro
otOf(_Z^6-54)^6*x+1152*RootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-54)^2-24*_Z*Roo
tOf(_Z^6-54)+576*_Z^2)^2*(-3*x^2+2)^(1/3)*x-72*RootOf(_Z^6-54)^4*RootOf(Roo
tOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*(-3*x^2+2)^(1/3)*x+(-3*x^2+2
)^(1/3)*RootOf(_Z^6-54)^5*x+36*RootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-54)^2-2
4*_Z*RootOf(_Z^6-54)+576*_Z^2)*x^2+72*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf
(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3+432*RootOf(_Z^6-54)*RootOf(RootOf(_Z^
6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*(-3*x^2+2)^(1/3)-18*(-3*x^2+2)^(1/3
)*RootOf(_Z^6-54)^2+54*(-3*x^2+2)^(2/3))/(x^2-6))*RootOf(_Z^6-54))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx^2 - 6\,d)(-3\,x^2 + 2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="maxima")
[Out] integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(2 - 3\,x^2\right)^{1/3}\,\left(6\,d - d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2 - 3*x^2)^(1/3)*(6*d - d*x^2)),x)
[Out] -int(1/((2 - 3*x^2)^(1/3)*(6*d - d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2\sqrt[3]{2-3x^2} - 6\sqrt[3]{2-3x^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/3)/(d*x**2-6*d),x)
[Out] Integral(1/(x**2*(2 - 3*x**2)**(1/3) - 6*(2 - 3*x**2)**(1/3)), x)/d
```

3.128 $\int \frac{1}{\sqrt[3]{-2+3x^2}\left(-6d+dx^2\right)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\,2^{5/6}d}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}$$

Rubi [A] time = 0.02, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {395}

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\,2^{5/6}d}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] ArcTan[(2^(1/6)*(2^(1/3) + (-2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d) + ArcTanh[
x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/3) + (-2 + 3*x^2)^(1/3))^2
/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\left(-6d+dx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)}{x}\right)}{4\,2^{5/6}d}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}$$

Mathematica [C] time = 0.10, size = 136, normalized size = 1.14

$$\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)}{d\left(x^2-6\right)\sqrt[3]{3x^2-2}\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] (9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(-6 + x^2)*(-2 + 3*x^
2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6] + x^2*(AppellF1[3/
2, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (3*x^2)/2,
 x^2/6])))
```

IntegrateAlgebraic [F] time = 4.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\left(-6d+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^2-6\,d\right)\left(3\,x^2-2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="giac")
[Out] integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

maple [C] time = 58.82, size = 1063, normalized size = 8.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x)
[Out] -1/24*(24*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*ln(-(460
8*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54
)^5*x-288*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_
Z^6-54)^6*x+4*RootOf(_Z^6-54)^7*x+6912*(3*x^2-2)^(1/3)*RootOf(_Z^6-54)^3*Ro
otOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*x-144*(3*x^2-2)^(1
/3)*RootOf(_Z^6-54)^4*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z
^2)*x+216*RootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+
576*_Z^2)*x^2-9*x^2*RootOf(_Z^6-54)^4+432*RootOf(RootOf(_Z^6-54)^2-24*_Z*Ro
otOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3-18*RootOf(_Z^6-54)^4-2592*(3*x^2-
2)^(1/3)*RootOf(_Z^6-54)*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576
*_Z^2)-324*(3*x^2-2)^(2/3))/(x^2-6))+24*ln((768*RootOf(RootOf(_Z^6-54)^2-24
*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)^5*x-16*RootOf(RootOf(_Z^6-5
4)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^6*x+1152*(3*x^2-2)^(1/
3)*RootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^
2)^2*x-72*(3*x^2-2)^(1/3)*RootOf(_Z^6-54)^4*RootOf(RootOf(_Z^6-54)^2-24*_Z*
RootOf(_Z^6-54)+576*_Z^2)*x+RootOf(_Z^6-54)^5*(3*x^2-2)^(1/3)*x+36*RootOf(_
Z^6-54)^3*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*x^2+72*R
ootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3-4
32*(3*x^2-2)^(1/3)*RootOf(_Z^6-54)*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z
^6-54)+576*_Z^2)+18*RootOf(_Z^6-54)^2*(3*x^2-2)^(1/3)+54*(3*x^2-2)^(2/3))/(
x^2-6))*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)-ln((768*Ro
otOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)^5*
```

```
x-16*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-5
4)^6*x+1152*(3*x^2-2)^(1/3)*RootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-54)^2-24*_
Z*RootOf(_Z^6-54)+576*_Z^2)^2*x-72*(3*x^2-2)^(1/3)*RootOf(_Z^6-54)^4*RootOf
(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*x+RootOf(_Z^6-54)^5*(3*x
^2-2)^(1/3)*x+36*RootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z
^6-54)+576*_Z^2)*x^2+72*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*
_Z^2)*RootOf(_Z^6-54)^3-432*(3*x^2-2)^(1/3)*RootOf(_Z^6-54)*RootOf(RootOf(_
Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)+18*RootOf(_Z^6-54)^2*(3*x^2-2)^(1
/3)+54*(3*x^2-2)^(2/3))/(x^2-6))*RootOf(_Z^6-54))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^2 - 6\,d\right)\left(3\,x^2 - 2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(3\,x^2 - 2\right)^{1/3}\,\left(6\,d - d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((3*x^2 - 2)^(1/3)*(6*d - d*x^2)),x)
```

```
[Out] -int(1/((3*x^2 - 2)^(1/3)*(6*d - d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2\,\sqrt[3]{3x^2-2} - 6\,\sqrt[3]{3x^2-2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)**(1/3)/(d*x**2-6*d),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 - 2)**(1/3) - 6*(3*x**2 - 2)**(1/3)), x)/d
```

3.129 $\int \frac{1}{\sqrt[3]{-2-3x^2}\left(6d+dx^2\right)}\,dx$

Optimal. Leaf size=119

$$-\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\,2^{5/6}d}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}$$

Rubi [A] time = 0.02, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {394}

$$-\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\,2^{5/6}d}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] -ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTan[(2^(1/3) + (-2 - 3*x^2)^(
1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/6)*(2^
(1/3) + (-2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\left(6d+dx^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)}{x}\right)}{4\,2^{5/6}d}$$

Mathematica [C] time = 0.11, size = 136, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\sqrt[3]{-3x^2-2}\left(x^2+6\right)\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2])/(d*(-2 - 3*x^2)^(1/
3)*(6 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2] + x^2*(Ap
pellF1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -1/6*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/
2, (-3*x^2)/2, -1/6*x^2])))
```

IntegrateAlgebraic [F] time = 4.63, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\left(6d+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^2+6\,d\right)\left(-3\,x^2-2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

maple [C] time = 77.11, size = 725, normalized size = 6.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x)
```

```
[Out] -1/24*(RootOf(_Z^6+54)*ln(-(4*RootOf(_Z^6+54)^7*x-288*RootOf(RootOf(_Z^6+54
)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^6*x+4608*RootOf(RootOf(
_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*RootOf(_Z^6+54)^5*x+144*RootOf
(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^4*(-3*x^
2-2)^(1/3)*x-6912*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)^
2*RootOf(_Z^6+54)^3*(-3*x^2-2)^(1/3)*x-9*x^2*RootOf(_Z^6+54)^4+216*RootOf(R
ootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3*x^2+18*R
ootOf(_Z^6+54)^4-432*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^
2)*RootOf(_Z^6+54)^3-2592*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+57
6*_Z^2)*RootOf(_Z^6+54)*(-3*x^2-2)^(1/3)+324*(-3*x^2-2)^(2/3))/(x^2+6))+ln(
-(-4*RootOf(_Z^6+54)^7*x+192*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)
+576*_Z^2)*RootOf(_Z^6+54)^6*x-6*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)^5*x+288*R
ootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^4*(
-3*x^2-2)^(1/3)*x+9*x^2*RootOf(_Z^6+54)^4-18*RootOf(_Z^6+54)^4+108*(-3*x^2-
2)^(1/3)*RootOf(_Z^6+54)^2+324*(-3*x^2-2)^(2/3))/(x^2+6))*RootOf(_Z^6+54)-2
4*ln(-(-4*RootOf(_Z^6+54)^7*x+192*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^
6+54)+576*_Z^2)*RootOf(_Z^6+54)^6*x-6*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)^5*x+
288*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54
)^4*(-3*x^2-2)^(1/3)*x+9*x^2*RootOf(_Z^6+54)^4-18*RootOf(_Z^6+54)^4+108*(-3
*x^2-2)^(1/3)*RootOf(_Z^6+54)^2+324*(-3*x^2-2)^(2/3))/(x^2+6))*RootOf(RootO
f(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^2+6\,d\right)\left(-3\,x^2-2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="maxima")
[Out] integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(-3\,x^2-2\right)^{1/3}\,\left(d\,x^2+6\,d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- 3*x^2 - 2)^(1/3)*(6*d + d*x^2)),x)
[Out] int(1/((- 3*x^2 - 2)^(1/3)*(6*d + d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2\sqrt[3]{-3x^2-2}+6\sqrt[3]{-3x^2-2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)**(1/3)/(d*x**2+6*d),x)
[Out] Integral(1/(x**2*(-3*x**2 - 2)**(1/3) + 6*(-3*x**2 - 2)**(1/3)), x)/d
```

3.130 $\int \frac{1}{\sqrt[3]{1+x^2}\left(9+x^2\right)}\,dx$

Optimal. Leaf size=70

$$\frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{x^2+1}\right)^2}{3x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{x^2+1}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {394}

$$\frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{x^2+1}\right)^2}{3x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{x^2+1}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] ArcTan[x/3]/12 + ArcTan[(1 - (1 + x^2)^(1/3))^2/(3*x)]/12 - ArcTanh[(Sqrt[3
]*(1 - (1 + x^2)^(1/3)))/x]/(4*Sqrt[3])
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1+x^2}\left(9+x^2\right)}\,dx=\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)+\frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+x^2}\right)^2}{3x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+x^2}\right)}{x}\right)}{4\sqrt{3}}$$

Mathematica [C] time = 0.10, size = 124, normalized size = 1.77

$$-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,-\frac{x^2}{9}\right)}{\sqrt[3]{x^2+1}\left(x^2+9\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,-\frac{x^2}{9}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,-\frac{x^2}{9}\right)\right)-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,-\frac{x^2}{9}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] (-27*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -1/9*x^2])/((1 + x^2)^(1/3)*(9 + x^
2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -1/9*x^2] + 2*x^2*(AppellF1[3/2, 1
/3, 2, 5/2, -x^2, -1/9*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -x^2, -1/9*x^2])
))
```

IntegrateAlgebraic [F] time = 3.90, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{1+x^2}\left(9+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((1 + x^2)^(1/3)*(9 + x^2)), x]
```

fricas [B] time = 8.02, size = 1395, normalized size = 19.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="fricas")
```

```
[Out] 1/144*sqrt(3)*log(4*(x^6 + 1647*x^4 + 891*x^2 + 18*(3*x^4 + 32*sqrt(3)*x^3
+ 126*x^2 + 27)*(x^2 + 1)^(2/3) + 108*sqrt(3)*(x^5 + 10*x^3 + 9*x) + 6*(81*
x^4 + 162*x^2 + sqrt(3)*(x^5 + 210*x^3 + 81*x) + 81)*(x^2 + 1)^(1/3) - 243)
/(x^6 + 27*x^4 + 243*x^2 + 729)) - 1/144*sqrt(3)*log(4*(x^6 + 1647*x^4 + 89
1*x^2 + 18*(3*x^4 - 32*sqrt(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) - 108*sq
rt(3)*(x^5 + 10*x^3 + 9*x) + 6*(81*x^4 + 162*x^2 - sqrt(3)*(x^5 + 210*x^3 +
 81*x) + 81)*(x^2 + 1)^(1/3) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) - 1/36*
arctan((384*x^11 - 130320*x^9 + 2379456*x^7 - 629856*x^5 - 1259712*x^3 + 36
*(388*x^9 - 27864*x^7 + 303264*x^5 + 17496*x^3 + sqrt(3)*(x^10 + 549*x^8 -
8046*x^6 + 129762*x^4 - 19683*x^2 + 59049) - 236196*x)*(x^2 + 1)^(2/3) + sq
rt(3)*(x^12 - 234*x^10 + 229311*x^8 - 1214028*x^6 + 6816879*x^4 + 6022998*x
^2 + 531441) + 2*(x^12 + 50616*x^10 - 1869399*x^8 - 3773304*x^6 - 6908733*x
^4 + 72*(x^10 + 1620*x^8 - 63666*x^6 - 43740*x^4 + 59049*x^2 + 12*sqrt(3)*(
11*x^9 - 261*x^7 - 6075*x^5 - 2187*x^3))*(x^2 + 1)^(2/3) + 6*sqrt(3)*(43*x^
11 + 14055*x^9 - 563922*x^7 - 1307826*x^5 - 898857*x^3 + 177147*x) + 6*(453
*x^10 + 21141*x^8 - 1483758*x^6 - 1404054*x^4 - 885735*x^2 + sqrt(3)*(x^11
+ 8985*x^9 - 349110*x^7 + 118098*x^5 + 32805*x^3 - 177147*x) + 531441)*(x^2
 + 1)^(1/3) + 1594323)*sqrt((x^6 + 1647*x^4 + 891*x^2 + 18*(3*x^4 - 32*sqrt
(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) - 108*sqrt(3)*(x^5 + 10*x^3 + 9*x)
+ 6*(81*x^4 + 162*x^2 - sqrt(3)*(x^5 + 210*x^3 + 81*x) + 81)*(x^2 + 1)^(1/3
) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) + 12*(x^11 - 6423*x^9 + 225018*x^7
 - 1106622*x^5 - 1541835*x^3 + 3*sqrt(3)*(37*x^10 - 675*x^8 + 34722*x^6 - 9
7686*x^4 + 59049*x^2 + 59049) - 177147*x)*(x^2 + 1)^(1/3) - 8503056*x)/(x^1
2 - 48978*x^10 + 2332071*x^8 - 16419996*x^6 - 24151041*x^4 - 9565938*x^2 +
4782969)) + 1/36*arctan(-(384*x^11 - 130320*x^9 + 2379456*x^7 - 629856*x^5
- 1259712*x^3 + 36*(388*x^9 - 27864*x^7 + 303264*x^5 + 17496*x^3 - sqrt(3)*
(x^10 + 549*x^8 - 8046*x^6 + 129762*x^4 - 19683*x^2 + 59049) - 236196*x)*(x
^2 + 1)^(2/3) - sqrt(3)*(x^12 - 234*x^10 + 229311*x^8 - 1214028*x^6 + 68168
79*x^4 + 6022998*x^2 + 531441) + 2*(x^12 + 50616*x^10 - 1869399*x^8 - 37733
04*x^6 - 6908733*x^4 + 72*(x^10 + 1620*x^8 - 63666*x^6 - 43740*x^4 + 59049*
x^2 - 12*sqrt(3)*(11*x^9 - 261*x^7 - 6075*x^5 - 2187*x^3))*(x^2 + 1)^(2/3)
- 6*sqrt(3)*(43*x^11 + 14055*x^9 - 563922*x^7 - 1307826*x^5 - 898857*x^3 +
177147*x) + 6*(453*x^10 + 21141*x^8 - 1483758*x^6 - 1404054*x^4 - 885735*x^
2 - sqrt(3)*(x^11 + 8985*x^9 - 349110*x^7 + 118098*x^5 + 32805*x^3 - 177147
*x) + 531441)*(x^2 + 1)^(1/3) + 1594323)*sqrt((x^6 + 1647*x^4 + 891*x^2 + 1
8*(3*x^4 + 32*sqrt(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) + 108*sqrt(3)*(x^
5 + 10*x^3 + 9*x) + 6*(81*x^4 + 162*x^2 + sqrt(3)*(x^5 + 210*x^3 + 81*x) +
81)*(x^2 + 1)^(1/3) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) + 12*(x^11 - 642
3*x^9 + 225018*x^7 - 1106622*x^5 - 1541835*x^3 - 3*sqrt(3)*(37*x^10 - 675*x
^8 + 34722*x^6 - 97686*x^4 + 59049*x^2 + 59049) - 177147*x)*(x^2 + 1)^(1/3)
 - 8503056*x)/(x^12 - 48978*x^10 + 2332071*x^8 - 16419996*x^6 - 24151041*x^
4 - 9565938*x^2 + 4782969)) - 1/36*arctan(6*(11*x^5 + 30*x^3 + 6*(23*x^3 +
```

```
27*x)*(x^2 + 1)^(2/3) + (x^5 - 240*x^3 - 81*x)*(x^2 + 1)^(1/3) - 81*x)/(x^6
 - 1971*x^4 - 1701*x^2 - 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+9\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

maple [C] time = 10.59, size = 512, normalized size = 7.31

$$-\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)\ln\left(\frac{-x^2+48\left(x^2+1\right)^{\frac{1}{3}}x\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)+96x\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)+2\left(x^2+1\right)^{\frac{1}{3}}x\operatorname{RootOf}\left(\_Z^2+1\right)+4x\operatorname{RootOf}\left(\_Z^2+1\right)-6\left(x^2+1\right)^{\frac{2}{3}}-6\left(x^2+1\right)^{\frac{1}{3}}+3}{x^2+9}\right)-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-x^2+48\left(x^2+1\right)^{\frac{1}{3}}x\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)+96x\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)+2\left(x^2+1\right)^{\frac{1}{3}}x\operatorname{RootOf}\left(\_Z^2+1\right)+4x\operatorname{RootOf}\left(\_Z^2+1\right)-6\left(x^2+1\right)^{\frac{2}{3}}-6\left(x^2+1\right)^{\frac{1}{3}}+3}{x^2+9}\right)}{12}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-12x^2\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)\operatorname{RootOf}\left(\_Z^2+1\right)+576\left(x^2+1\right)^{\frac{1}{3}}x\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)^2\operatorname{RootOf}\left(\_Z^2+1\right)-1152x\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)^2\operatorname{RootOf}\left(\_Z^2+1\right)+24\left(x^2+1\right)^{\frac{1}{3}}x\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)\operatorname{RootOf}\left(\_Z^2+1\right)^2-48x\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)\operatorname{RootOf}\left(\_Z^2+1\right)^2+x^2+96x\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)+4x\operatorname{RootOf}\left(\_Z^2+1\right)+72\left(x^2+1\right)^{\frac{1}{3}}\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)\operatorname{RootOf}\left(\_Z^2+1\right)+36\operatorname{RootOf}\left(144\_Z^2+12\_Z\operatorname{RootOf}\left(\_Z^2+1\right)-1\right)\operatorname{RootOf}\left(\_Z^2+1\right)-6\left(x^2+1\right)^{\frac{2}{3}}-3}{x^2+9}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/3)/(x^2+9),x)
```

```
[Out] 1/12*RootOf(_Z^2+1)*ln((24*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)*RootOf(_
Z^2+1)^2*(x^2+1)^(1/3)*x+576*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)^2*Root
Of(_Z^2+1)*(x^2+1)^(1/3)*x-48*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)*RootO
f(_Z^2+1)^2*x-1152*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)^2*RootOf(_Z^2+1)
*x-12*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)*RootOf(_Z^2+1)*x^2+72*(x^2+1)
^(1/3)*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)*RootOf(_Z^2+1)+36*RootOf(12*
_Z*RootOf(_Z^2+1)+144*_Z^2-1)*RootOf(_Z^2+1)+4*RootOf(_Z^2+1)*x+96*RootOf(1
2*_Z*RootOf(_Z^2+1)+144*_Z^2-1)*x-6*(x^2+1)^(2/3)+x^2-3)/(x^2+9))-ln((2*(x^
2+1)^(1/3)*RootOf(_Z^2+1)*x+48*(x^2+1)^(1/3)*RootOf(12*_Z*RootOf(_Z^2+1)+14
4*_Z^2-1)*x+4*RootOf(_Z^2+1)*x+96*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)*x
-6*(x^2+1)^(2/3)-x^2-6*(x^2+1)^(1/3)+3)/(x^2+9))*RootOf(12*_Z*RootOf(_Z^2+1
)+144*_Z^2-1)-1/12*ln((2*(x^2+1)^(1/3)*RootOf(_Z^2+1)*x+48*(x^2+1)^(1/3)*Ro
otOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)*x+4*RootOf(_Z^2+1)*x+96*RootOf(12*_Z*
RootOf(_Z^2+1)+144*_Z^2-1)*x-6*(x^2+1)^(2/3)-x^2-6*(x^2+1)^(1/3)+3)/(x^2+9)
)*RootOf(_Z^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+9\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^2+1\right)}^{1/3}\,\left(x^2+9\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/3)*(x^2 + 9)),x)
```

```
[Out] int(1/((x^2 + 1)^(1/3)*(x^2 + 9)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^2+1}\,\left(x^2+9\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/3)/(x**2+9),x)
```

```
[Out] Integral(1/((x**2 + 1)**(1/3)*(x**2 + 9)), x)
```

3.131 $\int \frac{1}{\sqrt[3]{1+bx^2}\left(9+bx^2\right)}\,dx$

Optimal. Leaf size=104

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{bx^2+1}\right)^2}{3\sqrt{b}\,x}\right)}{12\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{bx^2+1}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{12\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {394}

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{bx^2+1}\right)^2}{3\sqrt{b}\,x}\right)}{12\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{bx^2+1}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{12\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/3]/(12*Sqrt[b]) + ArcTan[(1 - (1 + b*x^2)^(1/3))^2/(3*Sq
rt[b]*x)]/(12*Sqrt[b]) - ArcTanh[(Sqrt[3]*(1 - (1 + b*x^2)^(1/3)))/(Sqrt[b]
*x)]/(4*Sqrt[3]*Sqrt[b])
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1+bx^2}\left(9+bx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{12\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+bx^2}\right)^2}{3\sqrt{b}\,x}\right)}{12\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+bx^2}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,\sqrt{b}}$$

Mathematica [C] time = 0.11, size = 137, normalized size = 1.32

$$-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-bx^2,-\frac{bx^2}{9}\right)}{\sqrt[3]{bx^2+1}\left(bx^2+9\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-bx^2,-\frac{bx^2}{9}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-bx^2,-\frac{bx^2}{9}\right)\right)-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-bx^2,-\frac{bx^2}{9}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```

```
[Out] (-27*x*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2), -1/9*(b*x^2)])/((1 + b*x^2)^(1/
3)*(9 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2), -1/9*(b*x^2)] + 2*
b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -(b*x^2), -1/9*(b*x^2)] + 3*AppellF1[3/2,
 4/3, 1, 5/2, -(b*x^2), -1/9*(b*x^2)])))
```

IntegrateAlgebraic [F] time = 4.40, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((1 + b*x^2)^(1/3)*(9 + b*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^2+9)(bx^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+1)^{\frac{1}{3}}\,(b\,x^2+9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
[Out] int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^2+9)(bx^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\,x^2+1)^{1/3}\,(b\,x^2+9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2 + 1)^(1/3)*(b*x^2 + 9)),x)
[Out] int(1/((b*x^2 + 1)^(1/3)*(b*x^2 + 9)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{bx^2+1}\,(bx^2+9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+1)**(1/3)/(b*x**2+9),x)
[Out] Integral(1/((b*x**2 + 1)**(1/3)*(b*x**2 + 9)), x)
```

3.132 $\int \frac{1}{\sqrt[3]{1-x^2}\left(9-x^2\right)}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}-\frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)+\frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {395}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}-\frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)+\frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(1 - (1 - x^2)^(1/3)))/x]/(4*Sqrt[3]) + ArcTanh[x/3]/12 - A
rcTanh[(1 - (1 - x^2)^(1/3))^2/(3*x)]/12
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(9-x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right)-\frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)$$

Mathematica [C] time = 0.05, size = 125, normalized size = 1.69

$$\frac{\sqrt[3]{\frac{x-1}{x-3}}\sqrt[3]{\frac{x+1}{x-3}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{4}{x-3},-\frac{2}{x-3}\right)-\sqrt[3]{\frac{x-1}{x+3}}\sqrt[3]{\frac{x+1}{x+3}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{2}{x+3},\frac{4}{x+3}\right)}{4\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

```
[Out] (((-1 + x)/(-3 + x))^(1/3)*((1 + x)/(-3 + x))^(1/3)*AppellF1[2/3, 1/3, 1/3,
 5/3, -4/(-3 + x), -2/(-3 + x)] - ((-1 + x)/(3 + x))^(1/3)*((1 + x)/(3 + x)
)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, 2/(3 + x), 4/(3 + x)])/(4*(1 - x^2)^(1
/3))
```

IntegrateAlgebraic [F] time = 8.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{1-x^2}\,\left(9-x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((1 - x^2)^(1/3)*(9 - x^2)), x]
```

fricas [B] time = 3.72, size = 269, normalized size = 3.64

$$-\frac{1}{36}\,\sqrt{3}\arctan\left(\frac{36\,\sqrt{3}\left(x^4-32\,x^3-42\,x^2+9\right)\left(-x^2+1\right)^{\frac{2}{3}}+12\,\sqrt{3}\left(x^5+27\,x^4-210\,x^3-54\,x^2+81\,x+27\right)\left(-x^2+1\right)^{\frac{1}{3}}+\sqrt{3}\left(x^6-108\,x^5-567\,x^4+1080\,x^3+459\,x^2-972\,x-405\right)}{3\left(x^6+108\,x^5-1647\,x^4-1080\,x^3+891\,x^2+972\,x+243\right)}\right)-\frac{1}{72}\log\left(\frac{x^3+33\,x^2+18\left(-x^2+1\right)^{\frac{2}{3}}(x+1)-6\left(x^2+6\,x-3\right)\left(-x^2+1\right)^{\frac{1}{3}}-9\,x-9}{x^3+9\,x^2+27\,x+27}\right)+\frac{1}{36}\log\left(-\frac{x^3-33\,x^2+18\left(-x^2+1\right)^{\frac{2}{3}}(x-1)+6\left(x^2-6\,x-3\right)\left(-x^2+1\right)^{\frac{1}{3}}-9\,x+9}{x^3+9\,x^2+27\,x+27}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*arctan(1/3*(36*sqrt(3)*(x^4 - 32*x^3 - 42*x^2 + 9)*(-x^2 + 1)
^(2/3) + 12*sqrt(3)*(x^5 + 27*x^4 - 210*x^3 - 54*x^2 + 81*x + 27)*(-x^2 + 1
)^(1/3) + sqrt(3)*(x^6 - 108*x^5 - 567*x^4 + 1080*x^3 + 459*x^2 - 972*x - 4
05))/(x^6 + 108*x^5 - 1647*x^4 - 1080*x^3 + 891*x^2 + 972*x + 243)) - 1/72*
log((x^3 + 33*x^2 + 18*(-x^2 + 1)^(2/3)*(x + 1) - 6*(x^2 + 6*x - 3)*(-x^2 +
 1)^(1/3) - 9*x - 9)/(x^3 + 9*x^2 + 27*x + 27)) + 1/36*log(-(x^3 - 33*x^2 +
 18*(-x^2 + 1)^(2/3)*(x - 1) + 6*(x^2 - 6*x - 3)*(-x^2 + 1)^(1/3) - 9*x + 9
)/(x^3 + 9*x^2 + 27*x + 27))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(x^2-9\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

maple [C] time = 2.61, size = 539, normalized size = 7.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(-x^2+9),x)
```

```
[Out] -1/12*ln((288*RootOf(144*_Z^2+12*_Z+1)^2*(-x^2+1)^(1/3)*x-576*RootOf(144*_Z
^2+12*_Z+1)^2*x+36*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)*x-6*RootOf(144*_
Z^2+12*_Z+1)*x^2-36*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)-24*RootOf(144*_
Z^2+12*_Z+1)*x+3*(-x^2+1)^(2/3)+(-x^2+1)^(1/3)*x-18*RootOf(144*_Z^2+12*_Z+1
)-3*(-x^2+1)^(1/3))/(x-3)/(x+3))-ln((288*RootOf(144*_Z^2+12*_Z+1)^2*(-x^2+1
)^(1/3)*x-576*RootOf(144*_Z^2+12*_Z+1)^2*x+36*RootOf(144*_Z^2+12*_Z+1)*(-x^
2+1)^(1/3)*x-6*RootOf(144*_Z^2+12*_Z+1)*x^2-36*RootOf(144*_Z^2+12*_Z+1)*(-x
^2+1)^(1/3)-24*RootOf(144*_Z^2+12*_Z+1)*x+3*(-x^2+1)^(2/3)+(-x^2+1)^(1/3)*x
-18*RootOf(144*_Z^2+12*_Z+1)-3*(-x^2+1)^(1/3))/(x-3)/(x+3))*RootOf(144*_Z^2
+12*_Z+1)+RootOf(144*_Z^2+12*_Z+1)*ln((576*RootOf(144*_Z^2+12*_Z+1)^2*(-x^2
+1)^(1/3)*x-1152*RootOf(144*_Z^2+12*_Z+1)^2*x+24*RootOf(144*_Z^2+12*_Z+1)*(
-x^2+1)^(1/3)*x+12*RootOf(144*_Z^2+12*_Z+1)*x^2+72*RootOf(144*_Z^2+12*_Z+1)
*(-x^2+1)^(1/3)-144*RootOf(144*_Z^2+12*_Z+1)*x+6*(-x^2+1)^(2/3)+x^2+36*Root
Of(144*_Z^2+12*_Z+1)-4*x+3)/(x-3)/(x+3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(x^2 - 9\right)\left(-x^2 + 1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(1 - x^2\right)^{1/3}\left(x^2 - 9\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((1 - x^2)^(1/3)*(x^2 - 9)),x)
```

```
[Out] -int(1/((1 - x^2)^(1/3)*(x^2 - 9)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2 \sqrt[3]{1 - x^2} - 9\sqrt[3]{1 - x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(-x**2+9),x)
```

```
[Out] -Integral(1/(x**2*(1 - x**2)**(1/3) - 9*(1 - x**2)**(1/3)), x)
```

3.133 $\int \frac{\sqrt{-1+c^2x^2}}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{c^2x^2-1}\,\tanh^{-1}(cx)}{2cd^2\sqrt{d-c^2dx^2}}+\frac{x\sqrt{c^2x^2-1}}{2d\left(d-c^2dx^2\right)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {23, 199, 208}

$$\frac{x\sqrt{c^2x^2-1}}{2d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{\sqrt{c^2x^2-1}\,\tanh^{-1}(cx)}{2cd^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + c^2*x^2]/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[-1 + c^2*x^2])/(2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1
 + c^2*x^2]*ArcTanh[c*x])/(2*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+c^2x^2}}{\left(d-c^2dx^2\right)^{5/2}}\,dx &= \frac{\sqrt{-1+c^2x^2}\int \frac{1}{\left(d-c^2dx^2\right)^2}\,dx}{\sqrt{d-c^2dx^2}}\\
&= \frac{x\sqrt{-1+c^2x^2}}{2d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{\sqrt{-1+c^2x^2}\int \frac{1}{d-c^2dx^2}\,dx}{2d\sqrt{d-c^2dx^2}}\\
&= \frac{x\sqrt{-1+c^2x^2}}{2d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{\sqrt{-1+c^2x^2}\,\tanh^{-1}(cx)}{2cd^2\sqrt{d-c^2dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.72

$$\frac{\left(c^2x^2-1\right)\tanh^{-1}(cx)-cx}{2cd^2\sqrt{c^2x^2-1}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + c^2*x^2]/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (-(c*x) + (-1 + c^2*x^2)*ArcTanh[c*x])/(2*c*d^2*Sqrt[-1 + c^2*x^2]*Sqrt[d -
 c^2*d*x^2])
```

IntegrateAlgebraic [C] time = 0.11, size = 79, normalized size = 1.00

$$\frac{id^{5/2}\left(c^2x^2-1\right)^{5/2}\left(\frac{ix}{2d^{5/2}\left(c^2x^2-1\right)}-\frac{i\tanh^{-1}(cx)}{2cd^{5/2}}\right)}{\left(d-c^2dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + c^2*x^2]/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (I*d^(5/2)*(-1 + c^2*x^2)^(5/2)*(((I/2)*x)/(d^(5/2)*(-1 + c^2*x^2)) - ((I/2
)*ArcTanh[c*x])/(c*d^(5/2))))/(d - c^2*d*x^2)^(5/2)
```

fricas [A] time = 2.25, size = 315, normalized size = 3.99

$$\left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,cx-\left(c^4x^4-2c^2x^2+1\right)\sqrt{-d}\log\left(-\frac{c^6dx^6+5c^4dx^4-5c^2dx^2-4\left(c^3x^3+cx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\sqrt{-d}-d}{c^6x^6-3c^4x^4+3c^2x^2-1}\right)}{8\left(c^5d^3x^4-2c^3d^3x^2+cd^3\right)},\ \frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,cx-\left(c^4x^4-2c^2x^2+1\right)\sqrt{d}\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,c\sqrt{d}x}{c^4dx^4-d}\right)}{4\left(c^5d^3x^4-2c^3d^3x^2+cd^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 +
 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x
)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4
 + 3*c^2*x^2 - 1)))/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3), 1/4*(2*sqrt(-c^2
*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arcta
n(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/(c
^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2-1}}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 - 1)/(-c^2*d*x^2 + d)^(5/2), x)
```

maple [A] time = 0.03, size = 94, normalized size = 1.19

$$-\frac{\sqrt{-\left(c^2x^2-1\right)d}\,\left(-c^2x^2\ln\left(cx-1\right)+c^2x^2\ln\left(cx+1\right)-2cx+\ln\left(cx-1\right)-\ln\left(cx+1\right)\right)}{4\sqrt{c^2x^2-1}\,\left(cx+1\right)\left(cx-1\right)c\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x+1)*x^2*c^2-ln(c*x-1)*
x^2*c^2-2*c*x-ln(c*x+1)+ln(c*x-1))/d^3/c/(c*x+1)/(c*x-1)
```

maxima [A] time = 1.49, size = 70, normalized size = 0.89

$$-\frac{x}{2\left(c^2\sqrt{-d}\,d^2x^2-\sqrt{-d}\,d^2\right)}-\frac{\sqrt{-d}\,\log\left(cx+1\right)}{4\,cd^3}+\frac{\sqrt{-d}\,\log\left(cx-1\right)}{4\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2*x/(c^2*sqrt(-d)*d^2*x^2 - sqrt(-d)*d^2) - 1/4*sqrt(-d)*log(c*x + 1)/(c
*d^3) + 1/4*sqrt(-d)*log(c*x - 1)/(c*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c^2\,x^2-1}}{{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 - 1)^(1/2)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((c^2*x^2 - 1)^(1/2)/(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx-1\right)\left(cx+1\right)}}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2-1)**(1/2)/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(sqrt((c*x - 1)*(c*x + 1))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.134 $\int \frac{1}{\left(-1+c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=74

$$\frac{dx\sqrt{c^2x^2-1}}{2\left(d-c^2dx^2\right)^{3/2}}+\frac{\sqrt{c^2x^2-1}\tanh^{-1}(cx)}{2c\sqrt{d-c^2dx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {23, 199, 208}

$$\frac{x\left(d-c^2dx^2\right)^{3/2}}{2d^2\left(1-c^2x^2\right)\left(c^2x^2-1\right)^{3/2}}+\frac{\left(d-c^2dx^2\right)^{3/2}\tanh^{-1}(cx)}{2cd^2\left(c^2x^2-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (x*(d - c^2*d*x^2)^(3/2))/(2*d^2*(1 - c^2*x^2)*(-1 + c^2*x^2)^(3/2)) + ((d
- c^2*d*x^2)^(3/2)*ArcTanh[c*x])/(2*c*d^2*(-1 + c^2*x^2)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1+c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(d-c^2dx^2\right)^{3/2}\int \frac{1}{\left(d-c^2dx^2\right)^2}\,dx}{\left(-1+c^2x^2\right)^{3/2}}\\ &= \frac{x\left(d-c^2dx^2\right)^{3/2}}{2d^2\left(1-c^2x^2\right)\left(-1+c^2x^2\right)^{3/2}}+\frac{\left(d-c^2dx^2\right)^{3/2}\int \frac{1}{d-c^2dx^2}\,dx}{2d\left(-1+c^2x^2\right)^{3/2}}\\ &= \frac{x\left(d-c^2dx^2\right)^{3/2}}{2d^2\left(1-c^2x^2\right)\left(-1+c^2x^2\right)^{3/2}}+\frac{\left(d-c^2dx^2\right)^{3/2}\tanh^{-1}(cx)}{2cd^2\left(-1+c^2x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.73

$$\frac{\left(c^2x^2-1\right)\tanh^{-1}(cx)-cx}{2c\sqrt{c^2x^2-1}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (-(c*x) + (-1 + c^2*x^2)*ArcTanh[c*x])/(2*c*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2
*d*x^2])
```

IntegrateAlgebraic [C] time = 0.08, size = 79, normalized size = 1.07

$$\frac{i\sqrt{d}\sqrt{c^2x^2-1}\left(\frac{ix}{2\sqrt{d}\left(c^2x^2-1\right)}-\frac{i\tanh^{-1}(cx)}{2c\sqrt{d}}\right)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (I*Sqrt[d]*Sqrt[-1 + c^2*x^2]*(((I/2)*x)/(Sqrt[d]*(-1 + c^2*x^2)) - ((I/2)*
ArcTanh[c*x])/(c*Sqrt[d])))/Sqrt[d - c^2*d*x^2]
```

fricas [A] time = 1.61, size = 303, normalized size = 4.09

$$\left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,cx-\left(c^4x^4-2c^2x^2+1\right)\sqrt{-d}\log\left(-\frac{c^6dx^6+5c^4dx^4-5c^2dx^2-4\left(c^3x^3+cx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\sqrt{-d}-d}{c^6x^6-3c^4x^4+3c^2x^2-1}\right)}{8\left(c^5dx^4-2c^3dx^2+cd\right)},\ \frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,cx-\left(c^4x^4-2c^2x^2+1\right)\sqrt{d}\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,c\sqrt{d}x}{c^4dx^4-d}\right)}{4\left(c^5dx^4-2c^3dx^2+cd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 +
 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x
)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4
 + 3*c^2*x^2 - 1)))/(c^5*d*x^4 - 2*c^3*d*x^2 + c*d), 1/4*(2*sqrt(-c^2*d*x^2
 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arctan(2*sq
rt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/(c^5*d*x
^4 - 2*c^3*d*x^2 + c*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c^2dx^2+d}\left(c^2x^2-1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2 + d)*(c^2*x^2 - 1)^(3/2)), x)
```

maple [A] time = 0.02, size = 94, normalized size = 1.27

$$-\frac{\sqrt{-\left(c^2x^2-1\right)d}\,\left(-c^2x^2\ln\left(cx-1\right)+c^2x^2\ln\left(cx+1\right)-2cx+\ln\left(cx-1\right)-\ln\left(cx+1\right)\right)}{4\sqrt{c^2x^2-1}\,\left(cx+1\right)\left(cx-1\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(-c^2*x^2*ln(c*x-1)+c^2*x^2*l
n(c*x+1)-2*c*x+ln(c*x-1)-ln(c*x+1))/d/c/(c*x+1)/(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c^2dx^2 + d}\left(c^2x^2 - 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2 + d)*(c^2*x^2 - 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d - c^2\,d\,x^2}\,\left(c^2\,x^2 - 1\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d - c^2*d*x^2)^(1/2)*(c^2*x^2 - 1)^(3/2)),x)
```

```
[Out] int(1/((d - c^2*d*x^2)^(1/2)*(c^2*x^2 - 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}\sqrt{-d\left(cx - 1\right)\left(cx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2-1)**(3/2)/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(((c*x - 1)*(c*x + 1))**(3/2)*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.135 $$\int \frac{1}{\sqrt{-1+c^2x^2}\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Optimal. Leaf size=76

$$-\frac{x\sqrt{c^2x^2-1}}{2\left(d-c^2dx^2\right)^{3/2}}-\frac{\sqrt{c^2x^2-1}\,\tanh^{-1}(cx)}{2cd\sqrt{d-c^2dx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {23, 199, 208}

$$\frac{x\sqrt{d-c^2dx^2}}{2d^2\left(1-c^2x^2\right)\sqrt{c^2x^2-1}}+\frac{\sqrt{d-c^2dx^2}\,\tanh^{-1}(cx)}{2cd^2\sqrt{c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + c^2*x^2]*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (x*Sqrt[d - c^2*d*x^2])/(2*d^2*(1 - c^2*x^2)*Sqrt[-1 + c^2*x^2]) + (Sqrt[d
- c^2*d*x^2]*ArcTanh[c*x])/(2*c*d^2*Sqrt[-1 + c^2*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+c^2x^2}\left(d-c^2dx^2\right)^{3/2}}\,dx &= \frac{\sqrt{d-c^2dx^2}\int \frac{1}{\left(d-c^2dx^2\right)^2}\,dx}{\sqrt{-1+c^2x^2}}\\
&= \frac{x\sqrt{d-c^2dx^2}}{2d^2\left(1-c^2x^2\right)\sqrt{-1+c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{1}{d-c^2dx^2}\,dx}{2d\sqrt{-1+c^2x^2}}\\
&= \frac{x\sqrt{d-c^2dx^2}}{2d^2\left(1-c^2x^2\right)\sqrt{-1+c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\,\tanh^{-1}(cx)}{2cd^2\sqrt{-1+c^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.75

$$\frac{\left(1-c^2x^2\right)\tanh^{-1}(cx)+cx}{2cd\sqrt{c^2x^2-1}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + c^2*x^2]*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (c*x + (1 - c^2*x^2)*ArcTanh[c*x])/(2*c*d*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d
*x^2])
```

IntegrateAlgebraic [C] time = 0.10, size = 79, normalized size = 1.04

$$-\frac{id^{3/2}\left(c^2x^2-1\right)^{3/2}\left(\frac{i\tanh^{-1}(cx)}{2cd^{3/2}}-\frac{ix}{2d^{3/2}\left(c^2x^2-1\right)}\right)}{\left(d-c^2dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + c^2*x^2]*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] ((-I)*d^(3/2)*(-1 + c^2*x^2)^(3/2)*(((-1/2*I)*x)/(d^(3/2)*(-1 + c^2*x^2)) +
 ((I/2)*ArcTanh[c*x])/(c*d^(3/2))))/(d - c^2*d*x^2)^(3/2)
```

fricas [A] time = 1.76, size = 314, normalized size = 4.13

$$\left[-\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,cx+\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2+4\left(c^3x^3+cx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)}{8\left(c^5d^2x^4-2\,c^3d^2x^2+cd^2\right)},\ -\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,cx-\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{d}\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)}{4\left(c^5d^2x^4-2\,c^3d^2x^2+cd^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x + (c^4*x^4 - 2*c^2*x^2
+ 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*
x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^
4 + 3*c^2*x^2 - 1)))/(c^5*d^2*x^4 - 2*c^3*d^2*x^2 + c*d^2), -1/4*(2*sqrt(-c
^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arc
tan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/
(c^5*d^2*x^4 - 2*c^3*d^2*x^2 + c*d^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\sqrt{c^2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*d*x^2 + d)^(3/2)*sqrt(c^2*x^2 - 1)), x)
```

maple [A] time = 0.02, size = 94, normalized size = 1.24

$$\frac{\sqrt{-\left(c^2x^2-1\right)d}\,\left(-c^2x^2\ln\left(cx-1\right)+c^2x^2\ln\left(cx+1\right)-2cx+\ln\left(cx-1\right)-\ln\left(cx+1\right)\right)}{4\sqrt{c^2x^2-1}\,\left(cx+1\right)\left(cx-1\right)c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(-c^2*x^2*ln(c*x-1)+c^2*x^2*ln
(c*x+1)-2*c*x+ln(c*x-1)-ln(c*x+1))/d^2/c/(c*x+1)/(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\sqrt{c^2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-c^2*d*x^2 + d)^(3/2)*sqrt(c^2*x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(d-c^2\,d\,x^2\right)}^{3/2}\,\sqrt{c^2\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d - c^2*d*x^2)^(3/2)*(c^2*x^2 - 1)^(1/2)),x)
```

```
[Out] int(1/((d - c^2*d*x^2)^(3/2)*(c^2*x^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(cx-1\right)\left(cx+1\right)}\,\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2-1)**(1/2)/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(1/(sqrt((c*x - 1)*(c*x + 1))*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

3.136 $\int \sqrt{2+4x^2}\,\sqrt{3+6x^2}\,dx$

Optimal. Leaf size=20

$$2\sqrt{\frac{2}{3}}x^3+\sqrt{6}x$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {22}

$$2\sqrt{\frac{2}{3}}x^3+\sqrt{6}x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
```

```
[Out] Sqrt[6]*x + 2*Sqrt[2/3]*x^3
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] && !(IntegerQ[m] || IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+4x^2}\,\sqrt{3+6x^2}\,dx&=\sqrt{\frac{2}{3}}\int \left(3+6x^2\right)\,dx\\&=\sqrt{6}x+2\sqrt{\frac{2}{3}}x^3\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.75

$$\sqrt{6}\left(\frac{2x^3}{3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
```

```
[Out] Sqrt[6]*(x + (2*x^3)/3)
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \sqrt{2+4x^2}\,\sqrt{3+6x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2], x]
```

fricas [B] time = 0.91, size = 38, normalized size = 1.90

$$\frac{\left(2\,x^3+3\,x\right)\sqrt{6\,x^2+3}\,\sqrt{4\,x^2+2}}{3\,\left(2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*x^3 + 3*x)*sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2)/(2*x^2 + 1)
```

giac [A] time = 0.57, size = 17, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\sqrt{2}\left(2x^3 + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(3)*sqrt(2)*(2*x^3 + 3*x)
```

maple [C] time = 0.00, size = 38, normalized size = 1.90

$$\frac{\left(2x^2+3\right)\sqrt{4x^2+2}\,x}{\sqrt{6x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x)
[Out] 1/3*x*(2*x^2+3)*(4*x^2+2)^(1/2)*(6*x^2+3)^(1/2)/(2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{6x^2+3}\sqrt{4x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \sqrt{4x^2+2}\,\sqrt{6x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 2)^(1/2)*(6*x^2 + 3)^(1/2),x)
[Out] int((4*x^2 + 2)^(1/2)*(6*x^2 + 3)^(1/2), x)
```

sympy [A] time = 3.43, size = 17, normalized size = 0.85

$$\frac{2\sqrt{6}x^3}{3} + \sqrt{6}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+2)**(1/2)*(6*x**2+3)**(1/2),x)
[Out] 2*sqrt(6)*x**3/3 + sqrt(6)*x
```

3.137 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=8

$$\frac{\tanh^{-1}(x)}{\sqrt{2}}$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {22, 206}

$$\frac{\tanh^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] ArcTanh[x]/Sqrt[2]
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx &= \frac{\int \frac{1}{1-x^2}\,dx}{\sqrt{2}}\\ &= \frac{\tanh^{-1}(x)}{\sqrt{2}}\end{aligned}$$

Mathematica [B] time = 0.01, size = 26, normalized size = 3.25

$$-\frac{\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] -((Log[1 - x]/2 - Log[1 + x]/2)/Sqrt[2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] IntegrateAlgebraic[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]), x]
```

fricas [B] time = 1.30, size = 68, normalized size = 8.50

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{x^6+5x^4-2\sqrt{2}\left(x^3+x\right)\sqrt{-x^2+1}\sqrt{-2x^2+2}-5x^2-1}{x^6-3x^4+3x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(x^6 + 5*x^4 - 2*sqrt(2)*(x^3 + x)*sqrt(-x^2 + 1)*sqrt(-2*
x^2 + 2) - 5*x^2 - 1)/(x^6 - 3*x^4 + 3*x^2 - 1))
```

giac [B] time = 0.58, size = 19, normalized size = 2.38

$$\frac{1}{4}\sqrt{2}\log\left(x+1\right)-\frac{1}{4}\sqrt{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(x + 1) - 1/4*sqrt(2)*log(x - 1)
```

maple [A] time = 0.32, size = 8, normalized size = 1.00

$$\frac{\sqrt{2}\ \operatorname{arctanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arctanh(x)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.12

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(2 - 2*x^2)^(1/2)),x)
```

```
[Out] int(1/((1 - x^2)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

sympy [A] time = 2.32, size = 22, normalized size = 2.75

$$-\sqrt{2}\left(\begin{cases}-\frac{\operatorname{acoth}(x)}{2} & \text{for}\ x^2>1\\ -\frac{\operatorname{atanh}(x)}{2} & \text{for}\ x^2<1\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(2)*Piecewise((-acoth(x)/2, x**2 > 1), (-atanh(x)/2, x**2 < 1))
```

3.138 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=8

$$\frac{\tan^{-1}(x)}{\sqrt{2}}$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {22, 203}

$$\frac{\tan^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] ArcTan[x]/Sqrt[2]
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] && !(IntegerQ[m] || IntegerQ[n])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx &= \sqrt{2}\int \frac{1}{2+2x^2}\,dx\\ &= \frac{\tan^{-1}(x)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\tan^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] ArcTan[x]/Sqrt[2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] IntegrateAlgebraic[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]), x]
```

fricas [B] time = 1.29, size = 34, normalized size = 4.25

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{2x^2+2}\sqrt{x^2+1}\,x}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(2)*arctan(sqrt(2)*sqrt(2*x^2 + 2)*sqrt(x^2 + 1)*x/(x^4 - 1))
```

giac [B] time = 0.57, size = 26, normalized size = 3.25

$$\frac{1}{4}\sqrt{2}\,i\log(ix-1)-\frac{1}{4}\sqrt{2}\,i\log(-ix-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(2)*i*log(i*x - 1) - 1/4*sqrt(2)*i*log(-i*x - 1)
```

maple [A] time = 0.31, size = 8, normalized size = 1.00

$$\frac{\sqrt{2}\,\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x)
[Out] 1/2*arctan(x)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{2x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.12

$$\int\frac{1}{\sqrt{x^2+1}\,\sqrt{2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(2*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

sympy [A] time = 2.49, size = 8, normalized size = 1.00

$$\frac{\sqrt{2}\,\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
[Out] sqrt(2)*atan(x)/2
```

3.139 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{x^2-1}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {23, 207}

$$-\frac{\sqrt{x^2-1}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] -((Sqrt[-1 + x^2]*ArcTanh[x])/(Sqrt[2]*Sqrt[1 - x^2]))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx &= \frac{\sqrt{-1+x^2}\int \frac{1}{-1+x^2}\,dx}{\sqrt{2-2x^2}}\\ &= -\frac{\sqrt{-1+x^2}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.38

$$\frac{\left(x^2-1\right)(\log(1-x)-\log(x+1))}{2\sqrt{2}\sqrt{-\left(x^2-1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] ((-1 + x^2)*(Log[1 - x] - Log[1 + x]))/(2*Sqrt[2]*Sqrt[-(-1 + x^2)^2])
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
[Out] Defer[IntegrateAlgebraic][1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]), x]
```

fricas [A] time = 1.17, size = 34, normalized size = 1.17

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{-2x^2+2}x}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)*x/(x^4 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

maple [A] time = 0.01, size = 24, normalized size = 0.83

$$\frac{\sqrt{2}\sqrt{-x^2+1}\,\operatorname{arctanh}(x)}{2\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x)
[Out] 1/2*2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)^(1/2)*arctanh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{2-2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2-1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(1 - x**2)*sqrt(x**2 - 1)), x)/2
```

3.140 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{x^2+1}\tan^{-1}(x)}{\sqrt{2}\sqrt{-x^2-1}}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {23, 203}

$$\frac{\sqrt{x^2+1}\tan^{-1}(x)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*ArcTan[x])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx &= \frac{\sqrt{2+2x^2}\int \frac{1}{2+2x^2}\,dx}{\sqrt{-1-x^2}}\\
&= \frac{\sqrt{1+x^2}\tan^{-1}(x)}{\sqrt{2}\sqrt{-1-x^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$\frac{\left(x^2+1\right)\tan^{-1}(x)}{\sqrt{2}\sqrt{-\left(x^2+1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] ((1 + x^2)*ArcTan[x])/(Sqrt[2]*Sqrt[-(1 + x^2)^2])
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]), x]
```

fricas [B] time = 1.15, size = 104, normalized size = 3.71

$$\frac{1}{8}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2x^2+2}\sqrt{-x^2-1}\,x+\sqrt{2}\left(x^4-1\right)\right)}{x^4+2x^2+1}\right)-\frac{1}{8}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2x^2+2}\sqrt{-x^2-1}\,x-\sqrt{2}\left(x^4-1\right)\right)}{x^4+2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(2*(2*sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)*x + sqrt(2)*(x^4 - 1))/
(x^4 + 2*x^2 + 1)) - 1/8*sqrt(2)*log(2*(2*sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)*x
- sqrt(2)*(x^4 - 1))/(x^4 + 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

maple [A] time = 0.01, size = 24, normalized size = 0.86

$$-\frac{\sqrt{-x^2-1}\,\sqrt{2}\,\arctan(x)}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x)
```

```
[Out] -1/2*(-x^2-1)^(1/2)*2^(1/2)/(x^2+1)^(1/2)*arctan(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((- x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{-x^2-1}\,\sqrt{x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
[Out] sqrt(2)*Integral(1/(sqrt(-x**2 - 1)*sqrt(x**2 + 1)), x)/2
```

3.141 $\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx$

Optimal. Leaf size=129

$$-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{3x^2+2}+2\,2^{3/4}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\,2^{3/4}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\sqrt{3x^2+2}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {397}

$$-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{3x^2+2}+2\,2^{3/4}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\,2^{3/4}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\sqrt{3x^2+2}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1
/4))]/(2*2^(3/4)*Sqrt[3]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + 3*x^2])
/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx=-\frac{\tan^{-1}\left(\frac{2\,2^{3/4}+2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Mathematica [C] time = 0.11, size = 135, normalized size = 1.05

$$-\frac{4xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)}{\sqrt[4]{3x^2+2}\,(3x^2+4)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)\right)-4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

```
[Out] (-4*x*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/2, (-3*x^2)/4])/((2 + 3*x^2)^(1/4
)*(4 + 3*x^2)*(-4*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/2, (-3*x^2)/4] + x^2*
(2*AppellF1[3/2, 1/4, 2, 5/2, (-3*x^2)/2, (-3*x^2)/4] + AppellF1[3/2, 5/4,
1, 5/2, (-3*x^2)/2, (-3*x^2)/4])))
```

IntegrateAlgebraic [A] time = 0.33, size = 137, normalized size = 1.06

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{3}x^2}{2\sqrt[4]{2}}-\frac{\sqrt[4]{2}\sqrt{3x^2+2}}{\sqrt{3}}}{x\sqrt[4]{3x^2+2}}\right)}{4\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}\sqrt{3}x\sqrt[4]{3x^2+2}}{3\sqrt{2}x^2+4\sqrt{3x^2+2}}\right)}{4\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

```
[Out] ArcTan[((Sqrt[3]*x^2)/(2*2^(1/4)) - (2^(1/4)*Sqrt[2 + 3*x^2])/Sqrt[3])/(x*(
2 + 3*x^2)^(1/4))]/(4*2^(3/4)*Sqrt[3]) + ArcTanh[(2*2^(3/4)*Sqrt[3]*x*(2 +
3*x^2)^(1/4))/(3*Sqrt[2]*x^2 + 4*Sqrt[2 + 3*x^2])]/(4*2^(3/4)*Sqrt[3])
```

fricas [B] time = 9.14, size = 553, normalized size = 4.29

$$\frac{1}{72}\cdot 18^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}x^3+54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{3}{4}}x+12\sqrt{2}\left(3x^2+4\right)\sqrt{3x^2+2}+72x^2-\left(18^{\frac{3}{4}}\sqrt{2}\left(3x^3-4x\right)\sqrt{3x^2+2}+72\left(3x^2+2\right)^{\frac{1}{4}}x^2+6\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^3+4x\right)+48\sqrt{2}\left(3x^2+2\right)^{\frac{3}{4}}\right)\sqrt{\frac{3\sqrt{2}x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{3x^2+2}}{3x^2+4}}}{6\left(9x^4-24x^2-16\right)}\right)-\frac{1}{72}\cdot 18^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}x^3-54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{3}{4}}x-12\sqrt{2}\left(3x^2+4\right)\sqrt{3x^2+2}-72x^2-\left(18^{\frac{3}{4}}\sqrt{2}\left(3x^3-4x\right)\sqrt{3x^2+2}-72\left(3x^2+2\right)^{\frac{1}{4}}x^2+6\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^3+4x\right)-48\sqrt{2}\left(3x^2+2\right)^{\frac{3}{4}}\right)\sqrt{\frac{3\sqrt{2}x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{3x^2+2}}{3x^2+4}}}{6\left(9x^4-24x^2-16\right)}\right)+\frac{1}{288}\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{36\left(3\sqrt{2}x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{3x^2+2}\right)}{3x^2+4}\right)-\frac{1}{288}\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{36\left(3\sqrt{2}x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{3x^2+2}\right)}{3x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/72*18^(3/4)*sqrt(2)*arctan(-1/6*(6*18^(3/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x^3
 + 54*x^4 + 24*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(3/4)*x + 12*sqrt(2)*(3*x^2 + 4
)*sqrt(3*x^2 + 2) + 72*x^2 - (18^(3/4)*sqrt(2)*(3*x^3 - 4*x)*sqrt(3*x^2 + 2
) + 72*(3*x^2 + 2)^(1/4)*x^2 + 6*18^(1/4)*sqrt(2)*(3*x^3 + 4*x) + 48*sqrt(2
)*(3*x^2 + 2)^(3/4))*sqrt((3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(
1/4)*x + 4*sqrt(3*x^2 + 2))/(3*x^2 + 4)))/(9*x^4 - 24*x^2 - 16)) - 1/72*18^
(3/4)*sqrt(2)*arctan(1/6*(6*18^(3/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x^3 - 54*x^4
 + 24*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(3/4)*x - 12*sqrt(2)*(3*x^2 + 4)*sqrt(3*
x^2 + 2) - 72*x^2 - (18^(3/4)*sqrt(2)*(3*x^3 - 4*x)*sqrt(3*x^2 + 2) - 72*(3
*x^2 + 2)^(1/4)*x^2 + 6*18^(1/4)*sqrt(2)*(3*x^3 + 4*x) - 48*sqrt(2)*(3*x^2
+ 2)^(3/4))*sqrt((3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x +
4*sqrt(3*x^2 + 2))/(3*x^2 + 4)))/(9*x^4 - 24*x^2 - 16)) + 1/288*18^(3/4)*sq
rt(2)*log(36*(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x + 4*sq
rt(3*x^2 + 2))/(3*x^2 + 4)) - 1/288*18^(3/4)*sqrt(2)*log(36*(3*sqrt(2)*x^2
- 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x + 4*sqrt(3*x^2 + 2))/(3*x^2 + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+4\right)\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)
```

maple [C] time = 1.67, size = 186, normalized size = 1.44

$$\frac{\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+72\right)^2\right)\ln\left(-\frac{3x\operatorname{RootOf}\left(\_Z^4+72\right)^2+\left(3x^2+2\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+72\right)^2\right)\operatorname{RootOf}\left(\_Z^4+72\right)^2-18\sqrt{3x^2+2}\,x-6\left(3x^2+2\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+72\right)^2\right)}{3x^2+4}\right)}{24}+\frac{\operatorname{RootOf}\left(\_Z^4+72\right)\ln\left(\frac{3x\operatorname{RootOf}\left(\_Z^4+72\right)^2+\left(3x^2+2\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+72\right)^3+18\sqrt{3x^2+2}\,x+6\left(3x^2+2\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^4+72\right)}{3x^2+4}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(1/4)/(3*x^2+4),x)
```

```
[Out] 1/24*RootOf(_Z^2+RootOf(_Z^4+72)^2)*ln(-((3*x^2+2)^(1/4)*RootOf(_Z^4+72)^2*
RootOf(_Z^2+RootOf(_Z^4+72)^2)-6*(3*x^2+2)^(3/4)*RootOf(_Z^2+RootOf(_Z^4+72
)^2)+3*RootOf(_Z^4+72)^2*x-18*(3*x^2+2)^(1/2)*x)/(3*x^2+4))+1/24*RootOf(_Z^
4+72)*ln(((3*x^2+2)^(1/4)*RootOf(_Z^4+72)^3+6*(3*x^2+2)^(3/4)*RootOf(_Z^4+7
2)+3*RootOf(_Z^4+72)^2*x+18*(3*x^2+2)^(1/2)*x)/(3*x^2+4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+4\right)\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(3\,x^2+2\right)^{1/4}\,\left(3\,x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x^2 + 2)^(1/4)*(3*x^2 + 4)),x)
```

```
[Out] int(1/((3*x^2 + 2)^(1/4)*(3*x^2 + 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{3x^2+2}\,\left(3x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+2)**(1/4)/(3*x**2+4),x)
```

```
[Out] Integral(1/((3*x**2 + 2)**(1/4)*(3*x**2 + 4)), x)
```

3.142 $\int \frac{1}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {397}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Mathematica [C] time = 0.13, size = 135, normalized size = 1.12

$$-\frac{4xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{\sqrt[4]{2-3x^2}\left(3x^2-4\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-4*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((2 - 3*x^2)^(1/4)*
(-4 + 3*x^2)*(4*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*(2*A
ppellF1[3/2, 1/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + AppellF1[3/2, 5/4, 1, 5/2
, (3*x^2)/2, (3*x^2)/4])))
```

IntegrateAlgebraic [A] time = 0.32, size = 137, normalized size = 1.14

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{3}x^2}{2\sqrt[4]{2}}-\frac{\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}}}{x\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}\sqrt{3}\,x\sqrt[4]{2-3x^2}}{3\sqrt{2}x^2+4\sqrt{2-3x^2}}\right)}{4\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[((Sqrt[3]*x^2)/(2*2^(1/4)) - (2^(1/4)*Sqrt[2 - 3*x^2])/Sqrt[3])/(x*(
2 - 3*x^2)^(1/4))]/(4*2^(3/4)*Sqrt[3]) + ArcTanh[(2*2^(3/4)*Sqrt[3]*x*(2 -
3*x^2)^(1/4))/(3*Sqrt[2]*x^2 + 4*Sqrt[2 - 3*x^2])]/(4*2^(3/4)*Sqrt[3])
```

fricas [B] time = 9.05, size = 553, normalized size = 4.61

$$\frac{1}{72}\cdot 18^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x^3+54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}x+12\sqrt{2}\left(3x^2-4\right)\sqrt{-3x^2+2}-72x^2+\left(18^{\frac{3}{4}}\sqrt{2}\left(3x^3+4x\right)\sqrt{-3x^2+2}-72\left(-3x^2+2\right)^{\frac{1}{4}}x^2-6\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^3-4x\right)-48\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}\right)\sqrt{-\frac{3\sqrt{2}x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}}{3x^2-4}}}{6\left(9x^4+24x^2-16\right)}\right)-\frac{1}{72}\cdot 18^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x^3-54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}x-12\sqrt{2}\left(3x^2-4\right)\sqrt{-3x^2+2}+72x^2+\left(18^{\frac{3}{4}}\sqrt{2}\left(3x^3+4x\right)\sqrt{-3x^2+2}+72\left(-3x^2+2\right)^{\frac{1}{4}}x^2-6\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^3-4x\right)+48\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}\right)\sqrt{-\frac{3\sqrt{2}x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}}{3x^2-4}}}{6\left(9x^4+24x^2-16\right)}\right)+\frac{1}{288}\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(-\frac{36\left(3\sqrt{2}x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}\right)}{3x^2-4}\right)-\frac{1}{288}\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(-\frac{36\left(3\sqrt{2}x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}\right)}{3x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/72*18^(3/4)*sqrt(2)*arctan(-1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 + 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x + 12*sqrt(2)*(3*x^2 -
 4)*sqrt(-3*x^2 + 2) - 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) - 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) - 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) -
 1/72*18^(3/4)*sqrt(2)*arctan(1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 - 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x - 12*sqrt(2)*(3*x^2 -
 4)*sqrt(-3*x^2 + 2) + 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) + 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) + 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) +
 1/288*18^(3/4)*sqrt(2)*log(-36*(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)) - 1/288*18^(3/4)*sqrt(2)*l
og(-36*(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 4*sqrt(-3
*x^2 + 2))/(3*x^2 - 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(3x^2-4\right)\left(-3x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

maple [C] time = 1.60, size = 187, normalized size = 1.56

$$-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+72\right)^2\right)\ln\left(-\frac{-3x\,\mathrm{RootOf}\left(\_Z^4+72\right)^2+\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+72\right)^2\right)\mathrm{RootOf}\left(\_Z^4+72\right)^2-18\sqrt{-3x^2+2}\,x+6\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+72\right)^2\right)}{3x^2-4}\right)}{24}+\frac{\mathrm{RootOf}\left(\_Z^4+72\right)\ln\left(-\frac{3x\,\mathrm{RootOf}\left(\_Z^4+72\right)^2+\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+72\right)^3-18\sqrt{-3x^2+2}\,x-6\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+72\right)}{3x^2-4}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] 1/24*RootOf(_Z^4+72)*ln(-((-3*x^2+2)^(1/4)*RootOf(_Z^4+72)^3-6*(-3*x^2+2)^(
3/4)*RootOf(_Z^4+72)+3*RootOf(_Z^4+72)^2*x-18*(-3*x^2+2)^(1/2)*x)/(3*x^2-4)
```

```
)-1/24*RootOf(_Z^2+RootOf(_Z^4+72)^2)*ln(-((-3*x^2+2)^(1/4)*RootOf(_Z^4+72)
^2*RootOf(_Z^2+RootOf(_Z^4+72)^2)+6*(-3*x^2+2)^(3/4)*RootOf(_Z^2+RootOf(_Z^
4+72)^2)-3*RootOf(_Z^4+72)^2*x-18*(-3*x^2+2)^(1/2)*x)/(3*x^2-4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(2-3\,x^2\right)^{1/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
[Out] -int(1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
[Out] -Integral(1/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.143 $\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx$

Optimal. Leaf size=129

$$-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\,\sqrt{bx^2+2}+2\,2^{3/4}}{2\sqrt{b}\,x\,\sqrt[4]{bx^2+2}}\right)}{2\,2^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{bx^2+2}}{2\sqrt{b}\,x\,\sqrt[4]{bx^2+2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {397}

$$-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\,\sqrt{bx^2+2}+2\,2^{3/4}}{2\sqrt{b}\,x\,\sqrt[4]{bx^2+2}}\right)}{2\,2^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{bx^2+2}}{2\sqrt{b}\,x\,\sqrt[4]{bx^2+2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2)^(1
/4))]/(2*2^(3/4)*Sqrt[b]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + b*x^2])
/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx=-\frac{\tan^{-1}\left(\frac{2\,2^{3/4}+2\sqrt[4]{2}\,\sqrt{2+bx^2}}{2\sqrt{b}\,x\,\sqrt[4]{2+bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{2+bx^2}}{2\sqrt{b}\,x\,\sqrt[4]{2+bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.14, size = 144, normalized size = 1.12

$$-\frac{12xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)}{\sqrt[4]{bx^2+2}\,(bx^2+4)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)\right)-12F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

```
[Out] (-12*x*AppellF1[1/2, 1/4, 1, 3/2, -1/2*(b*x^2), -1/4*(b*x^2)])/((2 + b*x^2)
^(1/4)*(4 + b*x^2)*(-12*AppellF1[1/2, 1/4, 1, 3/2, -1/2*(b*x^2), -1/4*(b*x^
2)] + b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -1/2*(b*x^2), -1/4*(b*x^2)] + App
ellF1[3/2, 5/4, 1, 5/2, -1/2*(b*x^2), -1/4*(b*x^2)])))
```

IntegrateAlgebraic [A] time = 0.27, size = 137, normalized size = 1.06

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{b}\,x^2}{2\sqrt[4]{2}}-\frac{\sqrt[4]{2}\,\sqrt{bx^2+2}}{\sqrt{b}}}{x\sqrt[4]{bx^2+2}}\right)}{4\,2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}\sqrt{b}\,x\sqrt[4]{bx^2+2}}{\sqrt{2}\,bx^2+4\sqrt{bx^2+2}}\right)}{4\,2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

```
[Out] ArcTan[((Sqrt[b]*x^2)/(2*2^(1/4)) - (2^(1/4)*Sqrt[2 + b*x^2])/Sqrt[b])/(x*(
2 + b*x^2)^(1/4))]/(4*2^(3/4)*Sqrt[b]) + ArcTanh[(2*2^(3/4)*Sqrt[b]*x*(2 +
b*x^2)^(1/4))/(Sqrt[2]*b*x^2 + 4*Sqrt[2 + b*x^2])]/(4*2^(3/4)*Sqrt[b])
```

fricas [B] time = 28.02, size = 755, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*arctan(-(2*sqrt(2)*(1/2)^(1/4)*(b*x^
2 + 2)^(1/4)*b^2*(b^(-2))^(1/4)*x^3 + b^2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(b*x^
2 + 2)^(3/4)*b^2*(b^(-2))^(3/4)*x + 4*b*x^2 + 4*sqrt(1/2)*(b^2*x^2 + 4*b)*s
qrt(b*x^2 + 2)*sqrt(b^(-2)) - 2*sqrt(1/2)*(4*(b*x^2 + 2)^(1/4)*b*x^2 + 2*sq
rt(2)*(1/2)^(3/4)*(b^3*x^3 + 4*b^2*x)*(b^(-2))^(3/4) + 16*sqrt(1/2)*(b*x^2
+ 2)^(3/4)*b*sqrt(b^(-2)) + sqrt(2)*(1/2)^(1/4)*(b^2*x^3 - 4*b*x)*sqrt(b*x^
2 + 2)*(b^(-2))^(1/4))*sqrt((2*sqrt(2)*(1/2)^(3/4)*(b*x^2 + 2)^(1/4)*b^2*(b
^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 + 2*sqrt(b*x^2 + 2))/(b*x^2
 + 4)))/(b^2*x^4 - 8*b*x^2 - 16)) - 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*
arctan((2*sqrt(2)*(1/2)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(1/4)*x^3 - b^
2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(b*x^2 + 2)^(3/4)*b^2*(b^(-2))^(3/4)*x - 4*b*
x^2 - 4*sqrt(1/2)*(b^2*x^2 + 4*b)*sqrt(b*x^2 + 2)*sqrt(b^(-2)) + 2*sqrt(1/2
)*(4*(b*x^2 + 2)^(1/4)*b*x^2 - 2*sqrt(2)*(1/2)^(3/4)*(b^3*x^3 + 4*b^2*x)*(b
^(-2))^(3/4) + 16*sqrt(1/2)*(b*x^2 + 2)^(3/4)*b*sqrt(b^(-2)) - sqrt(2)*(1/2
)^(1/4)*(b^2*x^3 - 4*b*x)*sqrt(b*x^2 + 2)*(b^(-2))^(1/4))*sqrt(-(2*sqrt(2)*
(1/2)^(3/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x - sqrt(1/2)*b^2*sqrt(b^(
-2))*x^2 - 2*sqrt(b*x^2 + 2))/(b*x^2 + 4)))/(b^2*x^4 - 8*b*x^2 - 16)) + 1/1
6*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*log(1/2*(2*sqrt(2)*(1/2)^(3/4)*(b*x^2
+ 2)^(1/4)*b^2*(b^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 + 2*sqrt(b
*x^2 + 2))/(b*x^2 + 4)) - 1/16*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*log(-1/2*
(2*sqrt(2)*(1/2)^(3/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x - sqrt(1/2)*b
^2*sqrt(b^(-2))*x^2 - 2*sqrt(b*x^2 + 2))/(b*x^2 + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+4\right)\left(bx^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+2\right)^{\frac{1}{4}}\left(b\,x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
```

```
[Out] int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^2+4)(bx^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\,x^2+2)^{1/4}\,(b\,x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2 + 2)^(1/4)*(b*x^2 + 4)),x)
```

```
[Out] int(1/((b*x^2 + 2)^(1/4)*(b*x^2 + 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{bx^2+2}\,(bx^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2)**(1/4)/(b*x**2+4),x)
```

```
[Out] Integral(1/((b*x**2 + 2)**(1/4)*(b*x**2 + 4)), x)
```

3.144 $\int \frac{1}{\sqrt[4]{2-bx^2}\left(4-bx^2\right)}\,dx$

Optimal. Leaf size=124

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {397}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[b]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[
b]*x*(2 - b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-bx^2}\left(4-bx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.13, size = 145, normalized size = 1.17

$$-\frac{12xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)}{\sqrt[4]{2-bx^2}\left(bx^2-4\right)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)\right)+12F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

```
[Out] (-12*x*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/2, (b*x^2)/4])/((2 - b*x^2)^(1/4)
*(-4 + b*x^2)*(12*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/2, (b*x^2)/4] + b*x^2*
(2*AppellF1[3/2, 1/4, 2, 5/2, (b*x^2)/2, (b*x^2)/4] + AppellF1[3/2, 5/4, 1,
 5/2, (b*x^2)/2, (b*x^2)/4]))
```

IntegrateAlgebraic [A] time = 0.28, size = 141, normalized size = 1.14

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{b}\,x^2}{2\sqrt[4]{2}}-\frac{\sqrt[4]{2}\sqrt{2-bx^2}}{\sqrt{b}}}{x\sqrt[4]{2-bx^2}}\right)}{4\,2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}\sqrt{b}\,x\sqrt[4]{2-bx^2}}{\sqrt{2}\,bx^2+4\sqrt{2-bx^2}}\right)}{4\,2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

```
[Out] ArcTan[((Sqrt[b]*x^2)/(2*2^(1/4)) - (2^(1/4)*Sqrt[2 - b*x^2])/Sqrt[b])/(x*(
2 - b*x^2)^(1/4))]/(4*2^(3/4)*Sqrt[b]) + ArcTanh[(2*2^(3/4)*Sqrt[b]*x*(2 -
b*x^2)^(1/4))/(Sqrt[2]*b*x^2 + 4*Sqrt[2 - b*x^2])]/(4*2^(3/4)*Sqrt[b])
```

fricas [B] time = 27.76, size = 776, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*arctan(-(2*sqrt(2)*(1/2)^(1/4)*(-b*x
^2 + 2)^(1/4)*b^2*(b^(-2))^(1/4)*x^3 + b^2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(-b*
x^2 + 2)^(3/4)*b^2*(b^(-2))^(3/4)*x - 4*b*x^2 + 4*sqrt(1/2)*(b^2*x^2 - 4*b)
*sqrt(-b*x^2 + 2)*sqrt(b^(-2)) - 2*sqrt(1/2)*(4*(-b*x^2 + 2)^(1/4)*b*x^2 +
2*sqrt(2)*(1/2)^(3/4)*(b^3*x^3 - 4*b^2*x)*(b^(-2))^(3/4) + 16*sqrt(1/2)*(-b
*x^2 + 2)^(3/4)*b*sqrt(b^(-2)) - sqrt(2)*(1/2)^(1/4)*(b^2*x^3 + 4*b*x)*sqrt
(-b*x^2 + 2)*(b^(-2))^(1/4))*sqrt(-(2*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(1/4
)*b^2*(b^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 + 2*sqrt(-b*x^2 + 2
))/(b*x^2 - 4)))/(b^2*x^4 + 8*b*x^2 - 16)) + 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2
))^(1/4)*arctan(-(2*sqrt(2)*(1/2)^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(1/
4)*x^3 - b^2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(3/4)*b^2*(b^(-2))^(3
/4)*x + 4*b*x^2 - 4*sqrt(1/2)*(b^2*x^2 - 4*b)*sqrt(-b*x^2 + 2)*sqrt(b^(-2))
 + 2*sqrt(1/2)*(4*(-b*x^2 + 2)^(1/4)*b*x^2 - 2*sqrt(2)*(1/2)^(3/4)*(b^3*x^3
 - 4*b^2*x)*(b^(-2))^(3/4) + 16*sqrt(1/2)*(-b*x^2 + 2)^(3/4)*b*sqrt(b^(-2))
 + sqrt(2)*(1/2)^(1/4)*(b^2*x^3 + 4*b*x)*sqrt(-b*x^2 + 2)*(b^(-2))^(1/4))*s
qrt((2*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x - sqrt(1
/2)*b^2*sqrt(b^(-2))*x^2 - 2*sqrt(-b*x^2 + 2))/(b*x^2 - 4)))/(b^2*x^4 + 8*b
*x^2 - 16)) + 1/16*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*log(-1/2*(2*sqrt(2)*(
1/2)^(3/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(
-2))*x^2 + 2*sqrt(-b*x^2 + 2))/(b*x^2 - 4)) - 1/16*sqrt(2)*(1/2)^(1/4)*(b^(
-2))^(1/4)*log(1/2*(2*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(
3/4)*x - sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 - 2*sqrt(-b*x^2 + 2))/(b*x^2 - 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(bx^2-4\right)\left(-bx^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+2\right)^{\frac{1}{4}}\left(-b\,x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
[Out] int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(bx^2-4\right)\left(-bx^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="maxima")
[Out] -integrate(1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(2-b\,x^2\right)^{1/4}\,\left(b\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2 - b*x^2)^(1/4)*(b*x^2 - 4)),x)
[Out] -int(1/((2 - b*x^2)^(1/4)*(b*x^2 - 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{bx^2\sqrt[4]{-bx^2+2}-4\sqrt[4]{-bx^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+2)**(1/4)/(-b*x**2+4),x)
[Out] -Integral(1/(b*x**2*(-b*x**2 + 2)**(1/4) - 4*(-b*x**2 + 2)**(1/4)), x)
```

3.145 $\int \frac{1}{\sqrt[4]{a+3x^2}\left(2a+3x^2\right)}\,dx$

Optimal. Leaf size=120

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

Rubi [A] time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {397}

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^(1/4
))]/(2*Sqrt[3]*a^(3/4)) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + 3*x^2]/Sqrt[a]))/(
Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+3x^2}\left(2a+3x^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

Mathematica [C] time = 0.15, size = 155, normalized size = 1.29

$$-\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{\sqrt[4]{a+3x^2}\left(2a+3x^2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)-2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

```
[Out] (-2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/a, (-3*x^2)/(2*a)])/((a + 3*x^2
)^(1/4)*(2*a + 3*x^2)*(-2*a*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/a, (-3*x^2)
/(2*a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)] + A
ppellF1[3/2, 5/4, 1, 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)])))
```

IntegrateAlgebraic [A] time = 0.25, size = 136, normalized size = 1.13

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{3}\,\sqrt[4]{a}\,x\,\sqrt[4]{a+3x^2}}{2\sqrt{a}\,\sqrt{a+3x^2}+3x^2}\right)}{4\sqrt{3}\,a^{3/4}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\,\sqrt{a+3x^2}}{\sqrt{3}}-\frac{\sqrt{3}\,x^2}{2\sqrt[4]{a}}}{x\sqrt[4]{a+3x^2}}\right)}{4\sqrt{3}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

```
[Out] -1/4*ArcTan[(-1/2*(Sqrt[3]*x^2)/a^(1/4) + (a^(1/4)*Sqrt[a + 3*x^2])/Sqrt[3]
)/(x*(a + 3*x^2)^(1/4))]/(Sqrt[3]*a^(3/4)) + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(
a + 3*x^2)^(1/4))/(3*x^2 + 2*Sqrt[a]*Sqrt[a + 3*x^2])]/(4*Sqrt[3]*a^(3/4))
```

fricas [B] time = 24.63, size = 286, normalized size = 2.38

$$\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}}a^3\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(\frac{1}{36}\right)^{\frac{1}{4}}\sqrt{3x^2+a}\,a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)\sqrt{-a\sqrt{-\frac{1}{a^3}}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(3x^2+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)}{x}\right)-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}\,a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(3x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}+\left(3x^2+a\right)^{\frac{3}{4}}}{3x^2+2a}\right)+\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}\,a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-\left(3x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}-\left(3x^2+a\right)^{\frac{3}{4}}}{3x^2+2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] (1/36)^(1/4)*(-1/a^3)^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(-1/a^3
)^(3/4) + (1/36)^(1/4)*sqrt(3*x^2 + a)*a*(-1/a^3)^(1/4))*sqrt(-a*sqrt(-1/a^
3)) - (1/36)^(1/4)*(3*x^2 + a)^(1/4)*a*(-1/a^3)^(1/4))/x) - 1/4*(1/36)^(1/4
)*(-1/a^3)^(1/4)*log((18*(1/36)^(3/4)*sqrt(3*x^2 + a)*a^2*x*(-1/a^3)^(3/4)
+ (3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) - 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) +
(3*x^2 + a)^(3/4))/(3*x^2 + 2*a)) + 1/4*(1/36)^(1/4)*(-1/a^3)^(1/4)*log(-(1
8*(1/36)^(3/4)*sqrt(3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (3*x^2 + a)^(1/4)*a^2
*sqrt(-1/a^3) - 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) - (3*x^2 + a)^(3/4))/(3*x
^2 + 2*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\,a\right)\left(3\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2+a\right)^{\frac{1}{4}}\left(3x^2+2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
```

```
[Out] int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\,a\right)\left(3\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(3\,x^2+2\,a\right)\,{\left(3\,x^2+a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*a + 3*x^2)*(a + 3*x^2)^(1/4)),x)
```

```
[Out] int(1/((2*a + 3*x^2)*(a + 3*x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,\left(2a+3x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+a)**(1/4)/(3*x**2+2*a),x)
```

```
[Out] Integral(1/((a + 3*x**2)**(1/4)*(2*a + 3*x**2)), x)
```

3.146 $\int \frac{1}{\sqrt[4]{a-3x^2}\left(2a-3x^2\right)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

Rubi [A] time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {397}

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4)
)]/(2*Sqrt[3]*a^(3/4)) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - 3*x^2]/Sqrt[a]))/(S
qrt[3]*x*(a - 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-3x^2}\left(2a-3x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

Mathematica [C] time = 0.16, size = 155, normalized size = 1.29

$$-\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{\sqrt[4]{a-3x^2}\left(3x^2-2a\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)+2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

```
[Out] (-2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/a, (3*x^2)/(2*a)])/((a - 3*x^2)^
(1/4)*(-2*a + 3*x^2)*(2*a*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/a, (3*x^2)/(2*
a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (3*x^2)/a, (3*x^2)/(2*a)] + AppellF
1[3/2, 5/4, 1, 5/2, (3*x^2)/a, (3*x^2)/(2*a)])))
```

IntegrateAlgebraic [A] time = 0.25, size = 136, normalized size = 1.13

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{3}\,\sqrt[4]{a}\,x\,\sqrt[4]{a-3x^2}}{2\sqrt{a}\,\sqrt{a-3x^2}+3x^2}\right)}{4\sqrt{3}\,a^{3/4}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\,\sqrt{a-3x^2}}{\sqrt{3}}-\frac{\sqrt{3}\,x^2}{2\sqrt[4]{a}}}{x\,\sqrt[4]{a-3x^2}}\right)}{4\sqrt{3}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

```
[Out] -1/4*ArcTan[(-1/2*(Sqrt[3]*x^2)/a^(1/4) + (a^(1/4)*Sqrt[a - 3*x^2])/Sqrt[3]
)/(x*(a - 3*x^2)^(1/4))]/(Sqrt[3]*a^(3/4)) + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(
a - 3*x^2)^(1/4))/(3*x^2 + 2*Sqrt[a]*Sqrt[a - 3*x^2])]/(4*Sqrt[3]*a^(3/4))
```

fricas [B] time = 23.72, size = 286, normalized size = 2.38

$$\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}}a^3\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\sqrt{-3x^2+a}\,a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)\sqrt{a\sqrt{-\frac{1}{a^3}}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-3x^2+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)}{x}\right)+\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2+a}\,a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(-3x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}-\left(-3x^2+a\right)^{\frac{3}{4}}}{3x^2-2a}\right)-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2+a}\,a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-\left(-3x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}+\left(-3x^2+a\right)^{\frac{3}{4}}}{3x^2-2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] (1/36)^(1/4)*(-1/a^3)^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(-1/a^3
)^(3/4) - (1/36)^(1/4)*sqrt(-3*x^2 + a)*a*(-1/a^3)^(1/4))*sqrt(a*sqrt(-1/a^
3)) - (1/36)^(1/4)*(-3*x^2 + a)^(1/4)*a*(-1/a^3)^(1/4))/x) + 1/4*(1/36)^(1/
4)*(-1/a^3)^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/
4) + (-3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) + 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4
) - (-3*x^2 + a)^(3/4))/(3*x^2 - 2*a)) - 1/4*(1/36)^(1/4)*(-1/a^3)^(1/4)*lo
g((18*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (-3*x^2 + a)^(1/
4)*a^2*sqrt(-1/a^3) + 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (-3*x^2 + a)^(3/4
))/(3*x^2 - 2*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(3x^2-2a\right)\left(-3x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2+a\right)^{\frac{1}{4}}\left(-3x^2+2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

```
[Out] int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(3x^2-2a\right)\left(-3x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(2\,a-3\,x^2\right)\,{\left(a-3\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*a - 3*x^2)*(a - 3*x^2)^(1/4)),x)
```

```
[Out] int(1/((2*a - 3*x^2)*(a - 3*x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-2a\sqrt[4]{a-3x^2}+3x^2\sqrt[4]{a-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+a)**(1/4)/(-3*x**2+2*a),x)
```

```
[Out] -Integral(1/(-2*a*(a - 3*x**2)**(1/4) + 3*x**2*(a - 3*x**2)**(1/4)), x)
```

3.147 $\int \frac{1}{\sqrt[4]{a+bx^2}\left(2a+bx^2\right)}\,dx$

Optimal. Leaf size=120

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {397}

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
```

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1/4
))]/(2*a^(3/4)*Sqrt[b]) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + b*x^2]/Sqrt[a]))/(
Sqrt[b]*x*(a + b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^2}\left(2a+bx^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.15, size = 165, normalized size = 1.38

$$\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{\sqrt[4]{a+bx^2}\left(2a+bx^2\right)\left(6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)-bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
```

```
[Out] (6*a*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -1/2*(b*x^2)/a])/((a + b*x^
2)^(1/4)*(2*a + b*x^2)*(6*a*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -1/2*(
b*x^2)/a] - b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -1/2*(b*x^2)/
a] + AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -1/2*(b*x^2)/a])))
```

IntegrateAlgebraic [A] time = 0.26, size = 136, normalized size = 1.13

$$\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{a}\sqrt{b}\,x\sqrt[4]{a+bx^2}}{2\sqrt{a}\sqrt{a+bx^2}+bx^2}\right)}{4a^{3/4}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{a+bx^2}}{\sqrt{b}}-\frac{\sqrt{b}\,x^2}{2\sqrt[4]{a}}}{x\sqrt[4]{a+bx^2}}\right)}{4a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
```

```
[Out] -1/4*ArcTan[(-1/2*(Sqrt[b]*x^2)/a^(1/4) + (a^(1/4)*Sqrt[a + b*x^2])/Sqrt[b]
)/(x*(a + b*x^2)^(1/4))]/(a^(3/4)*Sqrt[b]) + ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(
a + b*x^2)^(1/4))/(b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2])]/(4*a^(3/4)*Sqrt[b])
```

fricas [B] time = 137.83, size = 337, normalized size = 2.81

$$\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(2\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{bx^2+a}\,a\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)\sqrt{-ab\sqrt{-\frac{1}{a^3b^2}}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(bx^2+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)}{x}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+\left(bx^2+a\right)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+\left(bx^2+a\right)^{\frac{3}{4}}}{bx^2+2a}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-\left(bx^2+a\right)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-\left(bx^2+a\right)^{\frac{3}{4}}}{bx^2+2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] (1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*arctan(2*(sqrt(1/2)*(2*(1/4)^(3/4)*a^3*b*(
-1/(a^3*b^2))^(3/4) + (1/4)^(1/4)*sqrt(b*x^2 + a)*a*(-1/(a^3*b^2))^(1/4))*s
qrt(-a*b*sqrt(-1/(a^3*b^2))) - (1/4)^(1/4)*(b*x^2 + a)^(1/4)*a*(-1/(a^3*b^2
))^(1/4))/x) - 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt
(b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) + (b*x^2 + a)^(1/4)*a^2*b*sqrt(-
1/(a^3*b^2)) - (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (b*x^2 + a)^(3/4))/
(b*x^2 + 2*a)) + 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*s
qrt(b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (b*x^2 + a)^(1/4)*a^2*b*sqr
t(-1/(a^3*b^2)) - (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) - (b*x^2 + a)^(3/4
))/(b*x^2 + 2*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+2\,a\right)\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{4}}\left(b\,x^2+2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
```

```
[Out] int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+2\,a\right)\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^2+a\right)^{1/4}\,\left(b\,x^2+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x)
[Out] int(1/((a + b*x^2)^(1/4)*(2*a + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,\left(2a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/4)/(b*x**2+2*a),x)
[Out] Integral(1/((a + b*x**2)**(1/4)*(2*a + b*x**2)), x)
```

3.148 $\int \frac{1}{\sqrt[4]{a-bx^2}\left(2a-bx^2\right)}\,dx$

Optimal. Leaf size=124

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {397}

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4)
)]/(2*a^(3/4)*Sqrt[b]) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - b*x^2]/Sqrt[a]))/(S
qrt[b]*x*(a - b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-bx^2}\left(2a-bx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.16, size = 162, normalized size = 1.31

$$\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{\sqrt[4]{a-bx^2}\left(2a-bx^2\right)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)+6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

```
[Out] (6*a*x*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/a, (b*x^2)/(2*a)])/((a - b*x^2)^(
1/4)*(2*a - b*x^2)*(6*a*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/a, (b*x^2)/(2*a)
] + b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (b*x^2)/a, (b*x^2)/(2*a)] + AppellF
1[3/2, 5/4, 1, 5/2, (b*x^2)/a, (b*x^2)/(2*a)])))
```

IntegrateAlgebraic [A] time = 0.26, size = 140, normalized size = 1.13

$$\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{a}\,\sqrt{b}\,x\,\sqrt[4]{a-bx^2}}{2\sqrt{a}\,\sqrt{a-bx^2}+bx^2}\right)}{4a^{3/4}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\,\sqrt{a-bx^2}}{\sqrt{b}}-\frac{\sqrt{b}\,x^2}{2\sqrt[4]{a}}}{x\,\sqrt[4]{a-bx^2}}\right)}{4a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

```
[Out] -1/4*ArcTan[(-1/2*(Sqrt[b]*x^2)/a^(1/4) + (a^(1/4)*Sqrt[a - b*x^2])/Sqrt[b]
)/(x*(a - b*x^2)^(1/4))]/(a^(3/4)*Sqrt[b]) + ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(
a - b*x^2)^(1/4))/(b*x^2 + 2*Sqrt[a]*Sqrt[a - b*x^2])]/(4*a^(3/4)*Sqrt[b])
```

fricas [B] time = 129.69, size = 343, normalized size = 2.77

$$\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(2\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{-bx^2+a}\,a\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)\sqrt{ab\sqrt{-\frac{1}{a^3b^2}}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-bx^2+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)}{x}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+\left(-bx^2+a\right)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^2-2a}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-\left(-bx^2+a\right)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^2-2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] (1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*arctan(2*(sqrt(1/2)*(2*(1/4)^(3/4)*a^3*b*(
-1/(a^3*b^2))^(3/4) - (1/4)^(1/4)*sqrt(-b*x^2 + a)*a*(-1/(a^3*b^2))^(1/4))*
sqrt(a*b*sqrt(-1/(a^3*b^2))) - (1/4)^(1/4)*(-b*x^2 + a)^(1/4)*a*(-1/(a^3*b^
2))^(1/4))/x) + 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sq
rt(-b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) + (-b*x^2 + a)^(1/4)*a^2*b*sq
rt(-1/(a^3*b^2)) + (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) - (-b*x^2 + a)^(3
/4))/(b*x^2 - 2*a)) - 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/
4)*sqrt(-b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (-b*x^2 + a)^(1/4)*a^2
*b*sqrt(-1/(a^3*b^2)) + (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (-b*x^2 +
a)^(3/4))/(b*x^2 - 2*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(bx^2-2\,a\right)\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}}\left(-b\,x^2+2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(bx^2-2\,a\right)\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a-b\,x^2\right)^{1/4}\,\left(2\,a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x)
```

```
[Out] int(1/((a - b*x^2)^(1/4)*(2*a - b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-2a\sqrt[4]{a-bx^2}+bx^2\sqrt[4]{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/4)/(-b*x**2+2*a),x)
```

```
[Out] -Integral(1/(-2*a*(a - b*x**2)**(1/4) + b*x**2*(a - b*x**2)**(1/4)), x)
```

3.149 $\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

Mathematica [C] time = 0.15, size = 127, normalized size = 2.08

$$\frac{2xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{\left(3x^2-2\right)\sqrt[4]{3x^2-1}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*(-1 + 3*x^
2)^(1/4)*(2*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*AppellF1[
3/2, 1/4, 2, 5/2, 3*x^2, (3*x^2)/2] + AppellF1[3/2, 5/4, 1, 5/2, 3*x^2, (3*
x^2)/2])))
```

IntegrateAlgebraic [A] time = 0.12, size = 63, normalized size = 1.03

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{3x^2-1}}{x}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2/3]*(-1 + 3*x^2)^(1/4))/x]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

fricas [B] time = 9.42, size = 104, normalized size = 1.70

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}}{3x}\right)+\frac{1}{24}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}\,x^2-4\sqrt{6}\left(3x^2-1\right)^{\frac{3}{4}}x+12x^2-4}{9x^4-12x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/24*sqrt(6)*log(-(9
*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6)
*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2-1\right)^{\frac{1}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

maple [C] time = 1.14, size = 138, normalized size = 2.26

$$\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\ln\left(-\frac{-3\sqrt{3x^2-1}\,x-3x+\left(3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2-6\right)+\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2-6\right)}{3x^2-2}\right)}{12}-\frac{\mathrm{RootOf}\left(\_Z^2+6\right)\ln\left(\frac{3\sqrt{3x^2-1}\,x-3x+\left(3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+6\right)-\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+6\right)}{3x^2-2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] -1/12*RootOf(_Z^2+6)*ln((RootOf(_Z^2+6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x
-RootOf(_Z^2+6)*(3*x^2-1)^(1/4)-3*x)/(3*x^2-2))+1/12*RootOf(_Z^2-6)*ln(-(Ro
otOf(_Z^2-6)*(3*x^2-1)^(3/4)-3*(3*x^2-1)^(1/2)*x+RootOf(_Z^2-6)*(3*x^2-1)^(
1/4)-3*x)/(3*x^2-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2-1\right)^{\frac{1}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(3\,x^2-1\right)^{1/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
[Out] int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.150 $\int \frac{1}{\left(-2-3x^2\right)\sqrt[4]{-1-3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}$$

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2-3x^2\right)\sqrt[4]{-1-3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}$$

Mathematica [C] time = 0.13, size = 127, normalized size = 2.08

$$\frac{2xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-3x^2,-\frac{3x^2}{2}\right)}{\sqrt[4]{-3x^2-1}\left(3x^2+2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-3x^2,-\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-3x^2,-\frac{3x^2}{2}\right)\right)-2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-3x^2,-\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*AppellF1[1/2, 1/4, 1, 3/2, -3*x^2, (-3*x^2)/2])/((-1 - 3*x^2)^(1/4)*(2
 + 3*x^2)*(-2*AppellF1[1/2, 1/4, 1, 3/2, -3*x^2, (-3*x^2)/2] + x^2*(2*Appel
lF1[3/2, 1/4, 2, 5/2, -3*x^2, (-3*x^2)/2] + AppellF1[3/2, 5/4, 1, 5/2, -3*x
^2, (-3*x^2)/2])))
```

IntegrateAlgebraic [A] time = 0.12, size = 72, normalized size = 1.18

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{-3x^2-1}}{x}\right)}{2\sqrt{6}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x\left(-3x^2-1\right)^{3/4}}{3x^2+1}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2/3]*(-1 - 3*x^2)^(1/4))/x]/(2*Sqrt[6]) + ArcTanh[(Sqrt[3/2]*x
*(-1 - 3*x^2)^(3/4))/(1 + 3*x^2)]/(2*Sqrt[6])
```

fricas [C] time = 9.29, size = 243, normalized size = 3.98

$$-\frac{1}{24}\sqrt{6}\log\left(\frac{\sqrt{6}\sqrt{-3x^2-1}x-\sqrt{6}x+2\left(-3x^2-1\right)^{\frac{3}{4}}-2\left(-3x^2-1\right)^{\frac{1}{4}}}{3\left(3x^2+2\right)}\right)+\frac{1}{24}\sqrt{6}\log\left(-\frac{\sqrt{6}\sqrt{-3x^2-1}x-\sqrt{6}x-2\left(-3x^2-1\right)^{\frac{3}{4}}+2\left(-3x^2-1\right)^{\frac{1}{4}}}{3\left(3x^2+2\right)}\right)+\frac{1}{24}i\sqrt{6}\log\left(\frac{i\sqrt{6}\sqrt{-3x^2-1}x+i\sqrt{6}x+2\left(-3x^2-1\right)^{\frac{3}{4}}+2\left(-3x^2-1\right)^{\frac{1}{4}}}{3\left(3x^2+2\right)}\right)-\frac{1}{24}i\sqrt{6}\log\left(\frac{-i\sqrt{6}\sqrt{-3x^2-1}x-i\sqrt{6}x+2\left(-3x^2-1\right)^{\frac{3}{4}}+2\left(-3x^2-1\right)^{\frac{1}{4}}}{3\left(3x^2+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*log(1/3*(sqrt(6)*sqrt(-3*x^2 - 1)*x - sqrt(6)*x + 2*(-3*x^2 -
 1)^(3/4) - 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2)) + 1/24*sqrt(6)*log(-1/3*(sqr
t(6)*sqrt(-3*x^2 - 1)*x - sqrt(6)*x - 2*(-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)
^(1/4))/(3*x^2 + 2)) + 1/24*I*sqrt(6)*log(1/3*(I*sqrt(6)*sqrt(-3*x^2 - 1)*x
 + I*sqrt(6)*x + 2*(-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2))
- 1/24*I*sqrt(6)*log(1/3*(-I*sqrt(6)*sqrt(-3*x^2 - 1)*x - I*sqrt(6)*x + 2*(
-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(3x^2+2\right)\left(-3x^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

maple [C] time = 1.11, size = 138, normalized size = 2.26

$$-\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\ln\left(\frac{3\sqrt{-3x^2-1}\,x-3x+\left(-3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2-6\right)-\left(-3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2-6\right)}{3x^2+2}\right)}{12}+\frac{\mathrm{RootOf}\left(\_Z^2+6\right)\ln\left(-\frac{-3\sqrt{-3x^2-1}\,x-3x+\left(-3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+6\right)+\left(-3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+6\right)}{3x^2+2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x)
```

```
[Out] -1/12*RootOf(_Z^2-6)*ln((RootOf(_Z^2-6)*(-3*x^2-1)^(3/4)+3*(-3*x^2-1)^(1/2)
*x-RootOf(_Z^2-6)*(-3*x^2-1)^(1/4)-3*x)/(3*x^2+2))+1/12*RootOf(_Z^2+6)*ln(-
(RootOf(_Z^2+6)*(-3*x^2-1)^(3/4)-3*(-3*x^2-1)^(1/2)*x+RootOf(_Z^2+6)*(-3*x^
2-1)^(1/4)-3*x)/(3*x^2+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(3x^2+2\right)\left(-3x^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\left(-3x^2-1\right)}^{1/4}\,\left(3x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((- 3*x^2 - 1)^(1/4)*(3*x^2 + 2)),x)
```

```
[Out] -int(1/((- 3*x^2 - 1)^(1/4)*(3*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2\sqrt[4]{-3x^2-1}+2\sqrt[4]{-3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)/(-3*x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(3*x**2*(-3*x**2 - 1)**(1/4) + 2*(-3*x**2 - 1)**(1/4)), x)
```

3.151 $\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b]) - Arc
Tanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

Mathematica [C] time = 0.17, size = 132, normalized size = 1.71

$$\frac{6xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};bx^2,\frac{bx^2}{2}\right)}{(bx^2-2)\sqrt[4]{bx^2-1}\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};bx^2,\frac{bx^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};bx^2,\frac{bx^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};bx^2,\frac{bx^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

```
[Out] (6*x*AppellF1[1/2, 1/4, 1, 3/2, b*x^2, (b*x^2)/2])/((-2 + b*x^2)*(-1 + b*x^
2)^(1/4)*(6*AppellF1[1/2, 1/4, 1, 3/2, b*x^2, (b*x^2)/2] + b*x^2*(2*AppellF
1[3/2, 1/4, 2, 5/2, b*x^2, (b*x^2)/2] + AppellF1[3/2, 5/4, 1, 5/2, b*x^2, (
b*x^2)/2])))
```

IntegrateAlgebraic [A] time = 0.13, size = 79, normalized size = 1.03

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx^2-1}}{\sqrt{b}\,x}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2]*(-1 + b*x^2)^(1/4))/(Sqrt[b]*x)]/(2*Sqrt[2]*Sqrt[b]) - ArcT
anh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

fricas [B] time = 29.66, size = 274, normalized size = 3.56

$$\left[\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(bx^2-1\right)^{\frac{1}{4}}}{\sqrt{b}x}\right)+\sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4-2\sqrt{2}\left(bx^2-1\right)^{\frac{1}{4}}b^{\frac{3}{2}}x^3+4\sqrt{bx^2-1}bx^2+4bx^2-4\sqrt{2}\left(bx^2-1\right)^{\frac{3}{4}}\sqrt{b}x-4}{b^2x^4-4bx^2+4}\right)}{8b}, \frac{2\sqrt{2}\sqrt{-b}\arctan\left(\frac{\sqrt{2}\left(bx^2-1\right)^{\frac{1}{4}}\sqrt{-b}}{bx}\right)-\sqrt{2}\sqrt{-b}\log\left(-\frac{b^2x^4+2\sqrt{2}\left(bx^2-1\right)^{\frac{1}{4}}\sqrt{-b}bx^3-4\sqrt{bx^2-1}bx^2+4bx^2-4\sqrt{2}\left(bx^2-1\right)^{\frac{3}{4}}\sqrt{-b}x-4}{b^2x^4-4bx^2+4}\right)}{8b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)/(sqrt(b)*x)) + sqr
t(2)*sqrt(b)*log(-(b^2*x^4 - 2*sqrt(2)*(b*x^2 - 1)^(1/4)*b^(3/2)*x^3 + 4*sq
rt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(2)*(b*x^2 - 1)^(3/4)*sqrt(b)*x - 4)/
(b^2*x^4 - 4*b*x^2 + 4)))/b, 1/8*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(b*x^2
- 1)^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 + 2*sqrt(2)*(b*
x^2 - 1)^(1/4)*sqrt(-b)*b*x^3 - 4*sqrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(
2)*(b*x^2 - 1)^(3/4)*sqrt(-b)*x - 4)/(b^2*x^4 - 4*b*x^2 + 4)))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2-1\right)^{\frac{1}{4}}\left(bx^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2-2\right)\left(b\,x^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
```

```
[Out] int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2-1\right)^{\frac{1}{4}}\left(bx^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2-1\right)}^{1/4}\,\left(b\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)),x)
```

```
[Out] int(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2-2\right)\sqrt[4]{bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-2)/(b*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((b*x**2 - 2)*(b*x**2 - 1)**(1/4)), x)
```

3.152 $\int \frac{1}{\left(-2-bx^2\right)\sqrt[4]{-1-bx^2}}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b]) - Arc
Tanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2-bx^2\right)\sqrt[4]{-1-bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

Mathematica [C] time = 0.15, size = 137, normalized size = 1.73

$$\frac{6xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-bx^2,-\frac{bx^2}{2}\right)}{\sqrt[4]{-bx^2-1}\left(bx^2+2\right)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-bx^2,-\frac{bx^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-bx^2,-\frac{bx^2}{2}\right)\right)-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-bx^2,-\frac{bx^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

```
[Out] (6*x*AppellF1[1/2, 1/4, 1, 3/2, -(b*x^2), -1/2*(b*x^2)])/((-1 - b*x^2)^(1/4
)*(2 + b*x^2)*(-6*AppellF1[1/2, 1/4, 1, 3/2, -(b*x^2), -1/2*(b*x^2)] + b*x^
2*(2*AppellF1[3/2, 1/4, 2, 5/2, -(b*x^2), -1/2*(b*x^2)] + AppellF1[3/2, 5/4
, 1, 5/2, -(b*x^2), -1/2*(b*x^2)])))
```

IntegrateAlgebraic [A] time = 0.14, size = 90, normalized size = 1.14

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-bx^2-1}}{\sqrt{b}\,x}\right)}{2\sqrt{2}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x\left(-bx^2-1\right)^{3/4}}{\sqrt{2}\left(bx^2+1\right)}\right)}{2\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2]*(-1 - b*x^2)^(1/4))/(Sqrt[b]*x)]/(2*Sqrt[2]*Sqrt[b]) + ArcT
anh[(Sqrt[b]*x*(-1 - b*x^2)^(3/4))/(Sqrt[2]*(1 + b*x^2))]/(2*Sqrt[2]*Sqrt[b
])
```

fricas [B] time = 29.24, size = 273, normalized size = 3.46

$$\left[\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(-bx^2-1\right)^{\frac{1}{4}}}{\sqrt{b}x}\right)+\sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4+4\sqrt{-bx^2-1}bx^2-4bx^2-2\sqrt{2}\left(\left(-bx^2-1\right)^{\frac{1}{4}}bx^3+2\left(-bx^2-1\right)^{\frac{3}{4}}x\right)\sqrt{b}-4}{b^2x^4+4bx^2+4}\right)}{8b}, \frac{2\sqrt{2}\sqrt{-b}\arctan\left(\frac{\sqrt{2}\left(-bx^2-1\right)^{\frac{1}{4}}\sqrt{-b}}{bx}\right)-\sqrt{2}\sqrt{-b}\log\left(-\frac{b^2x^4-4\sqrt{-bx^2-1}bx^2-4bx^2+2\sqrt{2}\left(\left(-bx^2-1\right)^{\frac{1}{4}}bx^3-2\left(-bx^2-1\right)^{\frac{3}{4}}x\right)\sqrt{-b}-4}{b^2x^4+4bx^2+4}\right)}{8b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)/(sqrt(b)*x)) + sq
rt(2)*sqrt(b)*log(-(b^2*x^4 + 4*sqrt(-b*x^2 - 1)*b*x^2 - 4*b*x^2 - 2*sqrt(2
)*((-b*x^2 - 1)^(1/4)*b*x^3 + 2*(-b*x^2 - 1)^(3/4)*x)*sqrt(b) - 4)/(b^2*x^4
 + 4*b*x^2 + 4)))/b, 1/8*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1
/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 4*sqrt(-b*x^2 - 1)*b
*x^2 - 4*b*x^2 + 2*sqrt(2)*((-b*x^2 - 1)^(1/4)*b*x^3 - 2*(-b*x^2 - 1)^(3/4)
*x)*sqrt(-b) - 4)/(b^2*x^4 + 4*b*x^2 + 4)))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(bx^2+2\right)\left(-bx^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2-2\right)\left(-b\,x^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
```

```
[Out] int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(bx^2+2\right)\left(-bx^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(-b\,x^2-1\right)^{1/4}\,\left(b\,x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((- b*x^2 - 1)^(1/4)*(b*x^2 + 2)),x)
```

```
[Out] -int(1/((- b*x^2 - 1)^(1/4)*(b*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{bx^2\sqrt[4]{-bx^2-1}+2\sqrt[4]{-bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-2)/(-b*x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(b*x**2*(-b*x**2 - 1)**(1/4) + 2*(-b*x**2 - 1)**(1/4)), x)
```

3.153 $\int \frac{1}{\left(-2a+3x^2\right)\sqrt[4]{-a+3x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}$$

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4)) - A
rcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2a+3x^2\right)\sqrt[4]{-a+3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

Mathematica [C] time = 0.17, size = 157, normalized size = 1.85

$$\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{\left(3x^2-2a\right)\sqrt[4]{3x^2-a}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)+2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

```
[Out] (2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/a, (3*x^2)/(2*a)])/((-2*a + 3*x^2
)*(-a + 3*x^2)^(1/4)*(2*a*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/a, (3*x^2)/(2*
a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (3*x^2)/a, (3*x^2)/(2*a)] + AppellF
1[3/2, 5/4, 1, 5/2, (3*x^2)/a, (3*x^2)/(2*a)])))
```

IntegrateAlgebraic [A] time = 0.16, size = 89, normalized size = 1.05

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{3x^2-a}}{x}\right)}{2\sqrt{6}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{3x^2-a}}{x}\right)}{2\sqrt{6}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2/3]*a^(1/4)*(-a + 3*x^2)^(1/4))/x]/(2*Sqrt[6]*a^(3/4)) - ArcT
anh[(Sqrt[2/3]*a^(1/4)*(-a + 3*x^2)^(1/4))/x]/(2*Sqrt[6]*a^(3/4))
```

fricas [B] time = 20.45, size = 276, normalized size = 3.25

$$-\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}}a^3\frac{1}{a^3}^{\frac{3}{4}}+\left(\frac{1}{36}\right)^{\frac{1}{4}}\sqrt{3x^2-a}\,a\frac{1}{a^3}^{\frac{1}{4}}\right)\sqrt{a\sqrt{\frac{1}{a^3}}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(3x^2-a\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}\right)}{x}\right)-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}\,a^2\frac{1}{a^3}^{\frac{3}{4}}x+\left(3x^2-a\right)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+\left(3x^2-a\right)^{\frac{3}{4}}}{3x^2-2a}\right)+\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}\,a^2\frac{1}{a^3}^{\frac{3}{4}}x-\left(3x^2-a\right)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x-\left(3x^2-a\right)^{\frac{3}{4}}}{3x^2-2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/36)^(1/4)*(a^(-3))^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(a^(-3
))^(3/4) + (1/36)^(1/4)*sqrt(3*x^2 - a)*a*(a^(-3))^(1/4))*sqrt(a*sqrt(a^(-3
))) - (1/36)^(1/4)*(3*x^2 - a)^(1/4)*a*(a^(-3))^(1/4))/x) - 1/4*(1/36)^(1/4
)*(a^(-3))^(1/4)*log((18*(1/36)^(3/4)*sqrt(3*x^2 - a)*a^2*(a^(-3))^(3/4)*x
+ (3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) + 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x +
(3*x^2 - a)^(3/4))/(3*x^2 - 2*a)) + 1/4*(1/36)^(1/4)*(a^(-3))^(1/4)*log(-(1
8*(1/36)^(3/4)*sqrt(3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (3*x^2 - a)^(1/4)*a^2
*sqrt(a^(-3)) + 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x - (3*x^2 - a)^(3/4))/(3*x
^2 - 2*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2-a\right)^{\frac{1}{4}}\left(3x^2-2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2-2a\right)\left(3x^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
```

```
[Out] int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2-a\right)^{\frac{1}{4}}\left(3x^2-2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(2\,a-3\,x^2\right)\,{\left(3\,x^2-a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2*a - 3*x^2)*(3*x^2 - a)^(1/4)),x)
[Out] -int(1/((2*a - 3*x^2)*(3*x^2 - a)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-2a+3x^2\right)\sqrt[4]{-a+3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2*a)/(3*x**2-a)**(1/4),x)
[Out] Integral(1/((-2*a + 3*x**2)*(-a + 3*x**2)**(1/4)), x)
```

3.154 $\int \frac{1}{\left(-2a-3x^2\right)\sqrt[4]{-a-3x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}\,a^{3/4}}$$

Rubi [A] time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4)) - A
rcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2a-3x^2\right)\sqrt[4]{-a-3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}\,a^{3/4}}$$

Mathematica [C] time = 0.15, size = 157, normalized size = 1.85

$$\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{\sqrt[4]{-a-3x^2}\left(2a+3x^2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)-2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

```
[Out] (2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/a, (-3*x^2)/(2*a)])/((-a - 3*x^2
)^(1/4)*(2*a + 3*x^2)*(-2*a*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/a, (-3*x^2)
/(2*a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)] + A
ppellF1[3/2, 5/4, 1, 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)])))
```

IntegrateAlgebraic [A] time = 0.16, size = 89, normalized size = 1.05

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}{x}\right)}{2\sqrt{6}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}{x}\right)}{2\sqrt{6}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2/3]*a^(1/4)*(-a - 3*x^2)^(1/4))/x]/(2*Sqrt[6]*a^(3/4)) - ArcT
anh[(Sqrt[2/3]*a^(1/4)*(-a - 3*x^2)^(1/4))/x]/(2*Sqrt[6]*a^(3/4))
```

fricas [B] time = 20.76, size = 278, normalized size = 3.27

$$-\left(\frac{1}{36}\right)^{\frac{1}{4}} \frac{1}{a^3}^{\frac{1}{4}} \arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}} a^3 \frac{1}{a^3}^{\frac{3}{4}} - \left(\frac{1}{36}\right)^{\frac{1}{4}} \sqrt{-3x^2-a}\, a \frac{1}{a^3}^{\frac{1}{4}}\right)\sqrt{-a\sqrt{\frac{1}{a^3}}} - \left(\frac{1}{36}\right)^{\frac{1}{4}} \left(-3x^2-a\right)^{\frac{1}{4}} a \frac{1}{a^3}^{\frac{1}{4}}\right)}{x}\right) + \frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}} \frac{1}{a^3}^{\frac{1}{4}} \log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}} \sqrt{-3x^2-a}\, a^2 \frac{1}{a^3}^{\frac{3}{4}} x + \left(-3x^2-a\right)^{\frac{1}{4}} a^2 \sqrt{\frac{1}{a^3}} - 3\left(\frac{1}{36}\right)^{\frac{1}{4}} a \frac{1}{a^3}^{\frac{1}{4}} x - \left(-3x^2-a\right)^{\frac{3}{4}}}{3x^2+2a}\right) - \frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}} \frac{1}{a^3}^{\frac{1}{4}} \log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}} \sqrt{-3x^2-a}\, a^2 \frac{1}{a^3}^{\frac{3}{4}} x - \left(-3x^2-a\right)^{\frac{1}{4}} a^2 \sqrt{\frac{1}{a^3}} - 3\left(\frac{1}{36}\right)^{\frac{1}{4}} a \frac{1}{a^3}^{\frac{1}{4}} x + \left(-3x^2-a\right)^{\frac{3}{4}}}{3x^2+2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/36)^(1/4)*(a^(-3))^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(a^(-3
))^(3/4) - (1/36)^(1/4)*sqrt(-3*x^2 - a)*a*(a^(-3))^(1/4))*sqrt(-a*sqrt(a^(
-3))) - (1/36)^(1/4)*(-3*x^2 - a)^(1/4)*a*(a^(-3))^(1/4))/x) + 1/4*(1/36)^(
1/4)*(a^(-3))^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/
4)*x + (-3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) - 3*(1/36)^(1/4)*a*(a^(-3))^(1/4
)*x - (-3*x^2 - a)^(3/4))/(3*x^2 + 2*a)) - 1/4*(1/36)^(1/4)*(a^(-3))^(1/4)*
log((18*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (-3*x^2 - a)^(
1/4)*a^2*sqrt(a^(-3)) - 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (-3*x^2 - a)^(3
/4))/(3*x^2 + 2*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(3x^2+2a\right)\left(-3x^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2-2a\right)\left(-3x^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
```

```
[Out] int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(3x^2+2a\right)\left(-3x^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(3x^2+2a\right)\left(-3x^2-a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2*a + 3*x^2)*(- a - 3*x^2)^(1/4)),x)
```

```
[Out] -int(1/((2*a + 3*x^2)*(- a - 3*x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{2a\sqrt[4]{-a-3x^2}+3x^2\sqrt[4]{-a-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2*a)/(-3*x**2-a)**(1/4),x)
```

```
[Out] -Integral(1/(2*a*(-a - 3*x**2)**(1/4) + 3*x**2*(-a - 3*x**2)**(1/4)), x)
```

3.155 $\int \frac{1}{\left(-2a+bx^2\right)\sqrt[4]{-a+bx^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(2*Sqrt[2]*a^(3/4
)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(2*S
qrt[2]*a^(3/4)*Sqrt[b])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2a+bx^2\right)\sqrt[4]{-a+bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.16, size = 163, normalized size = 1.61

$$-\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{\left(2a-bx^2\right)\sqrt[4]{bx^2-a}\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)+6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

```
[Out] (-6*a*x*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/a, (b*x^2)/(2*a)])/((2*a - b*x^2
)*(-a + b*x^2)^(1/4)*(6*a*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/a, (b*x^2)/(2*
a)] + b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (b*x^2)/a, (b*x^2)/(2*a)] + Appel
lF1[3/2, 5/4, 1, 5/2, (b*x^2)/a, (b*x^2)/(2*a)])))
```

IntegrateAlgebraic [A] time = 0.17, size = 105, normalized size = 1.04

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}{\sqrt{b}\,x}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}{\sqrt{b}\,x}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))/(Sqrt[b]*x)]/(2*Sqrt[2]*a^(3/4)
*Sqrt[b]) - ArcTanh[(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))/(Sqrt[b]*x)]/(2*Sq
rt[2]*a^(3/4)*Sqrt[b])
```

fricas [B] time = 111.06, size = 338, normalized size = 3.35

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(2\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{bx^2-a}\,a\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)\sqrt{ab\sqrt{\frac{1}{a^3b^2}}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(bx^2-a\right)^{\frac{1}{4}}a\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)}{x}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}\,a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+\left(bx^2-a\right)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+\left(bx^2-a\right)^{\frac{3}{4}}}{bx^2-2\,a}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}\,a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-\left(bx^2-a\right)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-\left(bx^2-a\right)^{\frac{3}{4}}}{bx^2-2\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*arctan(2*(sqrt(1/2)*(2*(1/4)^(3/4)*a^3*b*(
1/(a^3*b^2))^(3/4) + (1/4)^(1/4)*sqrt(b*x^2 - a)*a*(1/(a^3*b^2))^(1/4))*sqr
t(a*b*sqrt(1/(a^3*b^2))) - (1/4)^(1/4)*(b*x^2 - a)^(1/4)*a*(1/(a^3*b^2))^(1
/4))/x) - 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(b*x^2
 - a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) + (b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b
^2)) + (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (b*x^2 - a)^(3/4))/(b*x^2 -
2*a)) + 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(b*x^2
- a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) - (b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^
2)) + (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) - (b*x^2 - a)^(3/4))/(b*x^2 - 2
*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2-a\right)^{\frac{1}{4}}\left(bx^2-2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2-2a\right)\left(b\,x^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
```

```
[Out] int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2-a\right)^{\frac{1}{4}}\left(bx^2-2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(b\,x^2-a\right)}^{1/4}\,\left(2\,a-b\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((b*x^2 - a)^(1/4)*(2*a - b*x^2)),x)
[Out] -int(1/((b*x^2 - a)^(1/4)*(2*a - b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-2a+bx^2\right)\sqrt[4]{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-2*a)/(b*x**2-a)**(1/4),x)
[Out] Integral(1/((-2*a + b*x**2)*(-a + b*x**2)**(1/4)), x)
```

3.156 $\int \frac{1}{\left(-2a-bx^2\right)\sqrt[4]{-a-bx^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(2*Sqrt[2]*a^(3/4
)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(2*S
qrt[2]*a^(3/4)*Sqrt[b])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2a-bx^2\right)\sqrt[4]{-a-bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.16, size = 168, normalized size = 1.63

$$-\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{\sqrt[4]{-a-bx^2}\left(2a+bx^2\right)\left(6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)-bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

```
[Out] (-6*a*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -1/2*(b*x^2)/a])/((-a - b*
x^2)^(1/4)*(2*a + b*x^2)*(6*a*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -1/2
*(b*x^2)/a] - b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -1/2*(b*x^2
)/a] + AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -1/2*(b*x^2)/a]))
```

IntegrateAlgebraic [A] time = 0.17, size = 107, normalized size = 1.04

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}{\sqrt{b}\,x}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}{\sqrt{b}\,x}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))/(Sqrt[b]*x)]/(2*Sqrt[2]*a^(3/4)
*Sqrt[b]) - ArcTanh[(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))/(Sqrt[b]*x)]/(2*Sq
rt[2]*a^(3/4)*Sqrt[b])
```

fricas [B] time = 110.16, size = 350, normalized size = 3.40

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(2\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{-bx^2-a}\,a\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)\sqrt{-ab\sqrt{\frac{1}{a^3b^2}}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-bx^2-a\right)^{\frac{1}{4}}a\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)}{x}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}\,a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+\left(-bx^2-a\right)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-\left(-bx^2-a\right)^{\frac{3}{4}}}{bx^2+2\,a}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}\,a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-\left(-bx^2-a\right)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+\left(-bx^2-a\right)^{\frac{3}{4}}}{bx^2+2\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*arctan(2*(sqrt(1/2)*(2*(1/4)^(3/4)*a^3*b*(
1/(a^3*b^2))^(3/4) - (1/4)^(1/4)*sqrt(-b*x^2 - a)*a*(1/(a^3*b^2))^(1/4))*sq
rt(-a*b*sqrt(1/(a^3*b^2))) - (1/4)^(1/4)*(-b*x^2 - a)^(1/4)*a*(1/(a^3*b^2))
^(1/4))/x) + 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(-
b*x^2 - a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) + (-b*x^2 - a)^(1/4)*a^2*b*sqrt(1/
(a^3*b^2)) - (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) - (-b*x^2 - a)^(3/4))/(b
*x^2 + 2*a)) - 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(
-b*x^2 - a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) - (-b*x^2 - a)^(1/4)*a^2*b*sqrt(1
/(a^3*b^2)) - (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (-b*x^2 - a)^(3/4))/(
b*x^2 + 2*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(bx^2+2\,a\right)\left(-bx^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2-2a\right)\left(-b\,x^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
```

```
[Out] int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(bx^2+2\,a\right)\left(-bx^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(-b\,x^2-a\right)^{1/4}\,\left(b\,x^2+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((- a - b*x^2)^(1/4)*(2*a + b*x^2)),x)
[Out] -int(1/((- a - b*x^2)^(1/4)*(2*a + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{2a\sqrt[4]{-a-bx^2}+bx^2\sqrt[4]{-a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-2*a)/(-b*x**2-a)**(1/4),x)
[Out] -Integral(1/(2*a*(-a - b*x**2)**(1/4) + b*x**2*(-a - b*x**2)**(1/4)), x)
```

3.157 $\int \frac{1}{\left(2-x^2\right)\sqrt[4]{-1+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {398}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2]) + ArcTanh[x/(Sqrt[2]*(-1 +
 x^2)^(1/4))]/(2*Sqrt[2])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(2-x^2\right)\sqrt[4]{-1+x^2}}\,dx=\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.14, size = 115, normalized size = 2.17

$$-\frac{6xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{\left(x^2-2\right)\sqrt[4]{x^2-1}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (-6*x*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/((-2 + x^2)*(-1 + x^2)^(1/4)*
(6*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/
2, x^2, x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, x^2, x^2/2])))
```

IntegrateAlgebraic [A] time = 0.10, size = 55, normalized size = 1.04

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{x}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]

[Out] -1/2*ArcTan[(Sqrt[2]*(-1 + x^2)^(1/4))/x]/Sqrt[2] + ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2])

fricas [B] time = 8.37, size = 91, normalized size = 1.72

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{8}\sqrt{2}\log\left(-\frac{x^4+2\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}x^3+4\sqrt{x^2-1}\,x^2+4\sqrt{2}\left(x^2-1\right)^{\frac{3}{4}}x+4x^2-4}{x^4-4x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")

[Out] -1/4*sqrt(2)*arctan(sqrt(2)*(x^2 - 1)^(1/4)/x) + 1/8*sqrt(2)*log(-(x^4 + 2*sqrt(2)*(x^2 - 1)^(1/4)*x^3 + 4*sqrt(x^2 - 1)*x^2 + 4*sqrt(2)*(x^2 - 1)^(3/4)*x + 4*x^2 - 4)/(x^4 - 4*x^2 + 4))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(x^2-1\right)^{\frac{1}{4}}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")

[Out] integrate(-1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)

maple [C] time = 1.22, size = 121, normalized size = 2.28

$$-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{-\sqrt{x^2-1}\,x-x+\left(x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2-2\right)+\left(x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2-2\right)}{x^2-2}\right)}{4}-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{-\sqrt{x^2-1}\,x+x+\left(x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+2\right)-\left(x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+2\right)}{x^2-2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+2)/(x^2-1)^(1/4),x)

[Out] -1/4*RootOf(_Z^2-2)*ln(-(RootOf(_Z^2-2)*(x^2-1)^(3/4)-(x^2-1)^(1/2)*x+RootOf(_Z^2-2)*(x^2-1)^(1/4)-x)/(x^2-2))-1/4*RootOf(_Z^2+2)*ln(-(RootOf(_Z^2+2)*(x^2-1)^(3/4)-(x^2-1)^(1/2)*x-RootOf(_Z^2+2)*(x^2-1)^(1/4)+x)/(x^2-2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(x^2-1\right)^{\frac{1}{4}}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")

[Out] -integrate(1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\left(x^2-1\right)}^{1/4}\,\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 - 1)^(1/4)*(x^2 - 2)),x)
```

```
[Out] -int(1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+2)/(x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

3.158 $\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}} dx$

Optimal. Leaf size=53

$$\frac{x\left(a+bx^2\right)^{-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {381}

$$\frac{x\left(a+bx^2\right)^{-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-1 + (a*d)/(2*b*c
 - 2*a*d)),x]
```

```
[Out] (x*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d)))/(a*c*(a + b*x^2)^((b*c)/(2*b*c - 2*
a*d)))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}} dx = \frac{x\left(a+bx^2\right)^{-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.98

$$\frac{x\left(a+bx^2\right)^{\frac{bc}{2ad-2bc}}\left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-1 + (a*d)/
(2*b*c - 2*a*d)),x]
```

```
[Out] (x*(a + b*x^2)^((b*c)/(-2*b*c + 2*a*d))*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d))
)/(a*c)
```

IntegrateAlgebraic [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-1
 + (a*d)/(2*b*c - 2*a*d)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x
^2)^(-1 + (a*d)/(2*b*c - 2*a*d)), x]
```

fricas [A] time = 1.00, size = 91, normalized size = 1.72

$$\frac{bdx^5 + (bc + ad)x^3 + acx}{\left(bx^2 + a\right)^{\frac{3\,bc - 2\,ad}{2\,(bc - ad)}}\left(dx^2 + c\right)^{\frac{2\,bc - 3\,ad}{2\,(bc - ad)}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="fricas")
```

```
[Out] (b*d*x^5 + (b*c + a*d)*x^3 + a*c*x)/((b*x^2 + a)^(1/2*(3*b*c - 2*a*d)/(b*c
- a*d))*(d*x^2 + c)^(1/2*(2*b*c - 3*a*d)/(b*c - a*d))*a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{-\frac{bc}{2\,(bc - ad)} - 1}\left(dx^2 + c\right)^{\frac{ad}{2\,(bc - ad)} - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c
- a*d) - 1), x)
```

maple [A] time = 0.00, size = 71, normalized size = 1.34

$$\frac{x\left(b\,x^2 + a\right)^{1 - \frac{2ad - 3bc}{2(ad - bc)}}\left(d\,x^2 + c\right)^{1 - \frac{3ad - 2bc}{2(ad - bc)}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c)),x)
```

```
[Out] (b*x^2+a)^(1-1/2*(2*a*d-3*b*c)/(a*d-b*c))*(d*x^2+c)^(1-1/2*(3*a*d-2*b*c)/(a
*d-b*c))/a/c*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{-\frac{bc}{2\,(bc - ad)} - 1}\left(dx^2 + c\right)^{\frac{ad}{2\,(bc - ad)} - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c
- a*d) - 1), x)
```

mupad [B] time = 5.74, size = 131, normalized size = 2.47

$$\frac{x\left(b\,x^2 + a\right)^{\frac{b\,c}{2\,a\,d - 2\,b\,c} - 1} + \frac{x^3\left(b\,x^2 + a\right)^{\frac{b\,c}{2\,a\,d - 2\,b\,c} - 1}\,(a\,d + b\,c)}{a\,c} + \frac{b\,d\,x^5\left(b\,x^2 + a\right)^{\frac{b\,c}{2\,a\,d - 2\,b\,c} - 1}}{a\,c}}{\left(d\,x^2 + c\right)^{\frac{a\,d}{2\,a\,d - 2\,b\,c} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^((b*c)/(2*a*d - 2*b*c) - 1)/(c + d*x^2)^((a*d)/(2*a*d - 2*b
*c) + 1),x)
```

```
[Out] (x*(a + b*x^2)^((b*c)/(2*a*d - 2*b*c) - 1) + (x^3*(a + b*x^2)^((b*c)/(2*a*d
 - 2*b*c) - 1)*(a*d + b*c))/(a*c) + (b*d*x^5*(a + b*x^2)^((b*c)/(2*a*d - 2*
b*c) - 1))/(a*c))/(c + d*x^2)^((a*d)/(2*a*d - 2*b*c) + 1)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(-1-b*c/(-2*a*d+2*b*c))*(d*x**2+c)**(-1+a*d/(-2*a*d+2
*b*c)),x)
```

```
[Out] Timed out
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
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    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
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            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
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        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
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